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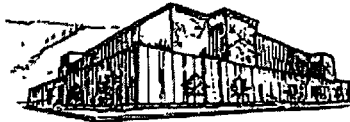
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**A HEIGHT GROWTH MODEL FOR YOUNG LODGEPOLE PINE
IN WESTERN MONTANA: QUANTIFYING SMALL TREE AND
COMPETING VEGETATION RELATIONSHIPS**

by

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B.A. in Political Science, Carroll College, Helena, Montana

Presented in partial fulfillment of the requirements for the

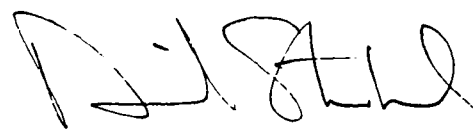
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Committee Chair


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Forestry

**A Height Growth Model for Young Lodgepole Pine in Western Montana:
Quantifying Small Tree and Competing Vegetation Relationships (49 pp.)**

Committee Chair: Kelsey S. Milner, Ph. D.



Abstract

Understory vegetation competes with conifer seedlings for resources, yet current growth and yield simulators do not adequately portray the effect competing vegetation has on small tree growth. Many data sets are not appropriate for creating growth models using competing vegetation as an independent variable, because site and vegetation effects upon tree growth cannot be separated. In response to these difficulties, a data set created from two Champion International research projects was used to build a height growth model for young lodgepole pine. The effects of competing vegetation are included in the model, because vegetation competition had been varied within each study site through various treatments. Data sets from six sites were analyzed individually to determine vegetation dynamics on each site. The effects of the independent variables initial tree height and percent vegetation cover on tree height growth were not found to be sufficiently similar so as to combine all data for analysis. Because a single model was desired, the data were combined and a single growth model using site index to describe site effects was then created and found to have realistic performance.

Keywords: growth and yield models, nonlinear regression, competing vegetation

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I. Introduction

Current Situation

The period of regeneration is the most crucial determinant of stand development (Smith 1986; Stewart 1987). During this critical period, silvicultural practices may have substantial impacts, either positive or negative. One important factor that influences seedling growth is competing vegetation. Under some circumstances, understory vegetation may be of benefit to young trees. On hot and dry sites for example, vegetation may increase seedling survival by providing shade. On other sites, certain types of vegetation such as red alder (*Alnus rubra*) or ceanothus (*Ceanothus* spp.) may serve to increase nitrogen available to the seedlings (Walstad and Kuch 1987). On many sites in the Inland Northwest, undergrowth vegetation competes with small trees for available resources and may indeed outcompete the trees causing reduced growth, damage or death. Vegetation growing in close proximity to small tree has the potential to compete with these trees, for even though the trees and their surrounding vegetation may have different nutritional needs, all plants need water and light to survive.

A number of studies have been conducted to determine whether vegetative competition has a deleterious effect upon small trees, or more precisely, whether control of understory vegetation enhances survival and growth. Many of these studies have focused upon woody shrub or hardwood competition, a major problem along the Pacific Coast (Oliver 1984) and in the Southeastern U. S. (Haywood 1993). Certainly shrub competition plays a role in the development of forests in western Montana, but it is often

herbaceous vegetation and grass which represents significant competition to small trees in this region (Milner 1997).

In recent years, researchers have found that the effect of vegetative competition upon trees is highly variable from year to year. One of the greatest factors affecting this variability is climate, including things such as precipitation and temperature. During years when climatic conditions are favorable, the effects of competing vegetation upon trees maybe quite minimal, while in years of drought, competing vegetation may compound the stress upon trees (Boyd 1985). These findings point to the need for long-term studies to clarify vegetation and tree interactions. In one such long-term study conducted in western Montana, it was found that reducing competing vegetation resulted in increased survival and growth of ponderosa and lodgepole pine (Keyser 1998). More importantly, increases in growth were still observed up to 15 years after planting and vegetation control treatment. Thus, the considerable influence competing vegetation exerts upon seedlings may extend many years past seedling establishment.

Even as the dynamic between small trees and competing vegetation has become more well known, most growth and yield simulators in current use do not allow the effects of vegetation to directly influence the growth of trees. One of the most widely used simulators, especially on public land, is the Forest Vegetation Simulator (FVS, *nee* Prognosis, Wykoff *et al* 1984) developed by the U. S. Forest Service. Subsequent extensions to the FVS simulator have allowed it to be utilized for a wider range of applications. The SHRUBS (Moeur 1985) and ESTAB (Ferguson and Crookston 1984) submodels allow understory vegetation information to be entered into the model and

project the understory development over time. There is however, no interaction between the vegetation and trees within the simulator (Milner 1997).

Another shortcoming of many current simulators is that the models contained within were developed from data taken in older stands with larger trees and less vegetation, making projections of young stands dubious (Powers *et al* 1989). To address these shortcomings and increase realism in current growth and yield simulators, models of small-tree growth in the presence of competing vegetation need to be incorporated into the simulators.

Creating such a growth model is not as simple as it would seem. Random sampling across the landscape will not necessarily yield a suitable data set. The problem is that site factors tend to confound the effects of competing vegetation. Sites that receive abundant moisture tend to have greater amounts of vegetation present as compared to relatively drier sites. If the trees on these sites grow faster than those on drier sites, analysis of data obtained from such sites will show increased vegetation cover associated with increased tree growth. To realistically portray small tree and vegetation dynamics, site and competing vegetation effects must be unlinked. That is, the effects of site must be held constant while amounts of vegetation are varied.

Two Champion International Corporation studies in western Montana provided data in which vegetation varied upon a given site. The objective of this study is to use that data to create an individual-tree height growth model that adequately describes the relationship between small trees and the surrounding vegetation.

II. Previous Work

Models

In the Pacific Coast region of Oregon and Northern California, a simulator entitled SYSTUM-1 (Simulating Young Stands Under Management, Phase 1) was created to address the aforementioned shortcomings (Powers *et al* 1989). This simulator predicts the growth of trees from establishment to a point where they may be incorporated into other simulators. Height growth is predicted by calculating potential height growth for an individual tree, then multiplying the potential height growth by modifiers for tree and vegetative competition (Ritchie and Powers 1993). Potential height growth is represented as a function of site (input as site index in feet at an index age of 50 years) and current tree height (Ritchie and Powers 1993). Competing vegetation is included as percent cover and height (Ritchie and Powers 1993). Tests of the simulator against real data showed that with regard to height growth, the simulator's predictions were within two percent of actual measurements, although predictions of diameter growth were somewhat less reliable (Powers *et al* 1989).

Uzoh (1992) developed a height-growth model using a modified Chapman-Richards function (Richards 1959), based on ponderosa pine data gathered in western Montana. Growth of ponderosa pine seedlings was predicted incorporating percent cover of all vegetation as a driving variable. Other variables found to be important predictors of height growth were competition from nearby trees, soil water holding capacity and depth, crown length, and precipitation (Uzoh 1992). Although age and height were

measured in the study, neither variable was incorporated into the final nonlinear model. It seems quite likely that crown length and total height were highly correlated, so one may speculate that crown length was the variable selected because this variable integrates both current height and some measure of tree vigor.

Other models have been created which represent vegetative competition discretely. An example of one such model is PLANT-PC, where total tree height is predicted by a function which includes manual and chemical weed control (Payandeh *et al* 1992). These two controls are included as variables input as either zero (no control) or one (Payandeh *et al* 1992).

Recent modeling efforts provide encouraging results that small tree growth is indeed influenced by vegetative competition, and once unknown relationships are currently being described in precise mathematical terms. The impact of these developments is that forest managers are gaining an important tool that will allow them to better evaluate alternatives in order to make the best management decisions.

Difficulties in Describing Vegetative Competition

Some researchers have pointed out problems quantifying the competition of understory vegetation. One important drawback is that one-time measurements of vegetation cannot fully describe competitive interaction with trees. The composition of the neighboring vegetation community changes throughout the growing season and from season to season, and affects trees differently at various times (Burton 1993). A series of vegetation measurements taken throughout the growing season would provide a better

description of system dynamics. When developing models with an eye toward practical use by professional land managers, one is confronted by the reality that large numbers of repeated measures present a high cost in time and money. It has been recommended that vegetation measurements be conducted over at least several growing seasons for the purpose of comparison (Burton 1993). To ensure consistency, it seems logical that these measurements should occur at the point in the growing season when the plants are at or near their fullest expression. This means that sampling must not occur too early in the season when the plants are growing rapidly, or too late in the season when many plants have died back.

Other problems are associated with the methods by which vegetation is measured. A simple ocular estimate of percent coverage is a fairly subjective technique, especially when all forms of vegetation are considered together. As with any subjective measure, experience and collaboration between data gatherers should reduce measurement error, and large sample sizes allow the detection of differences in cover. All vegetative cover is not equal, so depending on location and time of year, certain species of plants may have greater effects upon tree growth than others for an equivalent percent coverage. The modeler must consider trade-offs between practicality and accuracy. One solution may be to measure cover by classes of vegetation, such as shrubs, herbaceous plants (forbs) and grasses. Another solution is to place groups of species together, as was done in the SYSTUM-1 simulator, which recognizes “species complexes” such as manzanita and ceanothus (Powers *et al* 1989).

Previous Results of Champion International Trials

In 1983 and 1985, the Champion International Corporation initiated two studies to examine tree responses to various vegetation treatments. Two types of lodgepole pine growing stock were planted at various times in the 1983 study, entitled 83-10, and then subjected to several types of vegetation control treatments at three different times (McLeod and Mandzak 1990). The 1985 study, entitled 85 R2, was similar to the 83-10 study except that western larch seedlings were also planted (Thamarus and Milner 1989). The 83-10 study was conducted at two locations in western Montana, while the 85 R2 study was installed at four locations. While not explicitly intended for modeling purposes, these studies utilized a randomized block design so that treatment differences could be more readily detected.

The overall results of these two studies were encouraging. Six and seven year results of the 83-10 study showed that trees in the areas where vegetation was most effectively controlled (treatment with four pounds per acre of Velpar L) appeared to be very vigorous. Total tree heights were 2 to 2.5 times greater than those on either plots treated with lower amounts of Velpar L (two pounds per acre) or untreated control plots (McLeod and Mandzak 1990). Second year results in the 85 R2 study showed large increases in lodgepole pine tree volumes (plot means, ranging between 27% and 1270%) on plots where vegetation was most effectively controlled. The smallest average gains in growth were reported at the Gold Creek Site, where some control plots had mean tree volumes of 0.05 cubic inches, while treated plots with similar planting dates and stock had mean volumes of 0.08 cubic inches. At Smiley Creek, some control plots had mean

tree volumes of 0.07 cubic inches while treated plots showed mean tree volumes of 0.89 cubic inches (Thamarus and Milner 1989). Treatments in the 85 R2 study consisted of either two or four pounds per acre of Pronone[®] 10G, which contains the same active ingredient as Velpar L. In 1996 and 1997, surviving trees in both the 83-10 and 85 R2 studies were re-measured. These new data were analyzed to determine whether the gains in survival and growth previously found were continuing. Overall height growth trends were found to be diverging between plots with effective vegetation control and those with lesser or no control (Keyser 1998). These findings indicate that over longer time spans, trees that enjoy early gains in growth as a result of vegetation control continue to grow at a faster rate than those with uncontrolled vegetation, although the difference in growth rates currently seems to be lessening, but still significant (Keyser 1998). The trend seems to be that all trees on these sites will soon be growing at nearly the same rate. One reason for this trend may be that all trees will eventually grow to a size where they are able to escape the competitive effects of the surrounding vegetation (Burton 1993). Another plausible explanation is that the trees within these studies have grown to a size where the competition among trees is of greater significance than that of the surrounding vegetation. Whichever explanation is true is not as significant as the fact that trees given an early advantage continue to hold that advantage as the stand matures.

Given the positive results of the 83-10 and 85 R2 studies to date, the data seem to be appropriate for use in the construction of a model which will predict height growth in lodgepole pine using competing vegetation measures as independent variables. While the data do not represent a large geographic area, they include a wide variety of growth rates and levels of surrounding vegetation.

III. Methods

Data

Trees in the 85 R2 study were planted in the Spring and Fall of 1985 and the Spring of 1986. Measurements on the sites within the study were recorded in 1986, 1987 and 1988. Each tree was measured for height, basal diameter, and vigor. Vegetative cover was ocularly estimated around each tree. Individual trees were not tagged during this time, but the plots and treatment methods were marked. Early vegetation measurements were also taken on the two 83-10 study sites, but these data were lost.

Throughout the summer of 1997, all sites within the 85R2 and 83-10 studies were re-measured. Total height, basal and breast-height diameter, and crown ratio and width were recorded for all trees. Additionally, a 20% random subsample of trees was selected within each plot and measured for height increment and vegetative cover. Height increment was measured by analyzing the stem and branch whorls to determine the height at the end of each yearly growth period. Secondary or false whorls of limbs were ruled out as growth ending points by counting bud scars on limbs and matching their number with the corresponding year of growth on the stem. Vegetative cover was estimated using a 3.28 by 3.28-foot (one meter squared) plot centered at the base of the corresponding tree. Woody vegetation greater than three feet in height was classified as high shrub, while that less than three feet was deemed low shrub. Grass and forb cover were estimated separately. All live trees on each plot were tagged with unique numbers.

Construction of the Modeling Data Set

Lodgepole pine comprises the majority of observations when data sets for the two studies are combined. Lodgepole pine was the only species in the 83-10 study. In the 85R2 study, lodgepole pine seedlings were planted in greater numbers than western larch, which do not survive treatment with Pronone[®], the herbicide of main focus within the study. Numbers of surviving larch were even less because at the time the studies were initiated, Champion was having problems with its containerized larch seedlings and mortality was quite high (Milner 1997). Thus it was decided that the modeling effort would concentrate on the lodgepole pine data. The 85R2 data contain the smallest trees within the set. In an attempt to create a data set containing a wide range of tree sizes, larger trees from the 83-10 study were added. Early measurements in the 83-10 study were unavailable but all measurements recorded in 1997 were included here.

One of the greatest difficulties in creating the dataset was that the dependent variable height growth needed linkage to a preceding measure of vegetative cover. Trees were not individually numbered at the beginning of the study, so vegetation measured during the 1980's could not be tied to a specific tree. To overcome this limitation, coverage records were averaged for each plot. The results were plot-level measures of vegetation coverage at certain points in time, linked with subsequent measures of five-year height growth. The next problem to overcome was that for height growth during the period of 1992 through 1997, there was no measurement of vegetation in 1992. Therefore, vegetative cover measurements recorded in 1997 were chosen to represent the cover in 1992. Certainly a vegetation measurement period closer to 1992 would have been preferable to the 1997 measurements, but later vegetation measurements should

better represent the 1992-1997 tree growth period. Immediately after a disturbance (which in this case was a timber removal operation), vegetation changes rapidly as individual plants race to occupy available growing space. Once available space becomes occupied, the plant community tends to stabilize somewhat. This is not to say that the composition of the plant community becomes static, but rather that it tends to change less rapidly the further removed in time from the initial disturbance.

The potential for serial correlation problems was great. For each individual tree, there were often two observations of height growth and surrounding cover during the 1980's. Such observations are not independent, since past height growth likely influences subsequent height growth. While it could not be determined which trees measured in the 1980's and 1997 were the same individuals, trees measured in either the 1986, 1987 or 1988 periods could be distinguished. All plots were measured for cover in 1987, with one-third of these also measured in 1986 and another third in 1988. Therefore, trees with multiple measurements were randomly assigned to a single measurement period. The 1987 measurements could have been used exclusively, but the addition of the 1986 and 1988 measurements provided a necessary glimpse of early tree growth during a period when vegetation competition was likely changing rapidly. Since there were an abundant number of observations, the reduction in data was outweighed by reducing the serial correlation problem.

Model Formulation

The data were plotted to show the relationship between initial height and subsequent five-year growth. Figure 1 shows the relationship between initial tree height and subsequent three-year height increment for all observations and it appears that there exists a maximum height increment. The plot of the data also suggests that some form of a Chapman-Richards function may serve well as a model of height growth (Richards 1959). A family of curves generated by such a function would seem to fit the data points well. Another advantage of the Richards function is that it was originally formulated for biological growth applications, and has been utilized for tree growth models in various forms ever since it was first described (Richards 1959). The basic model form is as follows:

$$E(y) = \beta_1 * \left\{ 1 - \exp^{(-\beta_2 * x_1)} \right\}^{(\beta_3)} \quad [1]$$

where $E(y)$ is expected value of the response variable y , which in this case is an individual tree's five-year height growth in feet. In this model form, β_1 represents the maximum value of $E(y)$, while the rest of the expression is a proportion which reduces $E(y)$ until the proportion approaches one. The parameters β_1 through β_3 are unknown and their values are estimated. As the independent variable x_1 increases, the proportion also increases. This general model [Eq. 1] contains only one independent variable, so additions must be made to incorporate the Champion International Corporation data. Site variables should be placed next to the β_1 parameter because these are strong determinants

of maximum tree growth rates. Since initial tree height appears to have a strong influence upon height growth (Figure 1), this variable (x_1) was selected to occupy the central position in the model with the β_2 parameter. Vegetation cover measurements fit well with the β_3 parameter, because as this part of the equation increases, the proportion decreases. In [Eq. 1], exp is the base of natural logarithms ($exp \approx 2.71828\dots$).

Site characteristics, though likely a strong influence upon maximum tree growth rates, are commonly difficult to measure. Furthermore, it is doubtful that the full range of conditions found in this region could be captured on six sites. Data available on site conditions included elevation, slope, and aspect (Table 1). Initial model development using these site variables yielded poor results. The vegetation coverage coefficients estimated by these models were not likely different from zero, meaning there was no way of discerning coverage effects. Additionally, response surfaces did not exhibit a realistic shape. Height growth rates for trees on all sites reached a maximum when trees were still very small. The overall result was a height growth model that predicted less than half the variation found within the data, so these equations were discarded. For these kinds of continuous site variables to be useful, many more sites would be needed within the data set.

The four independent variables examined in this study are initial tree height, and shrub, forb, and grass coverage. The means, and maximum and minimum values of these variables are shown in Table 2.

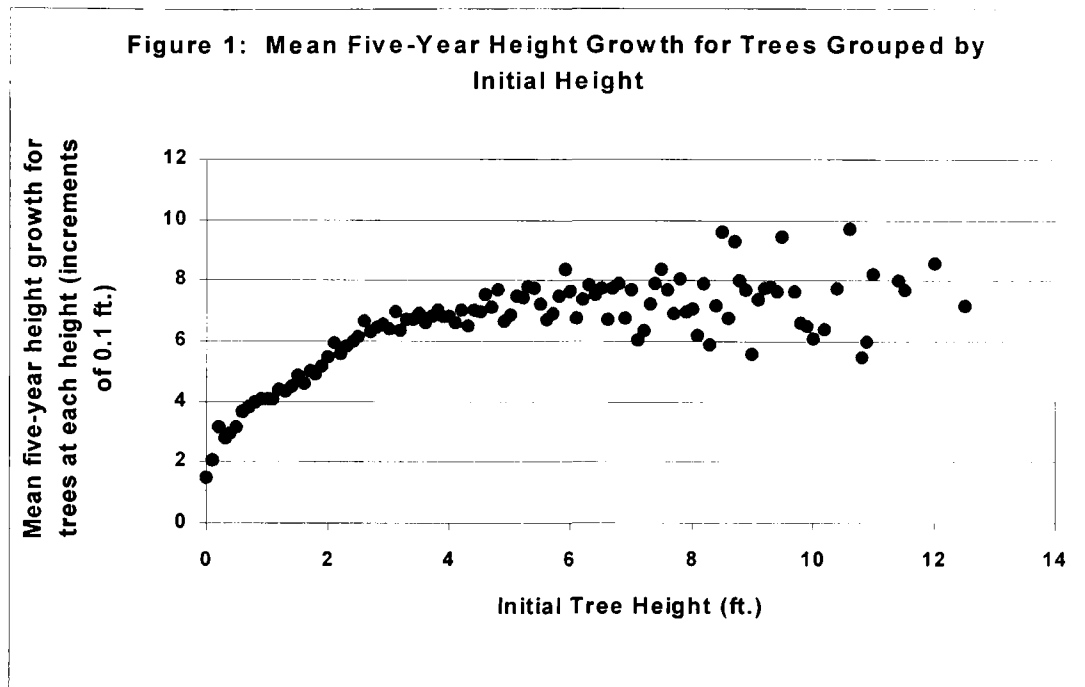


Table 1. Site descriptions for 1983 and 1985 trials.

Trial	Site	Elevation (feet)	Aspect	Slope (%)	Habitat Type ⁺
83-10*	Gold Creek	4100	NNE	2	PSME/VACA
	Boyd Mountain	5300	NW	40	PSME/VAGL
85 R2**	Gold Creek	4100	---	level	PSME/VACA
	Bear Creek	3050	---	level	ABGR/LIBO
	Lost Prairie	3600	SSW	8	PSME/VACA
	Smiley Creek	4300	NW	12	ABGR/LIBO

* - McLeod and Mandzak (1990)

** - Thamarus and Milner (1989)

+ - Pfister *et al* (1977)

Table 2. Means, and Maximum and Minimum Values for Four Independent Variables.

Site		Initial Height (feet)	Plot Mean Shrub Cover (percent)	Plot Mean Forb Cover (percent)	Plot Mean Grass Cover (percent)
<i>(Control Plots Only)</i>					
<i>(1985 Studies)</i>					
Smiley Creek	<i>mean</i>	2.4	31.0	24.7	36.8
	<i>max.</i>	7.8	112.3	42.0	63.5
	<i>min.</i>	0.1	1.0	6.0	18.0
Bear Creek	<i>mean</i>	2.0	12.8	29.5	37.5
	<i>max.</i>	8.8	85.0	50.7	75.0
	<i>min.</i>	0.0	0.0	10.0	4.0
Lost Prairie	<i>mean</i>	2.5	24.8	37.8	47.0
	<i>max.</i>	8.5	57.8	67.0	27.0
	<i>min.</i>	0.1	0.0	15.5	82.0
Gold Cr. (85 R2)	<i>mean</i>	1.9	30.1	23.7	12.0
	<i>max.</i>	7.5	49.3	38.3	26.0
	<i>min.</i>	0.1	8.0	13.3	1.0
<i>(1983 Studies)</i>					
Gold Cr. (83-10)	<i>mean</i>	6.5	35.6	15.7	39.8
	<i>max.</i>	12.5	59.3	26.5	23.3
	<i>min.</i>	1.7	14.5	7.0	52.3
Boyd Mtn.	<i>mean</i>	6.2	40.0	39.2	38.4
	<i>max.</i>	11.5	52.0	53.5	67.8
	<i>min.</i>	2.4	12.3	22.5	20.3

Model Development

Given the limitations of site variables within this study, three different approaches were followed to determine the best equation form. First, data from each site were analyzed individually. This approach is important to examine how the Chapman-Richards function fits each site's data. If the predicted height growth curves and vegetation cover coefficients appear similar, it may be reasonable to assume that in a model of the entire data set, only the first coefficient (β_1) would contain site influences. The entire data set was analyzed first using dummy variables for site quality and then by inserting site index, a quantitative measure of site. The dummy variables are qualitative variables, each of which has an associated unknown parameter. In this instance, the purpose of the dummy variables is to estimate the effect of site upon height growth when quantitative measures are unreliable. Model construction and statistical analysis were accomplished using SPSS® statistics software, and by following the procedures described by Bates and Watts (1988).

While site index provides a single quantitative measure of site quality, its use in this instance might prove problematic. Previous records of site index on the study sites could not be located, so site index was calculated according to equations developed for western Montana lodgepole pine by Milner (1992). These equations were developed from stem analysis of sample trees between 50 and 100 years of age, and may not be reliable for trees less than 20 years of age (Milner 1992). Since site index would likely be higher for trees where undergrowth vegetation had been reduced, only trees on control plots were utilized for site index calculations.

IV. Results

Site-specific Approach

Nonlinear regression analysis techniques were performed independently on each site's observations according to the following equation:

$$E(y) = \beta_1 * \left\{ 1 - \exp^{(-\beta_2 * x_2)} \right\} (\beta_3 * x_3 + \beta_4 * x_4 + \beta_5 * x_5) \quad [2]$$

where:

$E(y)$ = expected five-year height growth

β_1 = site parameter

β_2 = initial tree height parameter

β_3 through β_5 = vegetation cover parameters

x_2 = initial tree height in feet

x_3 = shrub cover in percent

x_4 = forb cover in percent

x_5 = grass cover in percent

All three types of vegetative cover were transformed using an arcsine transformation described by Sokal and Rohlf (1987). Parameter estimates and summary statistics for individual site analysis are shown in Table 3.

Table 3. Coefficient Estimates for Nonlinear Regression of Individual Site Data (*asymptotic standard errors in parentheses*)

SITE	β_1 estimated (site)	β_2 estimated (initial ht.)	β_3 estimated (shrub cover)	β_4 estimated (forb cover)	β_5 estimated (grass cover)	Mean Square Error	R ²
<i>(1985 Studies)</i>							
Smiley Creek	7.2518 (0.1545)	0.4928 (0.0803)	0.1470 (0.0842)	0.7153 (0.0907)	0.1940 (0.0747)	1.4893	0.5586
Bear Creek	8.0428 (0.1607)	0.7815 (0.0848)	0.0914 (0.1357)	1.3840 (0.1969)	0.2127 (0.1160)	2.0712	0.6273
Lost Prairie	8.0948 (0.1489)	0.6931 (0.0675)	0.5836 (0.1899)	1.0531 (0.1331)	0.4142 (0.1354)	1.7092	0.6589
Gold Cr. (85 R2)	6.2031 (0.2229)	0.4935 (0.0647)	0.8597 (0.0867)	0.6213 (0.1282)	0.0352 (0.1600)	0.8934	0.6538
<i>(1983 Studies)</i>							
Gold Cr. (83-10)	7.8093 (0.8356)	0.1173 (0.1162)	0.4908 (0.3838)	-0.1079 (0.3073)	0.1094 (0.1271)	1.8859	0.2423
Boyd Mountain	7.8114 (0.2685)	0.4809 (0.1060)	2.6639 (1.4685)	2.8837 (1.8640)	-0.8792 (1.1991)	1.0299	0.6303

The estimated parameters β_3 through β_5 are coefficients for shrub, forb and grass cover, respectively. Positive values for these parameters result in lower estimates for height growth. Note that β_4 at Gold Creek (83-10) and β_5 at Boyd Mountain show negative values. The large standard errors associated with these estimates mean that it cannot be statistically determined whether they differ from zero and have an effect on height growth. Several other sites show insignificant vegetation cover coefficients. Therefore, it is difficult to judge the significance of vegetation effects across sites. Also, there are

fairly large differences in the magnitudes of the β_2 coefficients across all sites, indicating a difference in the effects of initial heights by site.

It is important to determine whether similarities exist in the effects of the independent variables across all sites, in order to justify combining the data and creating a single height-growth model. Graphical displays of the individual site models placed over actual data points provide some evidence that the effects of the cover and initial height variables are similar across sites. The predicted height growth models created curves of similar shape at the four sites in the 1985 study and are displayed in Figures 2 and 3. Each curve was generated by inserting a range of tree heights into the growth equation while holding vegetation cover levels constant. The three curves shown in each chart show the effects of different vegetation cover levels. Hypothetical high and low vegetation values were selected and placed into each individual model to generate response surfaces. A third response surface was generated by determining the mean cover levels for each site. The 1985 Gold Creek site model generated curves similar in shape to the preceding three sites, except that the height increments are substantially less.

The two sites within the 1983 study (Gold Creek and Boyd Mountain) show markedly different height growth predictions than the four sites within the 1985 study, as seen in Figure 4. There are fewer small trees at the 1983 study sites because, as mentioned previously, these data points were included to increase the number of large trees (six to twelve feet in height) within the data set as a whole. The low number of small trees in the 1983 Gold Creek data set made regression analysis difficult as evidenced by the low R^2 value of 0.23. The data from the 1983 study sites cannot be used to determine whether it is appropriate to combine all sites' data.

Since the parameter estimates for each individual height growth model are questionable, each site's data were analyzed a second time with the three vegetation cover types combined into a single variable of total cover. This variable was the sum of all types of vegetation cover, and was not transformed. Some total vegetation cover values exceeded 100%, and only the 1985 study site data were analyzed in this instance, since the 1983 data sets did not contain a complete range of tree sizes. Table 4 shows that the β_2 coefficients (initial tree height) for the Smiley Creek, Lost Prairie and Gold Creek (85 R2) sites are fairly similar, while that for Bear Creek is significantly greater than the others. Confidence intervals ($\alpha = 0.05$) constructed for β_2 (Figure 5, top) show that the intervals for all sites contain common values. This is evidence that the effect of initial height may be similar on all four sites, but it is likely that the effects of initial height are different at Bear Creek when compared to the other three sites. The β_3 coefficients (vegetation cover) appear very similar for all sites except Smiley Creek. Confidence intervals show that the Smiley Creek site β_3 coefficient differs significantly from that for Gold Creek (Figure 5, bottom). This suggests that the effect of competing vegetation upon tree height growth differs between these sites. Though not statistically justified, practical considerations necessitate analyzing all sites together. The similarly shaped response surfaces in Figures 2 and 3 require a shorter assumptive leap.

Table 4. Coefficient Estimates for Nonlinear Regression of Individual Site Data with a Single Vegetation Cover Variable. (asymptotic standard errors in parentheses)

SITE	β_1 estimated (site)	β_2 estimated (initial height)	β_3 estimated (total veg. cover)	Mean Square Error	R ²
Smiley Creek	6.8208 (0.1025)	0.8705 (0.0977)	0.0114 (0.0011)	1.7117	0.4917
Bear Creek	7.2562 (0.1227)	1.3023 (0.1423)	0.0202 (0.0024)	2.6706	0.5176
Lost Prairie	7.5742 (0.1186)	0.9720 (0.0857)	0.0181 (0.0019)	2.0379	0.5920
Gold Creek (85 R2)	5.4904 (0.1191)	0.8701 (0.0882)	0.0212 (0.0019)	0.9644	0.6244

Figure 2. Actual and Predicted Five-Year Height Growth for 1985 Study Sites.

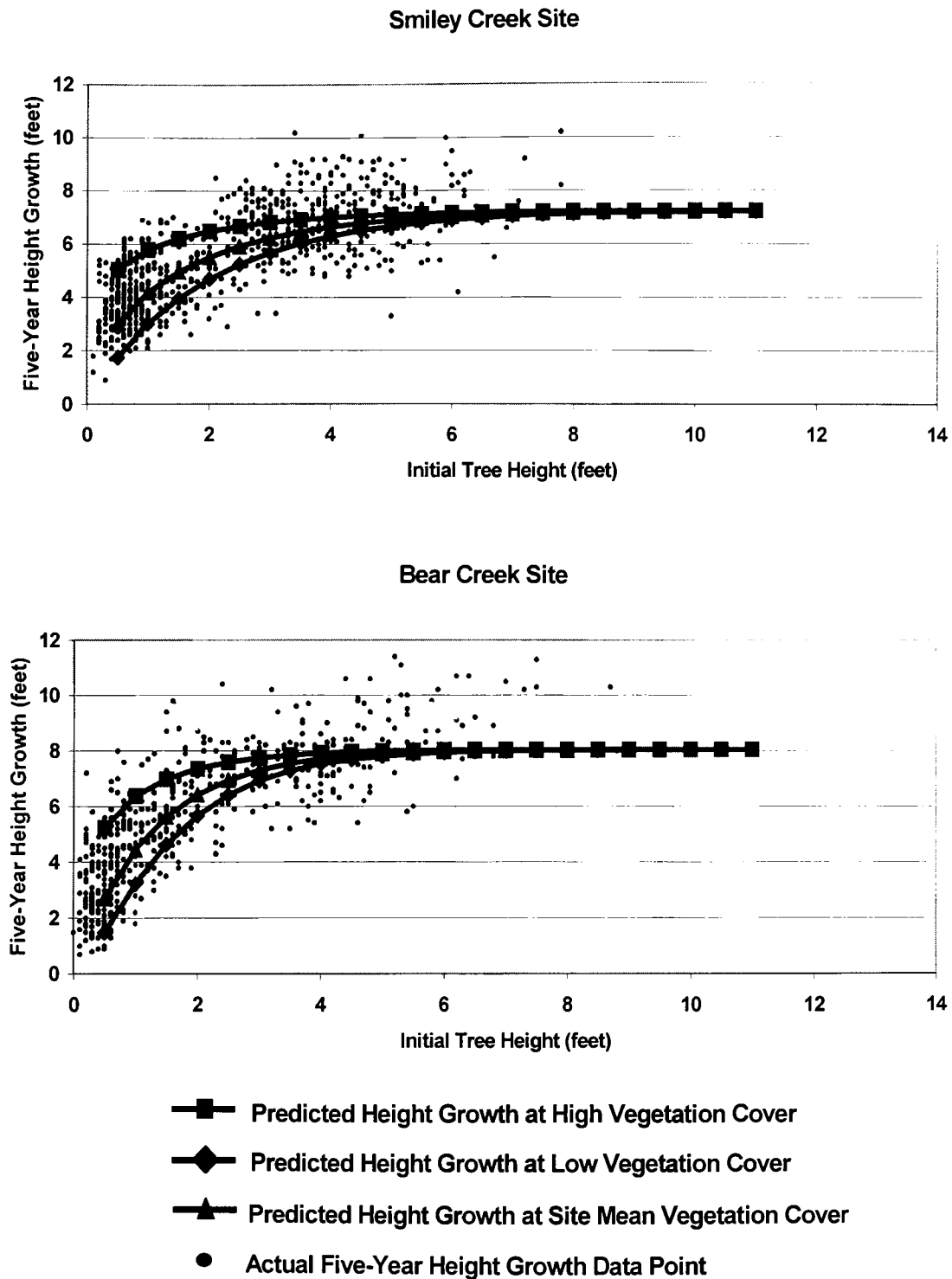


Figure 3. Actual and Predicted Five-Year Height Growth for 1985 Study Sites.

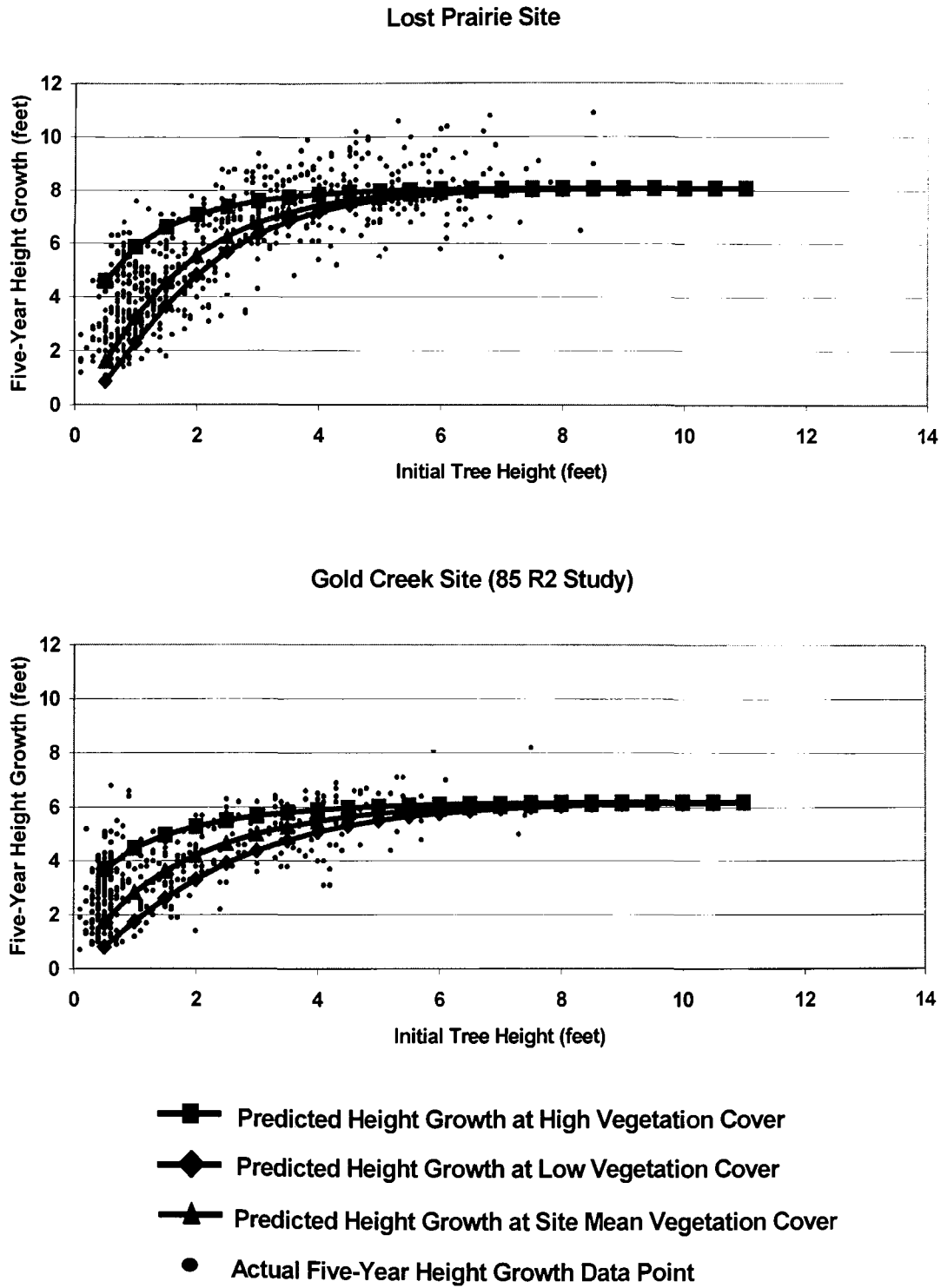


Figure 4. Actual and Predicted Five-Year Height Growth for 1983 Study Sites.

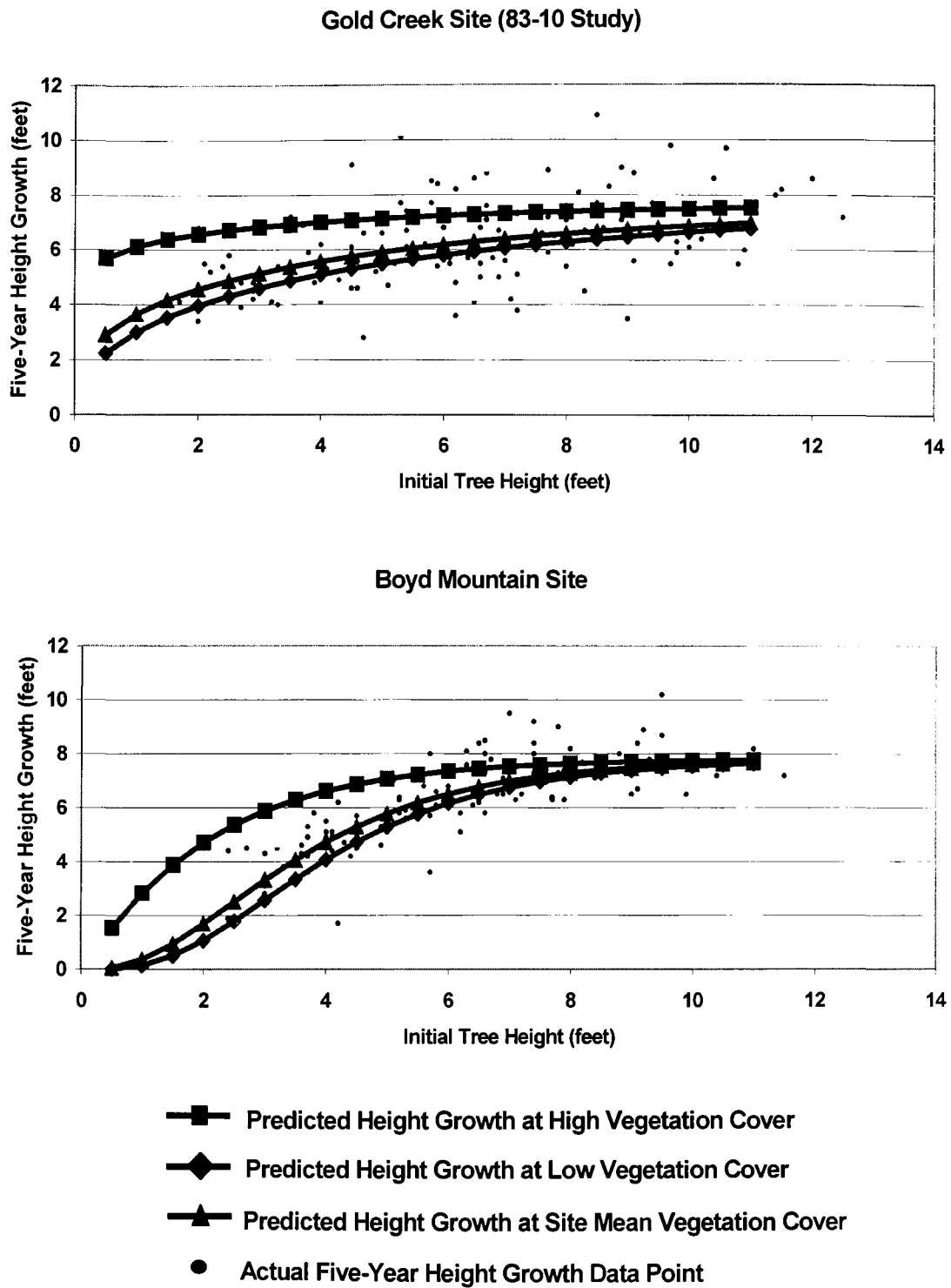
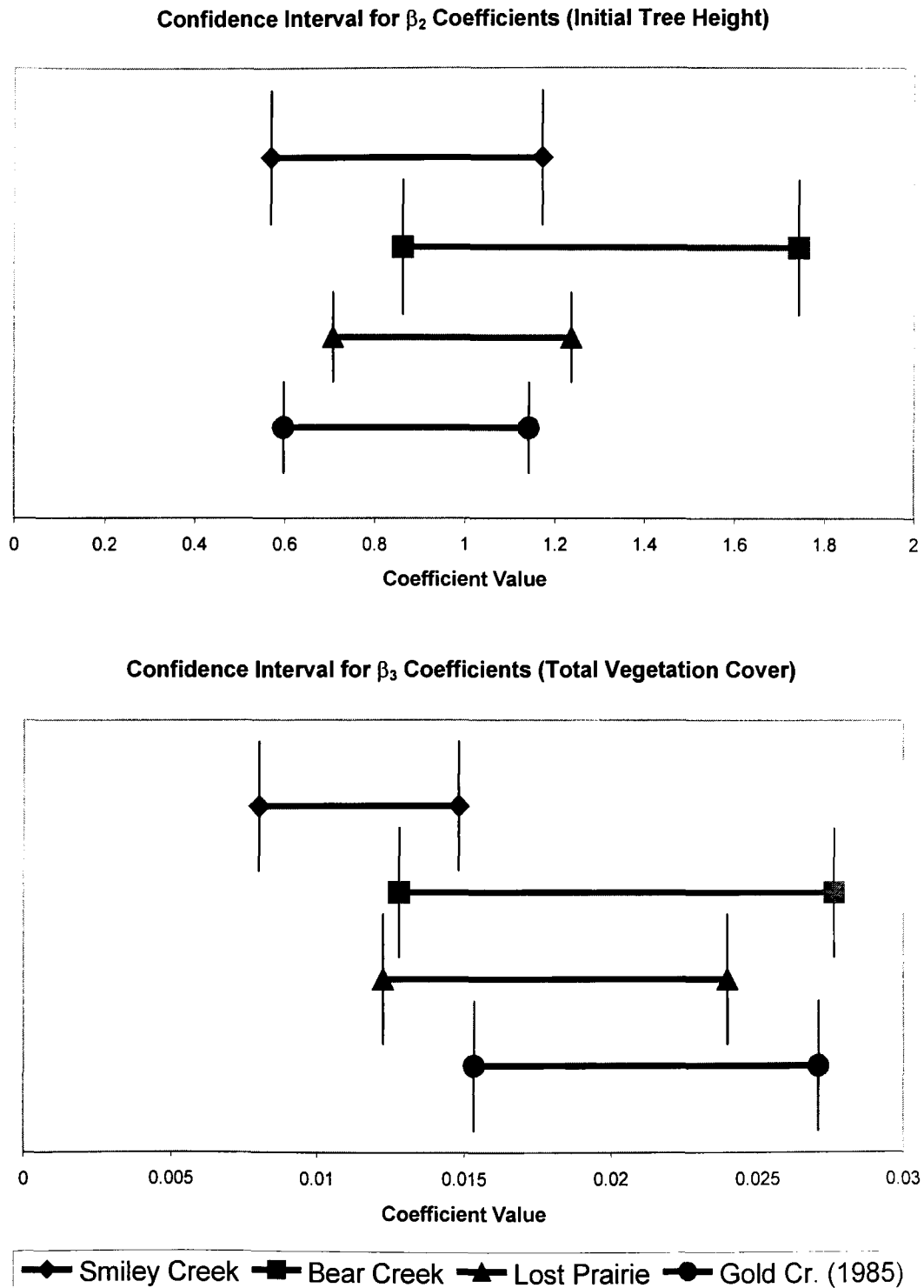


Figure 5. Individual Confidence Intervals for β_2 and β_3 Coefficient Estimates ($\alpha = 0.05$)



Entire Data Set Approach

Dummy variables for site influence are included in the Chapman-Richards function described in equation [1] resulting in the following expanded equation:

$$E(y) = (\beta_1 * x_1 + \beta_2 * x_2 + \dots + \beta_6 * x_6) * \left\{ 1 - \exp(-\beta_7 * x_7) \right\} (\beta_8 * x_8 + \beta_9 * x_9 + \beta_{10} * x_{10}) \quad [3]$$

where:

$E(y)$ = expected five-year height growth

β_1 through β_6 = site parameters

β_7 = initial tree height parameter

β_8 through β_{10} = vegetation cover parameters

x_1 through x_6 = site dummy variables (*Smiley Creek = Site 1, Bear Creek = Site 2,*

Lost Prairie = Site 3, Gold Creek (1985) = Site 4, Gold Creek (1983) = Site 5,

Boyd Mountain = Site 6)

x_7 = initial tree height in feet

x_8 = shrub cover in percent

x_9 = forb cover in percent

x_{10} = grass cover in percent

Site index values were calculated for each site and substituted for the dummy site variables in equation [3] as follows:

$$E(y) = \beta_1 * x_1 * \left\{ 1 - \exp^{(-\beta_2 * x_2)} \right\} (\beta_3 * x_3 + \beta_4 * x_4 + \beta_5 * x_5) \quad [4]$$

where:

$E(y)$ = expected five-year height growth

β_1 = site parameter

β_2 = initial tree height parameter

β_3 through β_5 = vegetation cover parameters

x_1 = site index variables in feet at index age 50 years

x_2 = initial tree height in feet

x_3 = shrub cover in percent

x_4 = forb cover in percent

x_5 = grass cover in percent

Calculated site index values are displayed in Table 5. The highest site index values indicate the best sites for tree growth. The summary statistics for both models are displayed in Table 6, and the estimated coefficients for both models are displayed in Table 7. All estimates shown to be different than zero with 95% confidence. If the asymptotic confidence interval contains zero, then it is questionable whether that independent variable has any discernible effect upon height growth.

A comparison of statistics between the two models indicates both models fit the data reasonably well. The R^2 value of 0.629 for the dummy variable model means that the model accounts for 63% of the variability in five-year height growth (Table 6). The dummy model also has a lower mean square error, indicating a better fit than the site index model.

Table 5. Site Indices for 1983 and 1985 Trials.

Trial	Site	<i>n</i>	Minimum Site Index*	Mean Site Index*	Maximum Site Index*	Standard Deviation
83-10	Gold Creek	15	56.93	65.81	80.36	6.70
	Boyd Mountain	12	59.70	63.24	69.03	2.93
85 R2	Smiley Creek	56	61.04	72.43	84.57	5.10
	Bear Creek	25	64.58	75.34	83.01	5.15
	Lost Prairie	44	61.71	73.66	81.87	4.67
	Gold Creek	39	58.66	65.40	70.06	3.39

* In feet, at an index age of 50 years at breast-height (Milner 1992)

Table 6. A Comparison of Summary Statistics for Two Height Growth Models.

Equation	Error sum of squares	Mean Square Error	R^2
Dummy variable	4135.047	1.681	0.629
Site index	4485.399	1.820	0.598

Table 7. A Comparison of Parameter Estimates for Two Height Growth Models.

Model	Parameter	Type	Estimate	Asymptotic Standard Error	Asymptotic 95% Confidence Interval	
					Lower	Upper
Dummy variable [Eq. 3]	β_1	<i>site</i>	7.4563	0.0940	7.2720	7.6405
	β_2	<i>site</i>	8.2233	0.1105	8.0066	8.4399
	β_3	<i>site</i>	8.0230	0.1102	7.8069	8.2390
	β_4	<i>site</i>	5.4204	0.1081	5.2084	5.6323
	β_5	<i>site</i>	6.5816	0.1234	6.3397	6.8236
	β_6	<i>site</i>	6.5990	0.1361	6.3322	6.8658
	β_7	<i>initial ht.</i>	0.5999	0.0349	0.5314	0.6684
	β_8	<i>shrub cov.</i>	0.3261	0.0526	0.2230	0.4293
	β_9	<i>forb cov.</i>	0.9280	0.0642	0.8021	1.0539
	β_{10}	<i>grass cov.</i>	0.2706	0.0506	0.1713	0.3699
Site index [Eq. 4]	β_1	<i>site</i>	0.1037	0.0010	0.1017	0.1056
	β_2	<i>initial ht.</i>	0.6335	0.0343	0.5662	0.7008
	β_3	<i>shrub cov.</i>	0.7087	0.0565	0.5978	0.8196
	β_4	<i>forb cov.</i>	0.8678	0.0660	0.7385	0.9972
	β_5	<i>grass cov.</i>	0.1753	0.0527	0.0719	0.2786

All coefficient estimates appear statistically significant in Table 7, as the asymptotic confidence intervals for all coefficients do not contain zero. The effects of site are clearly different in [Eq. 3] where the estimates for β_1 and β_6 are significantly different from the other four site coefficients and each other. The estimates of coefficients common to both models changed when site index replaced the dummy variables in [Eq. 3], but no pattern is evident. The effect of shrub competition and initial tree height increased from [Eq. 3] to [Eq. 4], but the effects of forb and grass competition decreased. Standard errors appear to change very little for common terms between models.

To determine whether the above inferences are to be accepted as reliable, the residuals must be examined for equal variance across the range of the data, and for normality. If the residuals exhibit unequal (heterogeneous) variance, then standard errors may be suspect. If residuals appear to have a non-normal distribution, then normal-based inferences will not be reliable.

Both a normal quantile (Q-Q) plot and a normal scores correlation test were performed on each of the two models. On the normal quantile plot, observed residual values of five-year height growth are plotted against expected (if the residuals were normally distributed) quantiles. If the actual residuals are normally distributed, they will cluster about the 45° line. For both models, the residuals are arranged closely about the lines in Figures 5 and 6 (bottom), with slight deviations at the ends of said lines. In the normal scores correlation test, residuals are assigned a score, based upon an expectation of normality, similar to the Q-Q plotting procedure. A coefficient is calculated for the correlation between the residuals and their corresponding normal scores, and is then compared to a critical value, which in this case has been determined to be 0.9935 (actually for a sample size of 300 and a significance level of 0.01). The calculated correlation for both models was approximately 1.000, which exceeded the aforementioned critical value and therefore provides strong evidence of normality.

Variations appear to increase slightly across the ranges of predicted values for both models (Figures 5 and 6, top). One reason this occurs is because there is greater variation in height growth amounts in large trees, as compared to smaller trees. Small trees have low maximum height growth rates compared to large trees, so the amounts a large tree may potentially grow are much more varied. A transformation on the dependent

variable of height growth may serve to alleviate this problem, but keeping the predicted height growth in meaningful units was a priority in this case. Although the Chapman-Richards function exhibits realistic performance and explains a large amount of the variation within the data, other model forms not examined here may also be appropriate. The residual plots for both models show a slightly greater number of points above zero at low predicted values, meaning that the models tend to slightly under predict height growth for smaller trees. Across the rest of the distribution, the models have no clear over or under prediction problem.

Figure 6. Plot of Residuals Versus Predicted Values and Normal Quantile Plot for the Dummy Variable Model [Eq. 3].

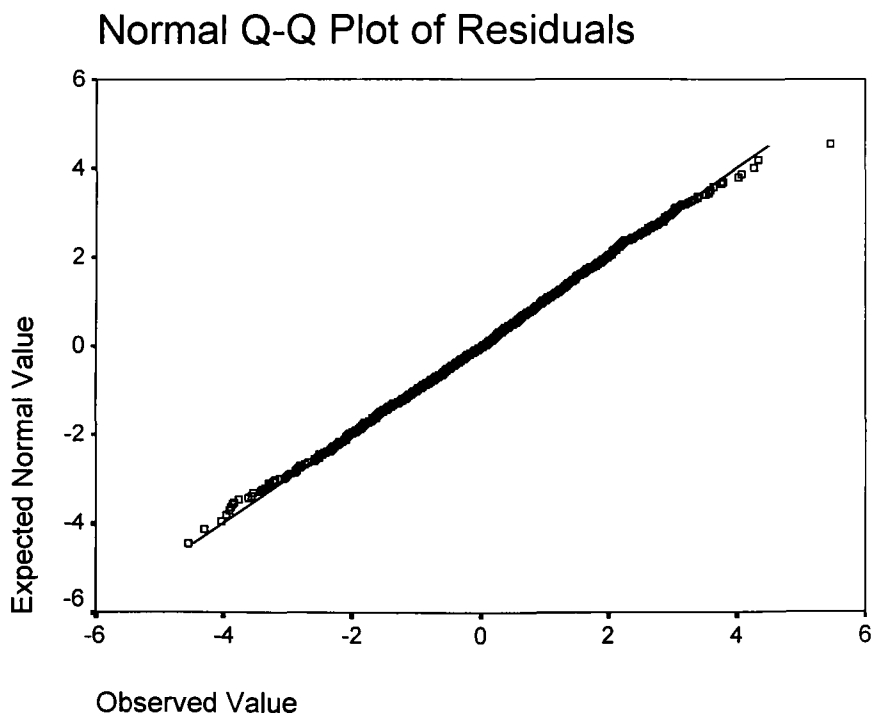
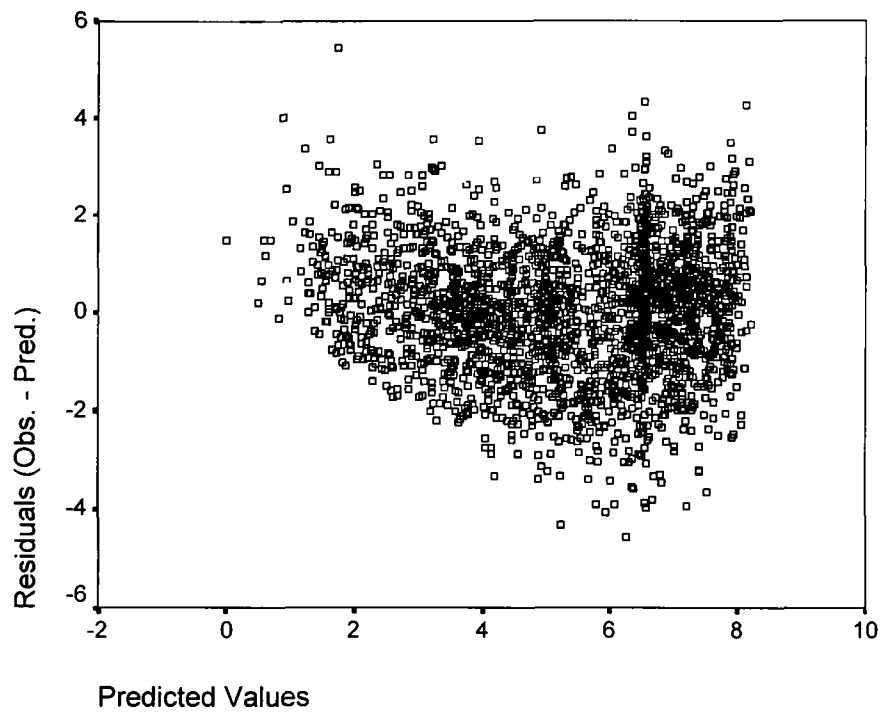
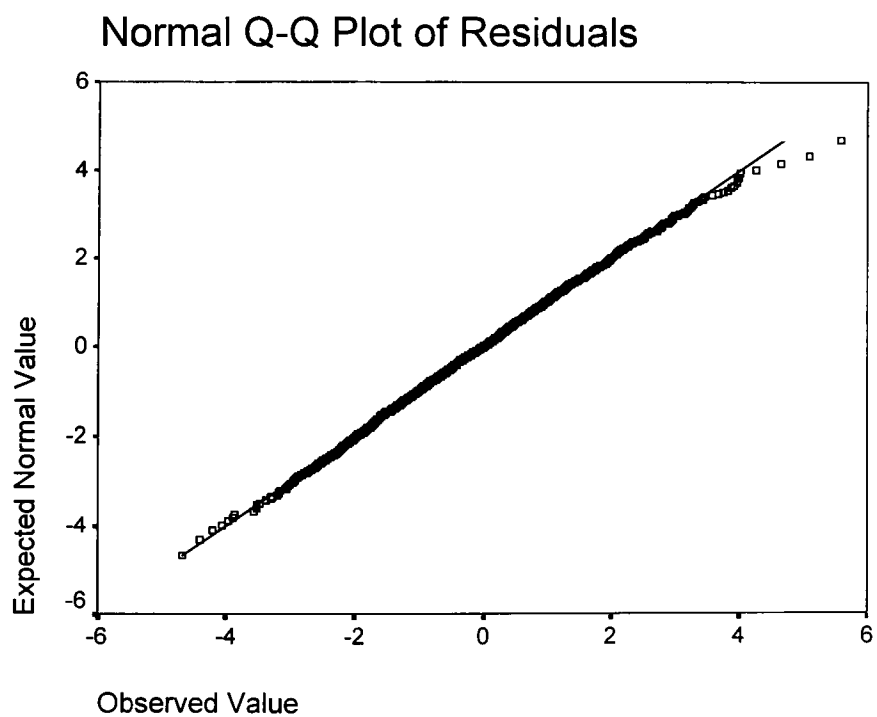
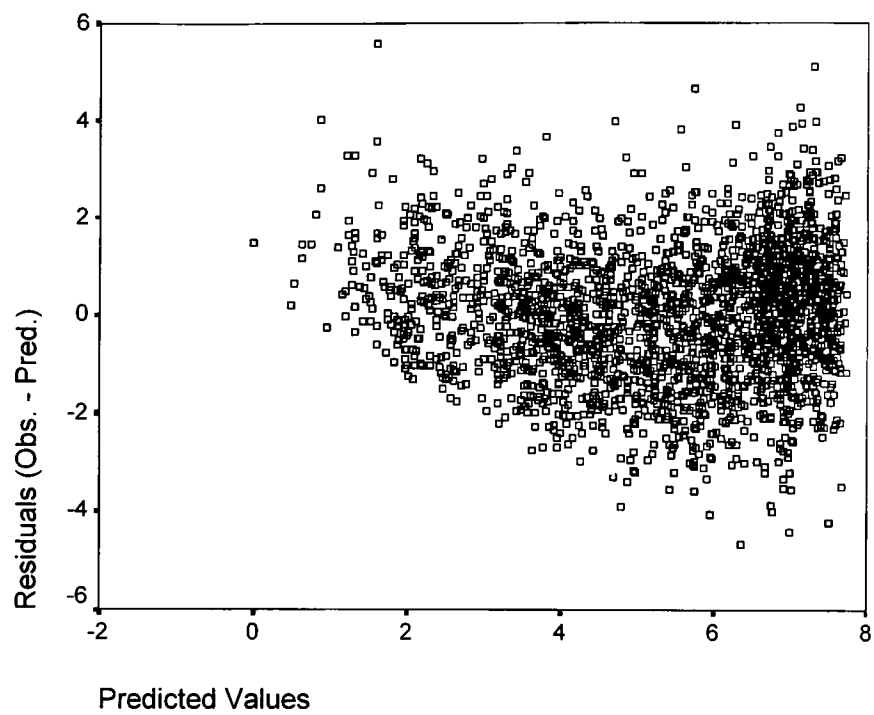


Figure 7. Plot of Residuals Versus Predicted Values and Normal Quantile Plot for the Site Index Variable Model [Eq. 4].



Model Performance

To evaluate how each model performs, a hypothetical distribution of tree heights was created and then five-year height growth was calculated for each tree. The response surfaces for the dummy variable [Eq. 3] and site index variable [Eq. 4] models are displayed in Figures 8 and 9, respectively. Curves are generated across a range of tree heights from 0.5 through 11 feet. The upper limit of tree size was selected based on the results found by Keyser (1998) which show that tree growth trajectories became parallel before reaching 10 feet in height. For the first model [Eq. 3], the Bear and Gold Creek (85 R2) sites were selected as examples since they rose to the highest and lowest asymptotes for height growth, respectively. As for the site index model [Eq. 4], the calculated site index of 66 feet for Bear Creek and 57 feet for Gold Creek (85 R2) were selected for representation.

The response surfaces shown in Figures 8 and 9 are almost identical in shape for the two models, but exhibit large differences in early height growth. In Figure 8, the response surfaces for Bear Creek show that increases in vegetative cover delay the rise to the asymptote (the maximum rate of growth) by nearly 3 feet of initial height. Trees growing in the conditions found at Bear Creek are predicted to achieve their maximum height growth when they reach about 7 feet in height under low vegetative competition and about 9 feet under higher competition. Maximum growth rates differ by more the 2.5 feet in five years between the Bear and Gold Creek sites.

When the site index values for the Bear Creek and Gold Creek sites are placed into the site index variable equation [Eq. 4], the overall shape of the height growth curves

remains similar, as shown in Figure 9. The asymptotes differ somewhat between [Eq. 3] and [Eq. 4], and the difference in maximum height growth between sites is less in [Eq. 4]. In fact, the effect of site differences has been compressed in [Eq. 4], where the maximum growth rate at Gold Creek was greater in [Eq. 4] than in [Eq. 3], while the maximum growth rate at Bear Creek was less.

To better visualize how both models predict tree growth, growth trajectories of hypothetical trees were generated and are displayed in Figures 10 and 11. Tree height was initialized at 0.5 feet, then five-year height growth was calculated and added to the initial height. Once again, five-year growth was calculated for the new height and added. This process was repeated for four growth periods, or twenty years of growth beyond the point when the tree was 0.5 feet tall.

The growth trajectories generated for the two models are similar in overall shape (Figures 10 and 11). However, the trajectories vary markedly among low, high, and very high vegetative cover levels for individual sites. For [Eq. 3], growth trajectories for trees under high competition at both sites approach the slope of those under low competition at around 5 years, while those under very high vegetative competition take at least 10 years to reach the maximum rate of growth. At the Gold Creek site, trees under the greatest amount of vegetative competition do not achieve maximum growth until after ten years. These results are similar to the findings of Keyser (1998).

Figure 8. Predicted five-year height growth for lodgepole pine at two sites and three levels of vegetative cover using dummy site variables [Eq. 3].

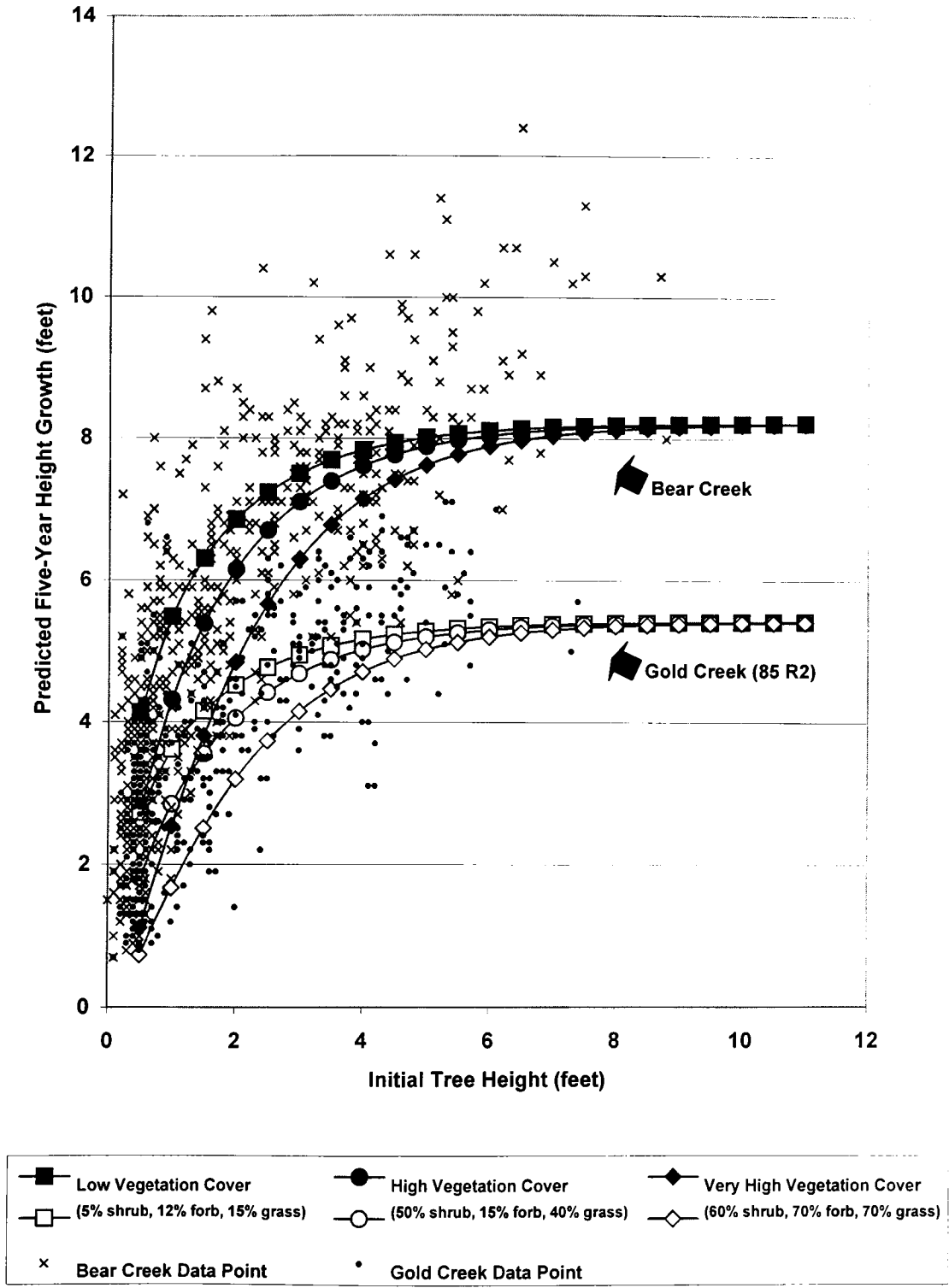


Figure 9. Predicted five-year height growth for lodgepole pine at two site indices and three levels of vegetative cover [Eq. 4].

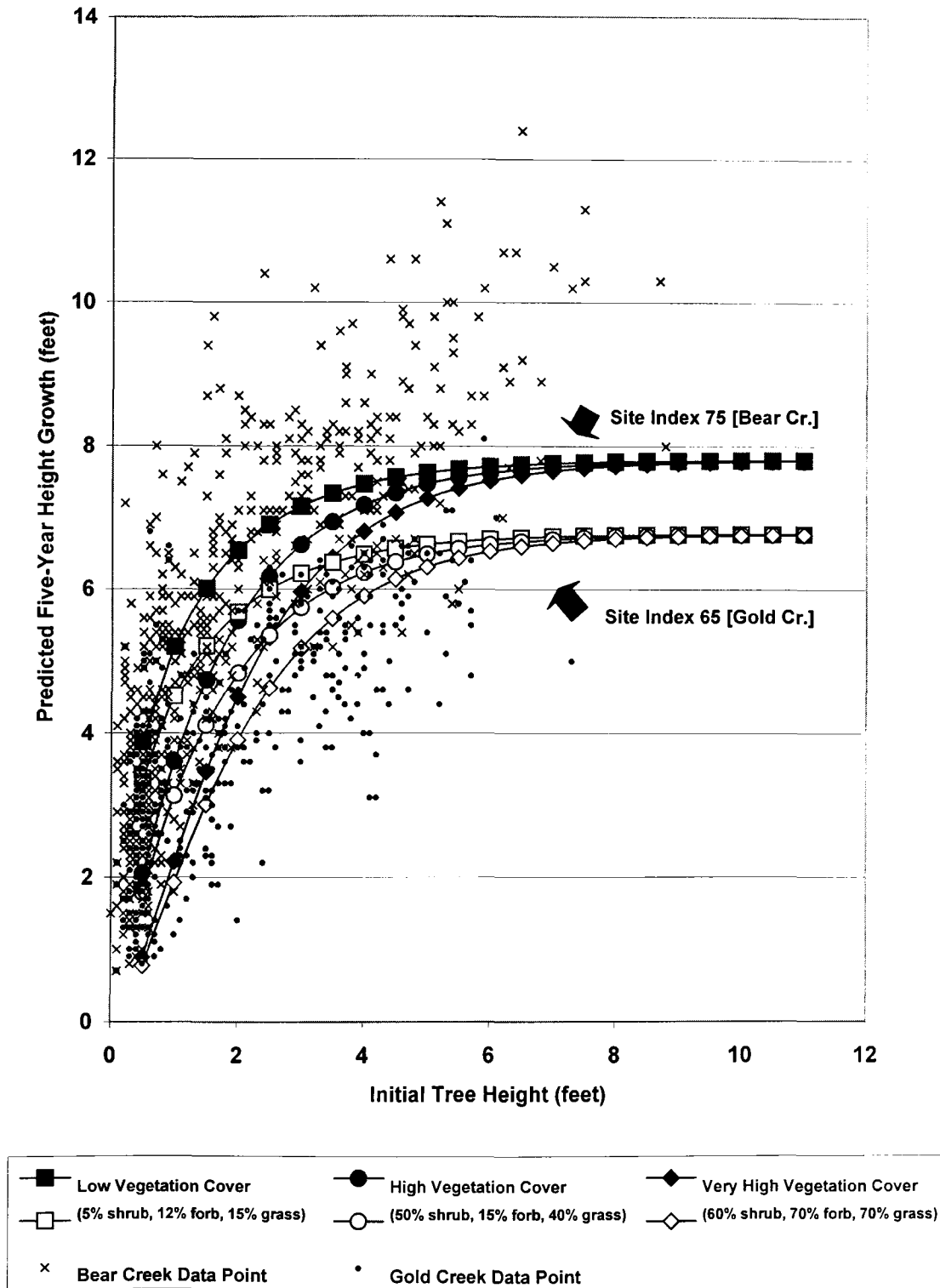


Figure 10. Height growth trajectory prediction for lodgepole pine at two sites and three levels of vegetative cover using the dummy variable model [Eq. 3].

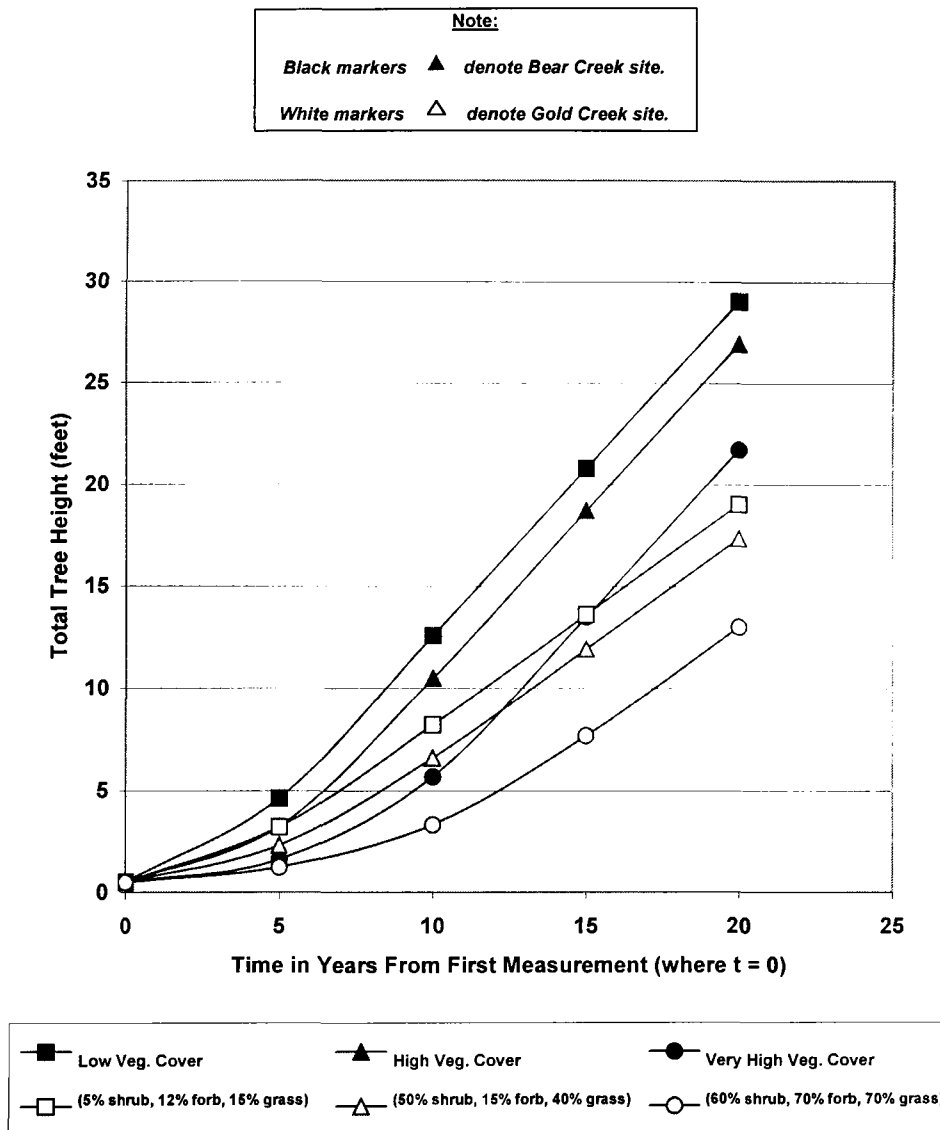
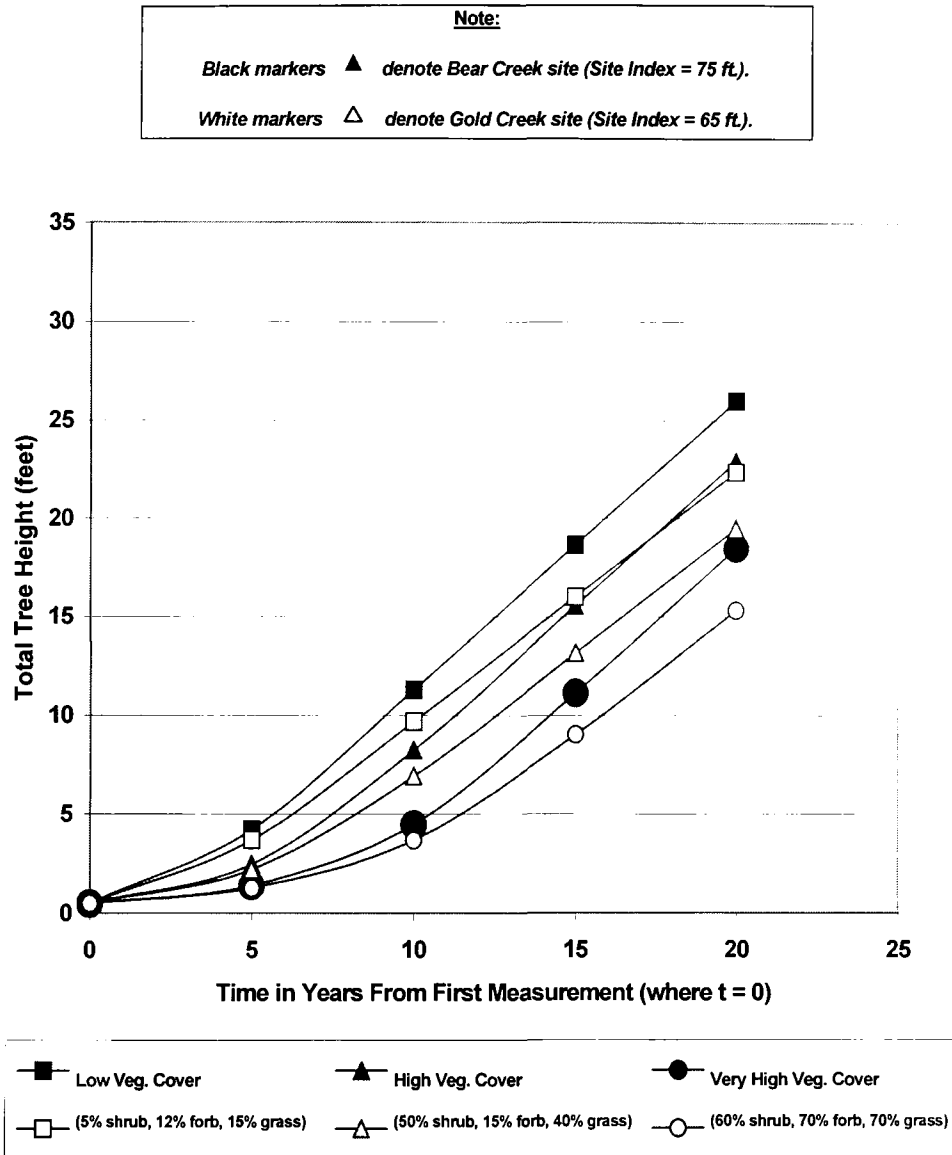


Figure 11. Height growth trajectory prediction for lodgepole pine at two sites and three levels of vegetative cover using the site index variable model [Eq. 4].



V. Discussion

Both of the models developed to predict height growth of young lodgepole pine fit the data reasonably well. The model using dummy site variables [Eq. 3] has a slightly better fit than the site index variable model [Eq. 4]. The R^2 value is slightly greater and the mean square error is less for [Eq. 3]. The greater R^2 value associated with [Eq. 3] may be the result of estimating a greater number of coefficients. The question is not ultimately which model to choose. The model shown in [Eq. 4] is the logical extension of the previous models to real-world applications. The model in [Eq. 3] has little practical value, except under site conditions similar to those used to build it. The use of site index allows extension of the model to all sites within this study's range of site quality.

Of greater concern is the model's loss of sensitivity when the dummy variables are changed to a single site index variable. Perhaps the reduction from six to one site coefficient estimate constrained the regression algorithm in assigning site effects to those coefficients. The difference in model performance may also be a result of problems with the site index variable. Since site index is generally calculated from larger trees than those found in this study, it is logical that the amount of variability in height among larger trees would be greater than among small trees. Large trees may also utilize site resources differently. An example of this phenomenon is that a large tree's root system extends wider and deeper into the soil allowing it to access resources unavailable to seedlings and small saplings. The overall impact is that the low variability in site index

would then be reflected in compressed maximum height growth predictions among sites, which has indeed occurred here.

The behavior of the site index model corresponds to a previous analysis of the Champion International Corporation data by Keyser (1998). At the Bear Creek site, for example, Keyser (1998) found that mean height of lodgepole pine was only 7 feet after 10 years on control plots with high vegetation cover, while on plots with low vegetation cover (Pronone[®] applied at four pounds per acre), mean tree height was nearly 12 feet. An examination of Figure 10 shows similar tree heights at year 10 for low and high vegetation cover levels in this study.

While the site index equation provides the widest range of applications, it must be recognized that this model was derived from data for only one species at six sites. If necessary, the model could conceivably be applied to other intolerant species found in western Montana. Western larch has often been found to have a similar growth trajectory to lodgepole pine (Cole and Schmidt 1986; O'Hara 1993; Oliver and Larson 1996). Ponderosa pine is another shade-intolerant species for which this model may be suited. Extending the model to Douglas-fir, a moderately tolerant species, may be incorrect. Evidence shows that within western Montana, Douglas-fir has much slower height growth rates than both western larch and lodgepole pine (Fiedler 1990). Also, this model should not be applied to more shade-tolerant species such as grand and subalpine fir. The model is unsuitable for predicting growth in any stands that have a history of partial cutting, or those that may have had substantial overstory at any time in recent history. Overstory trees compete with seedlings for nutrients, water, and light, which may reduce seedling growth where substantial competition exists.

This study shows that modeling height growth of small trees that are under vegetative competition has promise, and should be pursued more extensively. The use of initial tree height instead of age allows this type of model to be used in naturally regenerated stands where the age of individual trees may be difficult to determine, and where errors in estimating age may result in inaccurate growth predictions. This type of model also could be easily incorporated into existing simulators. Small trees would be grown with this model until they reach a certain maximum height, then transferred to a model for larger trees. A large-scale and long-term data collection effort would allow for the creation of small-tree models that would function well for many species over a wide variety of site conditions.

VI. Conclusion

The effects of cover broken into three types were not reliable at the individual site level, but when cover was combined into a single variable, sufficient explanatory power was found to provide evidence that vegetation effects were fairly similar across all sites in this study. It was previously mentioned that given amounts of cover have differing effects upon tree growth, depending on whether the cover is in shrub, forb or grass. However, the findings in this study indicate that at least on this narrow range of sites, certain amounts and types of vegetative cover result in similar effects upon tree height growth.

A more reliable site index measure, such as one taken before harvesting, would likely have improved model performance. The effects of vegetation cover upon height growth are reasonable. The problems inherent with using site index equations developed from larger trees affect predictions of maximum growth rates for smaller trees. Therefore, this model should be most reliable predicting the height growth of the less than 10 feet tall. The model is the logical extension of earlier work in western Montana that shows reductions in vegetation cover result in increased growth of seedlings. Although its range is limited by the data used in its construction, the model is appropriate for use within reasonable boundaries: that is, fairly dry grass-dominated sites throughout western Montana.

The broader import of this study is that it provides justification for proceeding with larger data collection efforts in order to improve future growth and yield modeling. The usefulness of the model will increase when sampling is conducted across a wider

range of sites and species. When the range of this small-tree model matches that of the large-tree simulators, it then may be incorporated into the simulator to make a great improvement in growth and yield prediction, ultimately resulting in improved land management decisions.

VII. Literature Cited

Bates, D. M. and D. G. Watts. 1988. *Nonlinear regression analysis and its applications*.

New York: John Wiley and Sons. 365 p.

Burton, P. J. 1993. Some limitations inherent to static indices of plant competition. *Can.*

J. For. Res. 23:2141-2152.

Cole, D. M. and W. C. Schmidt. 1986. Site treatments influence development of a

young mixed-species western larch stand. *USDA For. Serv. Res. Pap.*

INT-RP-364. Ogden, UT: Intermountain Research Station. 6 p.

Fiedler, C. E. 1990. Natural regeneration following clearcutting in the spruce-fir zone of

western Montana. Ph.D. diss. University of Minnesota, Twin Cities. 170 p.

Haywood, J. D. 1993. Effects of aerially applied glyphosate and hexazinone on

hardwoods and pines in a loblolly pine plantation. *USDA For. Serv. Res. Pap.*

SO-RP-277. New Orleans, LA: Southern Forest Experiment Station. 6p.

Keyser, C. E. 1998. Planted conifer and surface vegetation responses to herbaceous

vegetation control in western Montana. M. S. Thesis. Missoula, MT: The

University of Montana. 57 p.

- Knowe, S. A. and W. I. Stein. 1995. Predicting the effects of site preparation and protection on development of young Douglas-fir plantations. *Can. J. For. Res.* 25:1538-1547.
- Milner, K. S. 1992. Site index curves for ponderosa pine, western larch, lodgepole pine, and Douglas-fir in Western Montana. *West. J. Appl. For.* 7(1):9-14.
- Milner, K. S. 1997. Growth relationships between young ponderosa and lodgepole pine trees and herbaceous vegetation on dry habitat types in western Montana: a proposal for the 1997 McIntire-Stennis Program. School of Forestry, The University of Montana, Missoula, MT. 12 p.
- McLeod, S. D. and J. M. Mandzak. 1990. Site preparation using Velpar L. increases juvenile growth of lodgepole pine on Montana fee lands. Champion Inter. Corp. Res. Note WO90-1. Milltown, MT: Western Operations, Research and Development. 15 p.
- Moeur, M. 1985. COVER: a user's guide to the canopy and shrubs extension of the stand prognosis model. USDA For. Serv. Gen. Tech. Rep. INT-GTR-190. Ogden, UT: Intermountain Research Station. 49 p.
- O'Hara, K. L. 1993. Early height development and species stratification across five climax series in the eastern Washington Cascade range. *New Forests* 9:53-60.

- Oliver, C. D. and B. C. Larson. 1996. *Forest stand dynamics*. Update ed. New York: John Wiley and Sons. 520 p.
- Oliver, W. W. 1984. Brush reduces growth of thinned ponderosa pine in northern California. USDA For. Serv. Res. Pap. PSW-RP-172. Berkeley, CA: Pacific Southwest Research Station. 7 p.
- Payandeh, B., M. Punch and D. Basham. 1992. User's manual for PLANT-PC: a model for forest plantation establishment in Ontario. Can. For. Resource Dev. Agreement Rep. 3319. Sault Ste. Marie, Ont.: Forestry Canada, Ontario Region. 54 p.
- Pfister, R. D., B. L. Kovalchik, S. F. Arno and R. C. Presby. 1977. Forest habitat types of Montana. USDA For. Serv. Gen. Tech. Rep. INT-GTR-34. Ogden, UT: Intermountain Forest and Range Experiment Station. 174 p.
- Powers, R. F., M. W. Ritchie and L. O. Ticknor. 1989. SYSTUM-1: simulating the growth of young conifers under management. In: *A decade of forest vegetation management. Proc. 10th Ann. Forest Vegetation Management Conference, Nov. 1-3, 1988*. Eureka, California. Pp. 101-115.

- Richards, F. J. 1959. A flexible growth function for empirical use. *J. Exp. Bot.* 10:290-300.
- Ritchie, M.W. and R.F. Powers. 1988. Modeling height growth of young ponderosa pine plantations: influence of weed competition and crown ratio. P 79-91 In: *Proceedings 9th Annual Forest Vegetation Management Conference, Nov. 3-5, 1987, Redding CA.* Forest Vegetation Management Conference, Redding, CA. 226 p.
- Ritchie, M. W. and R. F. Powers. 1993. User's guide for SYSTUM-1 (version 2.0): a simulator of growth trends in young stands under management in California and Oregon. USDA For. Serv. Gen. Tech. Rep. PSW-GTR-147. Albany, CA: Pacific Southwest Research Station. 45 p.
- Smith, D. M. 1986. *The practice of silviculture*. 8th ed. New York: John Wiley and Sons. 514 p.
- Sokal, R. R. and F. J. Rohlf. 1987. *Introduction to biostatistics*. 2nd ed. New York: W.H. Freeman and Company. 363 p.
- Stewart, R. E. 1987. Seeing the forest for the weeds: a synthesis of forest vegetation management. In: *Forest vegetation management for conifer production*, eds. Walstad, J. D. and P. J. Kuch. New York: John Wiley and Sons. Pp. 431-474.

Thamarus, K. and K. S. Milner. 1989. Second year results of vegetation control, survival and volume response to chemical site preparation with Pronone 10G and Oust herbicides (Study 85R2). Champion Inter. Corp. Res. Note WO89-4. Milltown, MT: Western Operations, Research and Development. 26 p.

Uzoh, F. C. C. 1992. Height growth projection model for newly established stands of ponderosa pine under various levels of competing vegetation. Ph.D. diss., University of Montana--Missoula. 299 p.

Uzoh, F. C. C. 1999. Response of planted ponderosa pine seedlings to weed control by herbicide in Western Montana. *West. J. Appl. For.* 14: 48-52.

Uzoh, F. C. C. and M. W. Ritchie. 1996. Crown area equations for 12 species of shrubs and trees. USDA For. Serv. Res. Pap. PSW-RP-227. Albany, CA: Pacific Southwest Research Station. 13 p.

Walstad, J. D. and P. J. Kuch. 1987. Introduction to forest vegetation management. In: *Forest vegetation management for conifer production*, eds. Walstad, J. D. and P. J. Kuch. New York: John Wiley and Sons. Pp. 3-14.