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# Channel networks in ice: Application of river network research to ice streams

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# CHANNEL NETWORKS IN ICE
# APPLICATION OF RIVER NETWORK RESEARCH TO ICE STREAMS

by

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Many natural systems form branching, self-similar channel networks - from the vein pattern on a maple leaf to the system of rivers and tributaries comprising the Missouri river network. Since these networks are observed with such frequency in nature, many attempts have been made to understand why this organizational pattern is so prevalent. Indeed, a special classification now exists for networks that are directed (flow is well defined) as well as efficient (the network strives for energy minimization). These networks are known as *optimal channel networks.*

Most naturally occurring channel networks can be elegantly characterized with a mathematical technique known as *allometric scaling.* Allometric scaling is exhibited in systems that show similarity across many spatial scales. Comparing a sub-section of a system exhibiting this allometric scaling characteristic to other sub-sections of the same system, or to the overall system itself, will reveal strong similarity. For instance, a small tributary of a river system contains many of the same features as a larger tribtary, but scaled down.

It has recently been shown that aspects of a system's inherent characteristics may be discerned by examining its allometry. One of these characteristics is *dimensionality.* Dimensionality can be defined as the minimum number of spatial dimensions in which a system exists. Utilizing a process that invokes a series of algorithms, we can infer a system's dimension from scaling relations. The main algorithm in the process recursively visits all cells in a basin calculating the flow into each cell from all upslope neighbors.

The purpose of this thesis is to demonstrate that ice stream networks exist as allometric channel networks and to examine the similarities and differences between river networks (which have been shown to exist in two dimensions) and ice streams. Using current river analysis procedures, we show that the allometry for ice stream networks is dissimilar from the allometry for river networks. We conclude that any model of ice streams must encompass more than two dimensions in order to capture all relevant physical characteristics of the system.

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# CHAPTER 1 OVERVIEW

## Introduction

Antarctica provides a fertile environment for science. The extreme temperature in particular produces a massive continuum of ice and snow and with that comes an interesting dance between snowfall, accumulation, and runoff. Nature has provided this runoff function in the form of rivers of moving ice that discharge ice from the interior to the ocean. These so called *ice streams* perform the same transportation function that river networks perform; they provide an efficient mechanism for maintaining the system's mass balance.

Antarctica houses the largest source of fresh water in the world. If, for example, the Ross Ice Shelf in Western Antarctica collapsed the global ocean level may rise by four to six meters (Oppenheimer, 1998). Due to the magnitude of the system, the mass-balance of Antarctica is very significant to global climate, hence the need for careful study. Furthermore, ice streams are the primary mechanism by which Antarctic ice is deposited into the ocean (Oppenheimer, 1998), and thus are the focus of most studies concerning the Antarctic mass-balance.

Heightened interest in the global warming phenomenon has promoted research focused on the ice streams in Antarctica, particularly the Western Antarctic Ice Streams (WAIS) flowing into the Ross Ice Shelf. The speed at which these streams transport ice is of keen interest to scientists, and various techniques including satellite based radar interferometry have been utilized (Joughin et al., 1999) to gather relevant velocity data. Upon examination of this velocity data, the channelized character of these ice drainage networks is immediately apparent, however, no thorough investigation of the their fractal dimension and structure has been performed [1]. As an attempt to rigorously perform this investigation and to probe the character of these streams, this thesis appeals to similarities between ice streams, river networks, and general channel networks utilizing scaling laws for comparison.

### Scaling laws

A general study of channel networks reveals that the most efficient networks contain a property wherein the total system is *self similar* to sub-networks within itself (Banavar et al., 1999). This property can be characterized by a *scaling law* that has the general form:

$$Y \propto Y_0 X^n \tag{1.1}$$

which defines a relationship between two scalar quantities $X$ and $Y$. $Y_0$ functions as a proportionality constant and $n$ is the scaling exponent (Dreyer, 2001).

Scaling laws [2] can be found whenever a system exists that shows similarity across spatial scales. Perhaps the best known examples of scaling are "fractals" such as a Koch curve, where the fractal geometry at one level is the same at the next and previous scales. For the systems considered here, the scaling exponent $n$ is a fraction less than one, indicating that individual elements decrease in size as the system increases in depth.

Of course, when dealing with natural systems, we must impose reasonable con-

[1] Considering that such research is recent and ongoing (Maritan et al., 2002), this is not surprising.
[2] Scaling laws are also known as *power laws* - these terms will be used interchangeably throughout.

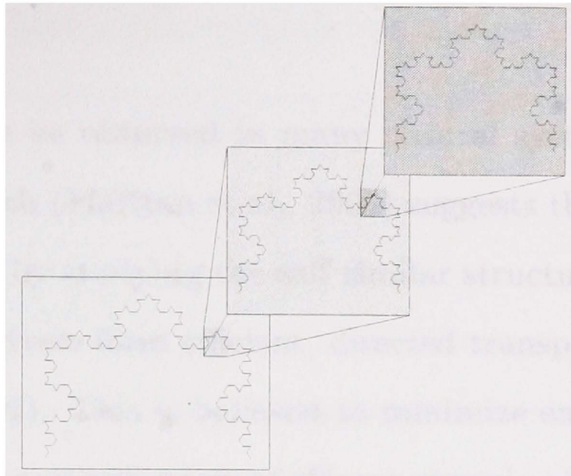

Figure 1.1 Koch Curve fractal has fractal dimension $\frac{4}{3}$

straints on the upper and lower bounds of where the exponent is measured [3]. Another consideration when dealing with natural systems are the artifacts introduced by using a finite spatial resolution. Since a dataset cannot contain infinitely many entries, there will always be some area between data points that must be interpolated. To minimize these artifacts, a dataset of high enough resolution to capture the significant attributes should be used.

An *allometric scaling law* is a scaling law (Equation 1.1) where the exponent $n \neq 1$ (Dodds and Rothman, 2000). Many natural channel networks can be described using an allometric scaling law. The *network allometry*, along with many other aspects of the network, can be examined by descerning the exponent of the scaling law describing the system.

[3]Only in theory do fractals span all scales. In practice finite size effects provide upper and lower bounds on the system. For instance there are no rivers larger than the Amazon, and none smaller than a channel head.

## River networks

Channel networks can be observed in many natural systems, including river networks, and recent research (Maritan et al., 2002) suggests that aspects of a network's character can be known by studying the self similar structure of the system. For example, it's known that rivers form efficient, directed transport networks (Rodriguez-Iturbe and Rinaldo, 2001). This is believed to minimize energy dissipation and this type of network falls into a specific class of efficent spanning tree topographic networks as defined by Banavar et al., 1999.

A river network can be categorized by an allometric scaling law because it shows self similarity (*i.e.*,small sub-basins of the drainage network resemble the entire drainage network when scaled appropriately). This allometric scaling law provides a mechanism to examine river drainage networks and is known as Hack's Law (Equation 1.2) - one of the earliest observations of river allometry. Proposed by John Hack in 1957 (Hack, 1957), Hack's Law states that the length $l$ of a drainage basin's main tributary is proportional to the basin's area $a$ taken to an exponent $h$ (Rodriguez-Iturbe and Rinaldo, 2001) - $h$ is known as Hack's exponent.

$$l \propto a^h \tag{1.2}$$

The seminal work on this relation studied the drainage basin for the Shenandoah Valley in Virginia, and found Hack's exponent to be approximately 0.6 (Hack, 1957). Further studies confirmed that for most river networks Hack's exponent falls between 0.52 and 0.6 (Rodriguez-Iturbe and Rinaldo, 2001).

Hack's Law provides an interesting observation of a river network, however, it is ultimately driven by empirical data that provides no real explanation of why the relation exists. The desire to couch such interesting scaling behavior in a more geometrically based mechanism is met using the concept of dimensionality. Maritan et al. (2002) shows a simple calculation that translates between network dimensionality and Hack's exponent.

## Network dimensionality

Recent research has produced convincing arguments that a relation between the river network's basin area and the total accumulation of water in the basin may provide a consistent and geometrically based derivation for Hack's exponent (Maritan et al., 2002). By analyzing a river network's drainage basin at a range of scales, a very close approximation to Hack's exponent is derived.

Combining the current research of channel networks and dimensionality with the surface elevation, bed elevation, and velocity data gathered in Antarctica, we can demonstrate that the scaling behavior of ice streams differs significantly from the behavior of river networks.

### Glaciology

The visual and compositional similarities between ice stream networks and river networks provides an impetus to analyze ice streams using river drainage analysis tools. Most river analysis tools use a digital elevation map to determine flow networks - after all, water is primarily subject to gravity. Similarly, ice stream networks can be closely approximated using elevation or surface slope only (a so called "first order approximation" of an ice stream). These approximate ice stream networks reveal a strong similarity when compared to measured ice stream networks, indicating that a first order approximation is appropriate for finding ice stream networks.

Many sources of data concerning the elevation and the velocity of the ice flowing in

Antarctica are available through The National Snow and Ice Data Center (NSIDC) website [4]. The RADARSAT Antarctic Mapping Mission (RAMP) dataset contains high resolution elevation data that can be used to find ice stream networks. This research also benefits from the availability of high resolution velocity data for most of the Siple Dome area in Antarctica including WAIS, and many sections of the Ronne-Filchner Ice Shelf provided by Joughin et al., 1999.

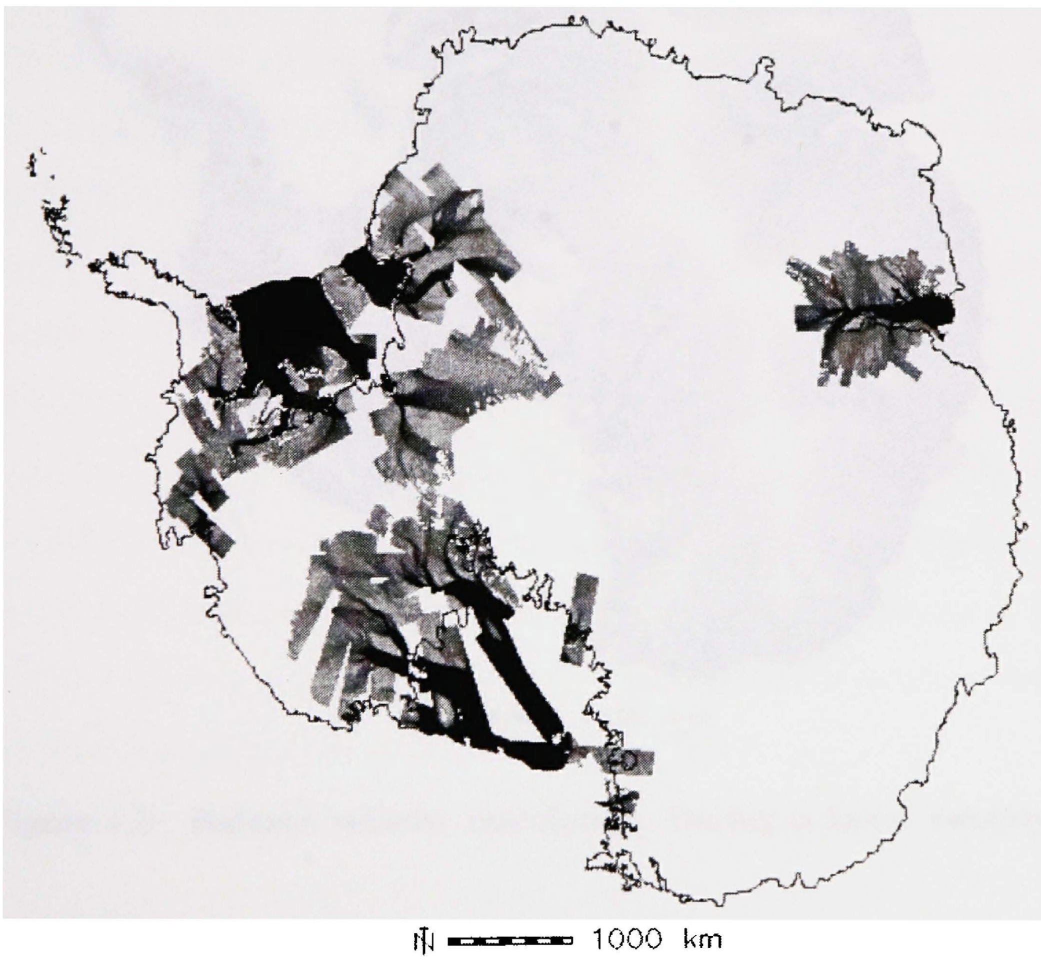


Figure 1.2 RADARSAT Interferometry velocity provided by Joughin et al., 1999 and personal communication. Darker is faster velocity.

A map of velocity can also be generated using a balance velocity calculation (Budd

[4]http://nsidc.org/

and Warner, 1996) over the whole continent's elevation data. While not as accurate as the measured velocity data, it does provide a complete and alternate view of the ice motion at a continental scale.

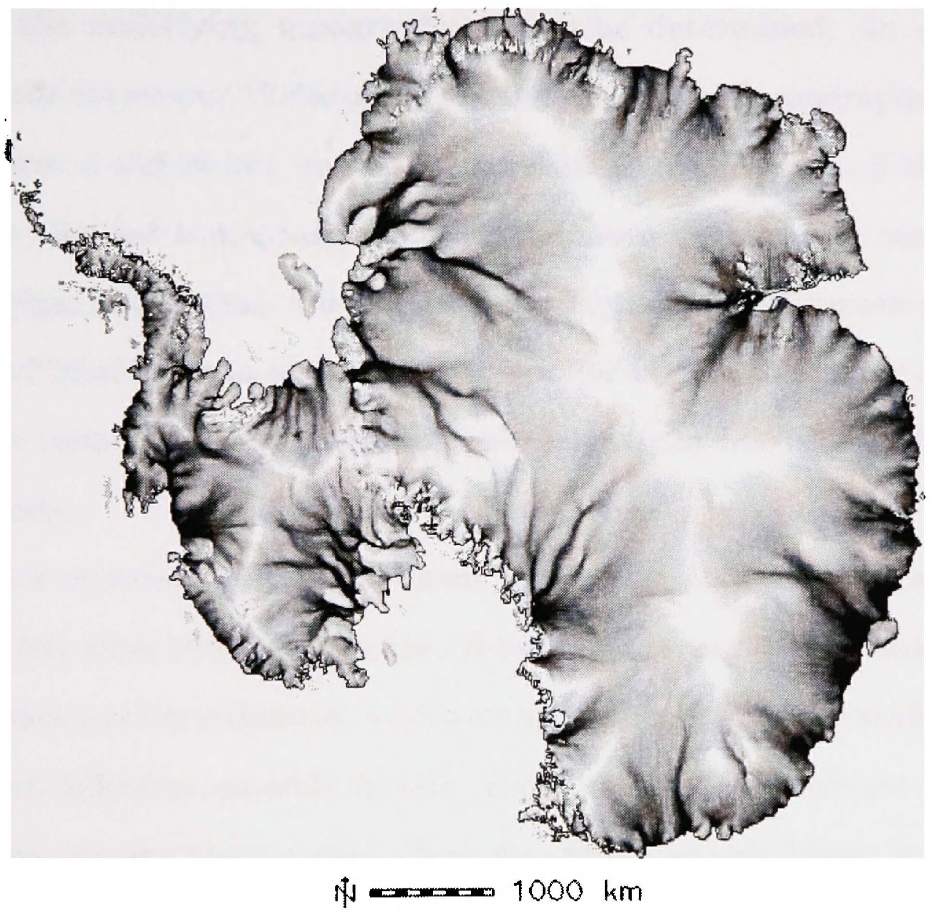


Figure 1.3 Balance velocity calculation. Darker is faster velocity.

## Antarctic analysis

With access to the most current and accurate data available and the river analysis tools provided by the open source community, the process turns into one of systematically breaking down the area in question into sub-basins spanning multiple spatial scales, analyzing the individual sub-basins, then plotting all the sub-basins data together to determine the overall dimension of the network.

This process is perfected and validated on multiple distinct river networks and finally applied to ice streams. However, before applying this procedure to ice streams, the impact of the underlying topography has to be determined. In short - Is the topography of the ice stream dictated by the underlying bed topography or do higher order effects have a significant impact? If the dominant features of the ice stream are dictated by the bed topography, it would be more beneficial to run the analysis on the bed surface rather than the ice surface. Significant differences exist between surface and bed analysis and show that, except for continental scale topographical features, higher order effects produce differences that can not be anticipated by bed only flow analysis.

Determining a system's scaling exponent is the first step. Once the exponent is known, we are left with several questions. How do ice streams of an unknown scaling exponent compare to river networks, with a known exponent? If the scaling exponents differ, does that difference provide insight about the status/structure of the overall network? What are the next steps? The remainder of this thesis is dedicated to performing the analysis and answering these questions.

# CHAPTER 2 METHODS

The focus of this research is to determine the dimensionality of ice stream networks. The term "dimensionality" in this context is taken to mean the scaling exponent that characterizes an allometrically scaled network. Determining dimensionality is not a straightforward procedure. Starting with an analysis of known, two-dimensional systems and comparing the results to an analysis of ice stream networks yields a process to determine the allometric scaling exponent. It is convenient to use Hack's Law exponent as a measurement of similarity since this can be easily determined from the dimensionality exponent (Maritan et al., 2002). This comparison acts as a litmus test for determining if river networks and ice stream networks exist primarily in the same dimensional space.

To complete this task, networks of ice streams and their respective drainage basins must be identified. This also requires a process that mimics the results found in Maritan et al., 2002. This process defines a mechanism for determining the dimensionality exponent of a channel network. Specifically, the dimensionality exponent is obtained by a log-log plot of many sub-basin catchment areas versus the sum of all *flowing medium*[1] in the sub-basin. This process uses the open source GIS tool Geographic Resources Analysis Support System (GRASS) to perform river network basin analysis and imports the result into MATLAB to aggregate and plot the basin data.

The first step is illustrating that ice stream networks can be reliably and accurately

[1]Flowing medium is defined as the material that is being transported by the network.

extracted using elevation data or velocity data. Before arriving at the final process, many efforts produced methods that were discarded, but are expounded upon here for completeteness. Included in these previous efforts are a user supervised hand-tracing process and an automated attempt at catchment delineation.

The user supervised drainage extraction attempts used ice-flow velocity data from RADARSAT interferometry (Joughin et al., 1999). Further attempts included the use of the Terraflow routing tool (Toma et al., 2003)[2] for drainage basin analysis which proved excessive for this application. Finally, the `r.watershed` (Neteler and Mitsova, 2002) module of **GRASS** was chosen as the primary tool of drainage basin and flow network extraction combined with MATLAB for the data manipulation and display.

In this chapter, each method is described and the potential or actual issues are identified. The results of the application of these methods are documented in Chapter Three.

## Verifying Hack's Law for ice streams

This research was initially performed in order to determine if the WAIS Ice Streams upheld Hack's Law. Prior to determining a more general approach, Hack's Law seemed like it would be enough to show the similarities between rivers and ice streams. Using gradually more sophisticated methods to determine basin catchment area and tributary length soon gave way to fully automated methods of drainage basin analysis. Methods of determining Hack's Law from manual measurement of the area and length

[2]Terraflow by default uses a Multi-Flow Direction (MFD) scheme where drainage from each cell is routed to multiple downslope neighbors proportionally dependant on the elevation gradient. While this produces smoother flow, tests showed that the additional computation time did not provide significantly better results, and given the uncertainty inherent in the elevation data, may not be better at all.

led to a more sophisticated computational method - finding network flow using least cost drainage analysis, and finding drainage sub-basins at multiple scales using a spectrum of initial conditions. Finally, the goal of finding Hack's Law exponent was supplanted by finding a more general network scaling exponent. Hack's Law exponent and this more general exponent are related in (Maritan et al., 2002).

The final result is a set of procedures that, when given an elevation dataset for an area, produces an allometric scaling exponent of the drainage network which corresponds to the network's dimension. Using these procedures, a critical examination can be performed on the similarities between river networks and ice streams.

Throughout, the goal has been to determine the similarities or differences between channel networks comprised of rivers versus those comprised of ice streams. A network's scaling exponent and Hack's Law provide the structure to perform a comparison.

## Supervised methods

Given the need to find the drainage basin area and the length of the longest tributary, it was determined as a good first step to manually draw the basin boundary. A system was needed that allowed the user to easily plot points on the basin boundary by studying elevation contours. MATLAB provided an environment in which to easily create a set of scripts allowing the user to view the velocity data of the ice streams as a backdrop to the elevation contours for better identification.

### Manually finding drainage area

The easiest approach was to delineate the basin catchment by selecting data points on the perimeter and using a convex hull method (Barber et al., 1997) on the resulting polygon to determine the catchment area.

The MATLAB environment allowed the user to manually inspect the velocity data (augmented with contour plots of the same area) to find the flow direction at each pixel. Using these data as a visual guide, selection points were placed on the boundary of the catchment area until the appropriate area was fully surrounded. Finally a convex hull algorithm was used to determine the outer points of the polygon and the area was computed using MATLAB.

The process consisted of three steps:

1. Manual inspection of velocity vector data augmented with contour plots to find the flow direction at each pixel

2. Approximate catchment by selecting points along the boundary

3. Sum the interior pixels of the resulting polygon

This visual approach is more difficult than it first appeared. Using only velocity to find catchment area is difficult due to the fact that velocity vectors are sparse and a ridge boundary may lie between two measurements. Other visual aids helped, such as displaying the velocity as a 3-D surface, but ultimately this method was subject to too much human influence. As shown in Figure 2.1 and Figure 2.2 , the results were widely varied[3]. In general, as the basin area got smaller, finding the boundary became more problematic. At a smaller scale, any missing data has a much more profound impact on the amount of human judgement that has to enter the basin definition. Hence, the basin areas became even more suspicious at smaller scales.

The overall boundary area was susceptible to a wide range of interpretation and it was difficult to get reproducible results. While most of the calculated values for Hack's Law seemed reasonable (between 0.52 and 0.65), since the analysis of Hack's

[3]Due to the subjective nature of the process, a user could perform successive runs on the same region, but produce results that differed by as much as 20% in total area.

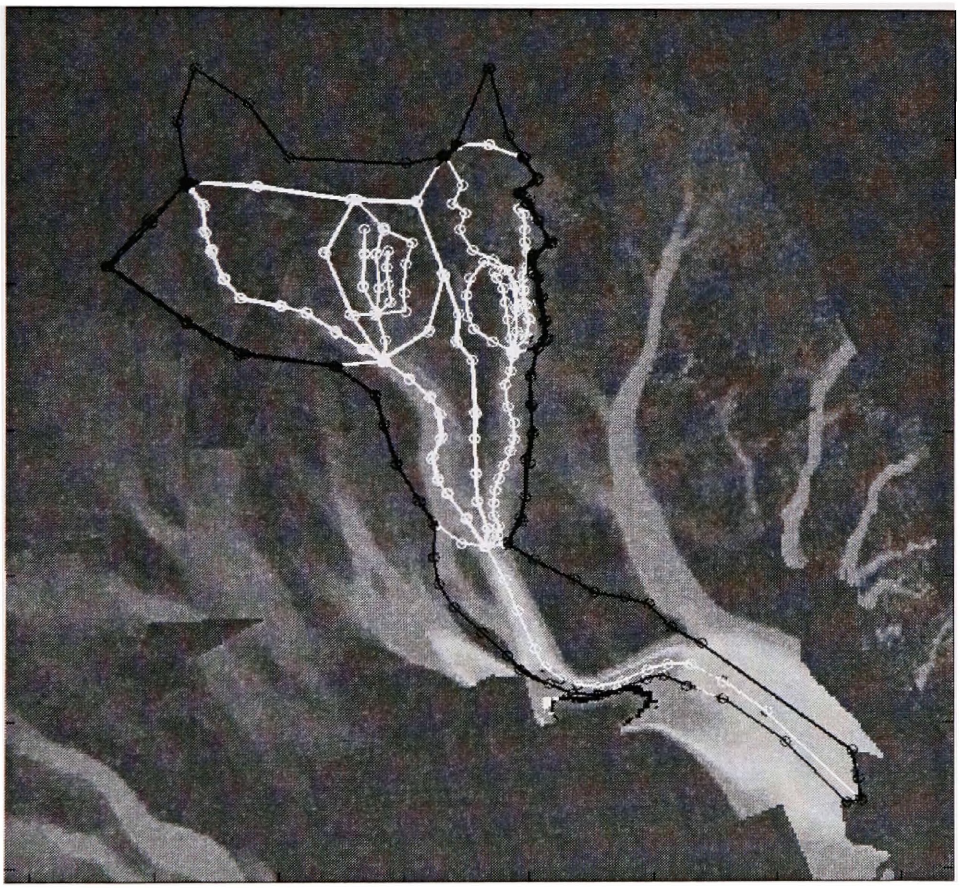

Figure 2.1 Manual basin trace one

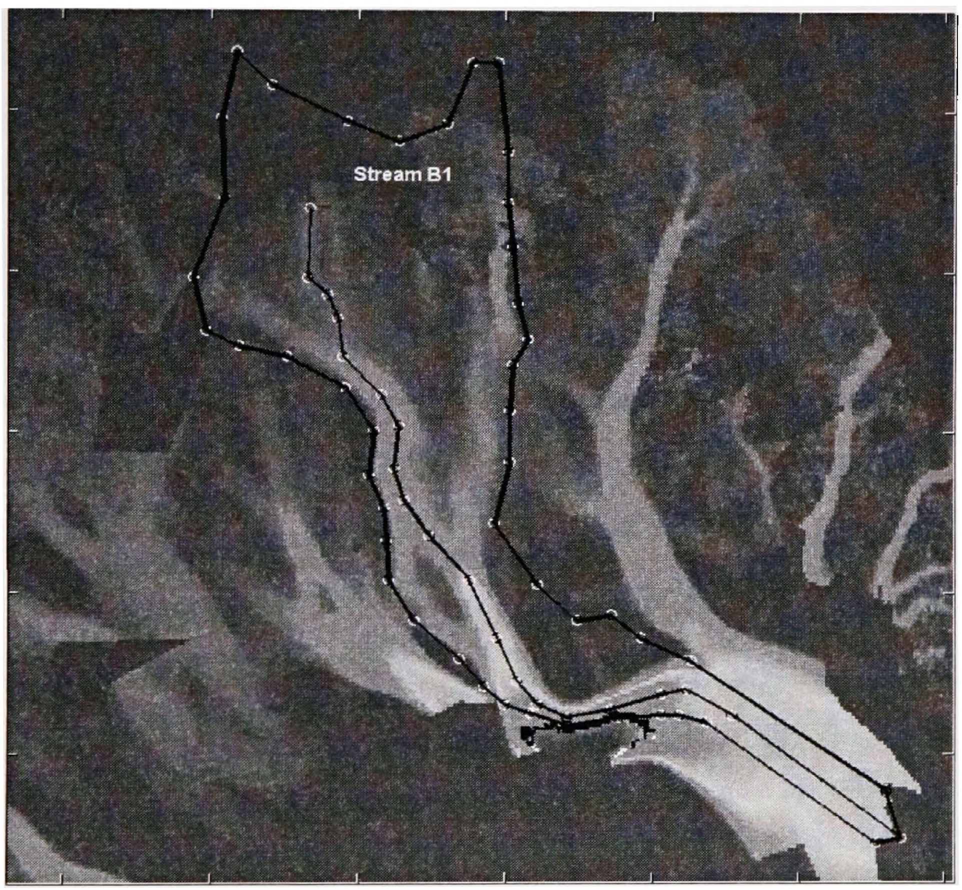


Figure 2.2 Manual basin trace two

Law on ice streams had not been performed before, there was no way to validate the accuracy of the results. Even though this method met with partial success, it was discarded in favor of a more automated approach, that would remove the subjectivity in evaluating basin boundaries.

### Manually finding tributary length

Early attempts at finding the longest tributary in a drainage basin included tracing up the center of the ice stream velocity by hand. With the assistance of flow vectors, a hand drawn line could be traced up the velocity channel with a certain amount of accuracy. Due to gaps in the data the traced tributary routes would always have an error factor associated with them. The results were also very susceptible to human interpretation since no exact measurement of the length could be made. Small variations in the stream would produce different exponents and could occasionally change which tributary was the longest.

Other functions such as contouring and displaying the surface in 3D were helpful, but ultimately this method proved too problematic. Fundamental difficulties were introduced by human judgment and the manual nature of input, so the process and results were discarded.

As shown in Figure 2.3 the results from a hand trace are crude and have the potential for drastic differences depending on how the data are interpreted.

### Semi-supervised catchment determination

Another attempt used to determine a basin's catchment area employed a more automated approach, but still maintained a user supervised component. The user would manually trace the ice streams on top of the velocity map, essentially providing a single pixel skeleton of the network along the center of the streams to their

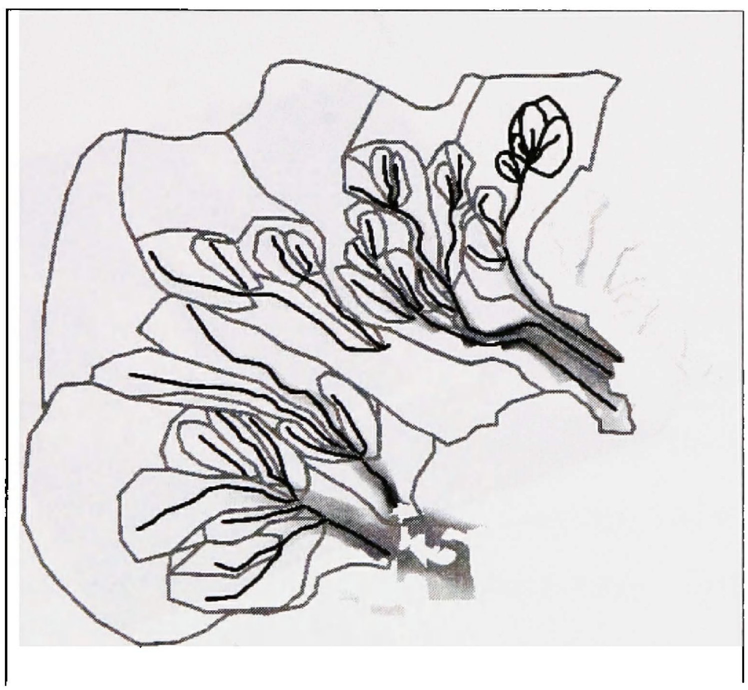

Figure 2.3 Manual determination of WAIS ice stream catchment areas

headwater, see Figure 2.4 [4]. This function still suffered from the problems of the previous unsupervised methods.

Due to missing data in the trace set, the basin detection step had to first thicken the network skeleton to jump over any gaps. Ultimately this thickening became a fatal flaw. Once the stream was thicker than a single pixel, it became very difficult to determine the endpoints and to trace to those endpoints without tracing sideways and backwards. This artificially added length to the tributaries, resulting in the abandonment of this method[5].

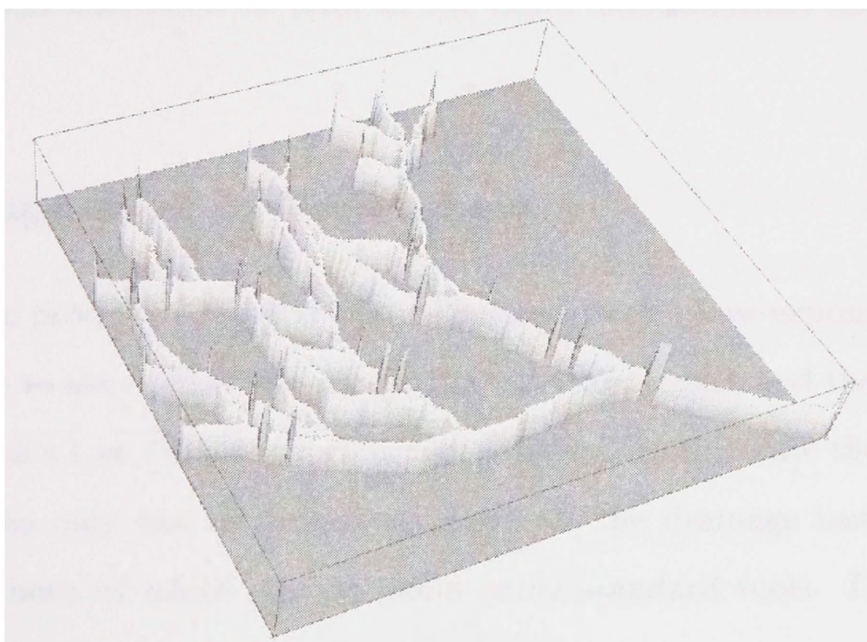

Figure 2.4 Automated discovery of tributary length

Once all endpoints were found, a systematic trace from every end point to every other endpoint would reveal the longest tributary in the system, keeping in mind flow

[4]The spikes indicate possible tributary endpoints. A path between every peak to every other peak is measured in the pathfinding step to determine the longest distance. Area units are in pixel units.

[5]This approach might still have merit if a path finding algorithm such as A* (Pearl, 1984) were to be used, however, more automated and standardized methods were discovered.

direction[6].

### Other methods

Other possible methods of determining catchment boundaries that were not attempted include starting at any downstream point and recursively tracing the path consisting of the steepest ascending elevation, essentially "walking" up the drainage basin ridge. Another method would be to use repetitive random walk analysis combined with velocity and elevation data (Price and Whillans, 1998). These approaches, while viable, were surpassed in favor of the basin determination functions found in `r.watershed`.

## Alternate approach to Hack's Law

The efforts to produce a method that calculated Hack's Law exponent exactly were abandoned due to the difficulties calculating tributary length and the relative ease of calculating Hack's Law from network dimensionality. To calculate the dimensionality of a system, the only two quantities required are the drainage basin area and the accumulation, both of which can be found using standard tools. Determination of the main tributary length is no longer a necessity.

### Dimensionality

As mentioned previously, recent research (Banavar et al., 1999) suggests that the dimension $D$ of a system can be used to determine Hack's exponent in spanning tree networks. Given that a system is directed and efficient and exists in D-dimensional

[6]The approach was abandoned before this result could be fully determined.

space, how can we determine D? Banavar et al., 1999 suggests that the theoretical minimum of D for such systems is determined by:

$$C \sim M^{\frac{D}{D+1}} \tag{2.1}$$

where $C$ is the *flow (metabolic)*[7] of the network and $M$ is the volume of "flowing medium" (Dreyer, 2001)[8] in the system.

For our purpose it is convenient and equivalent to invert the function and replace the variables with the river network equivalents to obtain from (Equation 2.1) :

$$M \sim A^{\frac{D+1}{D}} \tag{2.2}$$

For the case of river networks, $A$ is taken to be the area drained and $M$ is equivalent to drainage accumulation (Banavar et al., 1999). Using (Equation 2.2) , Banavar et al., 1999 shows that river networks exist in two dimensions - the theoretical minimum exponent is $\frac{3}{2}$ or 1.5 (Banavar et al., 1999). It is important to note that every point in a drainage system has an associated $M$ and $A$ - Every point has an amount that flows through it, and every point also has an upslope area drained, even if both amounts are unit amounts.

The *dimensionality* exponent can be defined from (Equation 2.2) as:

$$\alpha = \frac{D+1}{D} \tag{2.3}$$

The dimensionality exponent $\alpha$ can then be determined by examining the slope of the line described by the log-log plot of the total accumulation $M$ (Equation 2.5)

[7]This quantity can be taken to mean either metabolic rate in animals (Kleiber's Law) or river flow rate (Dreyer, 2001).

[8]Defined as blood, water, or whatever medium for which the network is providing transport.

versus the total contributing area $A_x$ (Equation 2.4) .

$$A_x = \sum_{z \in nn(x)} A_z + 1 \tag{2.4}$$

where $A_x$ is a recursive quantity that includes the entire area connected to, but not including, the outlet pixel[9] (Banavar et al., 1999). One area is connected to another area if there exists a steepest descent drainage direction from one point to the other. $nn(x)$ denotes the nearest upslope neighbors of site $x$ (*i.e.*,neighbors that drain into site $x$). $A$ can also be obtained by finding the drainage basin boundary, adding a unit area for each element in the drainage, then subtracting one unit area from the total to account for the spillpoint (Maritan et al., 2002).

Next, to determine the flow accumulation for the sub-basin, $M$:

$$M = \sum_{z \in \gamma} A_z \tag{2.5}$$

where $M$ is the accumulation of flowing medium in the sub-basin flowing into the spillpoint and $\gamma$ is the set of all pixel areas that eventually flow through the spillpoint. The spillpoint area and accumulation are not included in the calculations to act as a scaling correction (Maritan et al., 2002). $M$ may also be thought of as the sum of fluxes over every site in the drainage basin (Dreyer, 2001).

Knowing the dimensionality exponent $\alpha$, Hack's exponent $h$ is easily derived using (Equation 2.6) (Maritan et al., 2002):

$$h = \alpha - 1 \tag{2.6}$$

The dimensionality approach to determining Hack's exponent is provided in Ba-

[9]The outlet pixel is also known as the "spillpoint".

navar et al., 1999. A comparison of results from traditional methods of determining Hack's exponent and the new dimensionality method is performed in Martitan et al., 2002 and indicates that the processes produce equivalent results.

### Applying this approach to ice streams

Hack's exponent for ice stream networks is a relatively unknown quantity - their allometric nature has not been studied. This approach provides a geometrically based process by which the dimension of ice stream networks can be explored and Hack's exponent may be found for these streams. This approach is also free from the human interpretation error that previous attempts included.

## An automated process for determining system dimension

Since every point in a drainage network is associated with a spatially distinct catchment area[10], it is necessary to evaluate the basins at many different scales to determine the overall scaling factor. Some basins are encompassed by other larger basins, but the allometric nature of the overall system is defined as an aggregate of all basins, and so remains relatively constant. To gather basin data at multiple scales, an automated process was needed.

### GIS methods

**GRASS** GIS provides half the functionality needed to asses dimensionality. The `r.watershed` subprogram uses the term *threshold* to indicate the minimum number of pixels per basin. For instance, if a large threshold value is used, the result is a small

[10] A point that has no upslope neighbors is in a watershed consisting of only a single unit area or a point on the ridgeline between watersheds.

number of sub-basins. To provide access to multiple spatial scales, the process must include analysis at multiple threshold values that span at least an order of magnitude.

Each threshold value produces a patchwork of basins (the exact number depends on the topography of the area being analyzed and the threshold value itself). Two maps are needed, the basin map and the associated accumulation map. The `r.watershed` module of **GRASS** provides both of these maps. A script powers the automated process wherein many basins[11] at multiple spatial scales are generated and analyzed one at a time.

## Overall process

The process requires a drainage and accumulation analysis step (performed with GRASS) and a data aggregation and plotting step (accomplished with MATLAB).

[11] Some of the analysis includes 3500 or more basins and sub-basins.

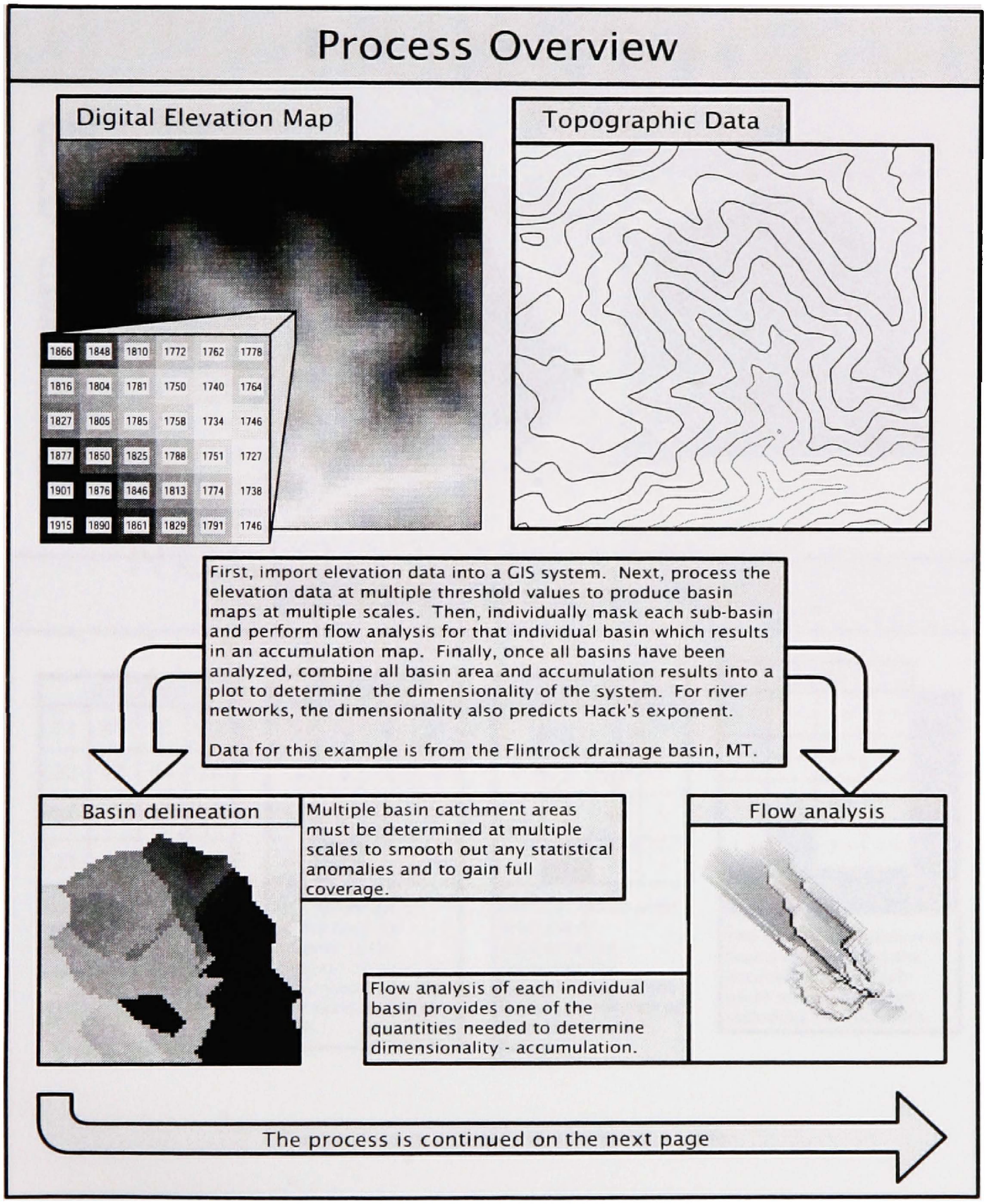


Figure 2.5 High level process overview

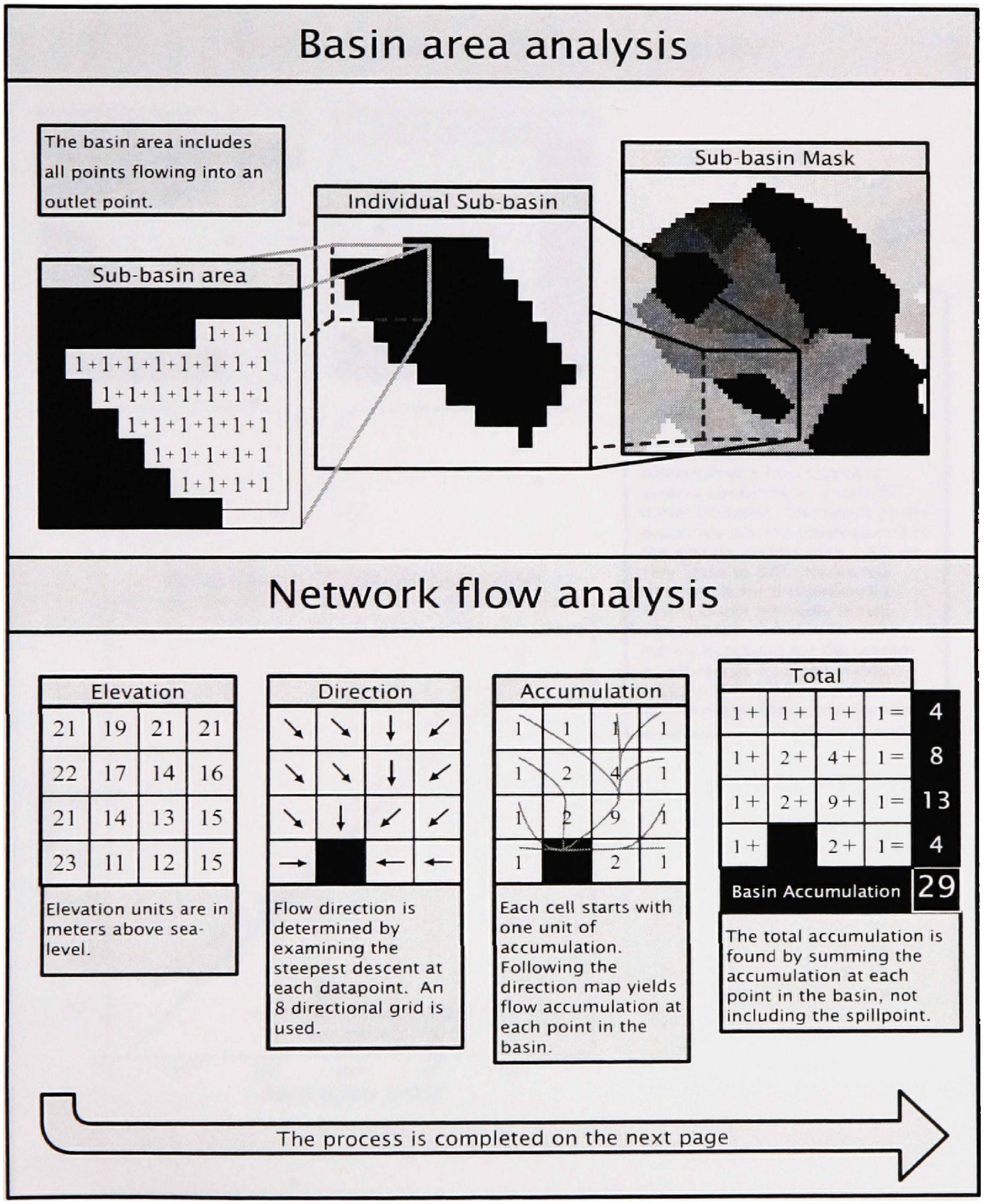


Figure 2.6 High level process overview (con't)

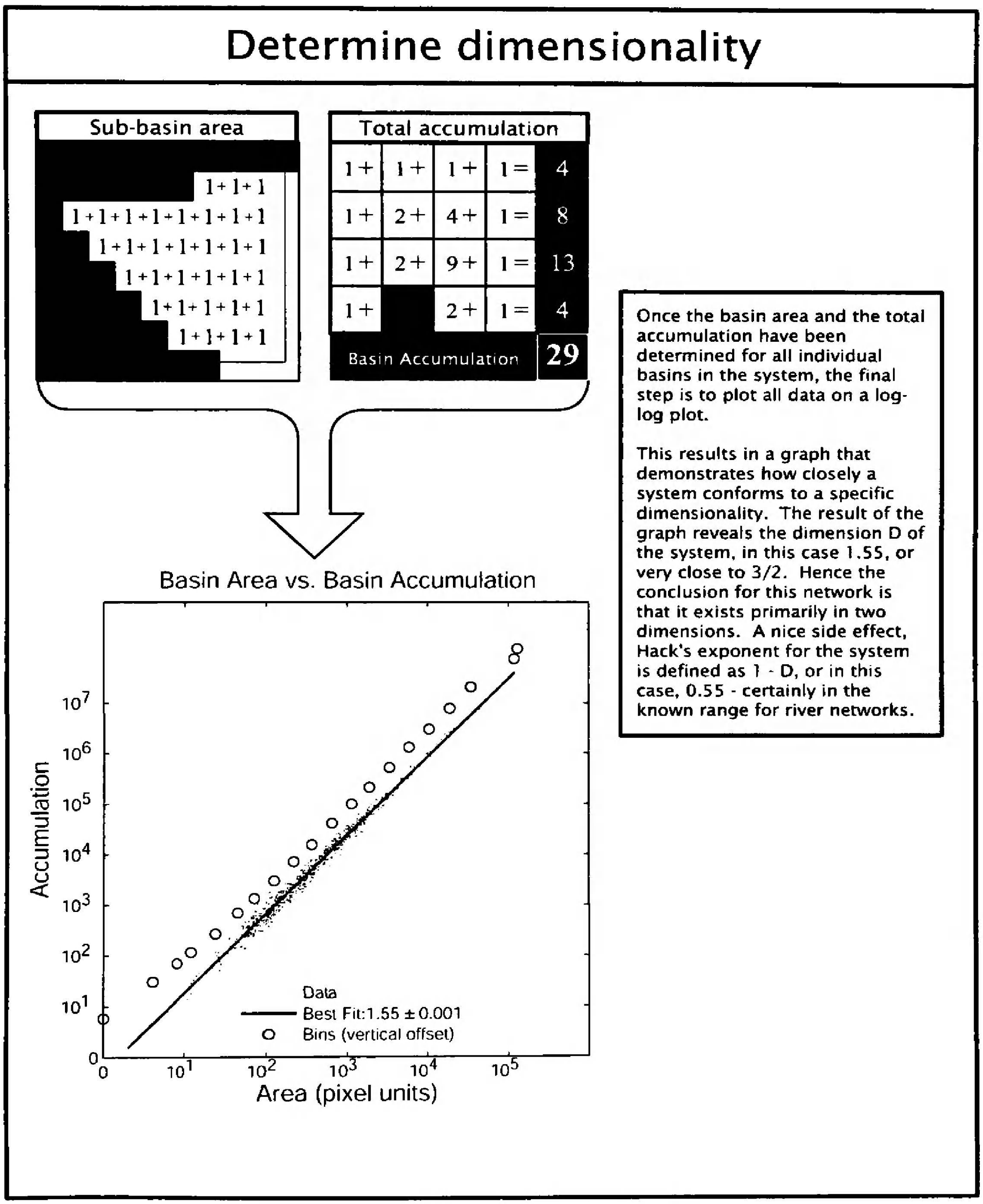


Figure 2.7 Flow analysis algorithm overview

### GRASS - Determine catchment areas and accumulations

#### Drainage basin map accumulation

The **GRASS** module `r.watershed` takes as input an adjusted[12] elevation map and a threshold value. It produces a basin map that contains unique basins from that elevation data. Each basin is identified with a unique color value where all pixels of that color belong to that basin. The basins can be processed individually by using another **GRASS** function, `r.mapcalc`, to prepare a MASK that includes only a single basin. This process is shown in Figure 2.6 .

The drainage network map is generated by repeatedly overlaying smaller sub-basins on a surface until the entire drainage system is covered. Any "no data" values have been either interpolated or masked prior to performing the analysis.

#### Drainage flow accumulation map generation

To perform the accumulation map generation, again `r.watershed` is again invoked. Accumulation is calculated for every individual basin over all spatial scales and saved to a file that MATLAB can import. The accumulation process is shown in Figure 2.6 .

### MATLAB - Combine basin data into graphs

Finally all the generated files are imported one at a time into MATLAB where the accumulation map values are summed and the drainage basin area calculated according to (Equation 2.5) and (Equation 2.4) , respectively.

[12] Elevation maps sometimes contain artifacts called *pits* that form a non-escapable flow pattern. A standard practice to keep the algorithm from getting stuck in these pits is to fill the pits before attempting any flow routing (Toma et al., 2003). This fill procedure is performed on the input elevation data.

Each basin's accumulation and area results are stored in co-registered arrays and a map of the data is continually updated. The final graph is a log-log plot of area versus accumulation with the slope of the best fit line (the dimensionality exponent $\alpha$) printed with its uncertainty as well. Binning is included to help the eye follow the trend.

## Error analysis

A general error analysis is performed noting where errors might exist in the process. Data fitting error analysis is systematically performed on every graph.

### Data fitting uncertainty

The data generated is fitted with a line using the least squares method. Least squares fitting allows for an estimation of the error to be performed[13] (Gould and Tobochnik, 1996). For each basin system analyzed, the least squares fitted line with error estimation is included.

### Network estimation error

The largest source of uncertainty with this process is associated with the simple elevation model for finding ice stream networks. An elevation only approach is a widely accepted method for determining ice flow; however, there are many cases where it produces obvious error.

In addition, the underlying data may not be entirely accurate. The RAMP 200M elevation data (Liu et al., 1999) used for Antarctica is the most current and accurate available, but its accuracy is derived from an ensemble approach. Multiple data sets

[13]The error analysis method is that described in Gould and Tobochnik, 1996 pg. 204.

Figure 2.8 Joughin velocity overlayed on RAMP DEM

Figure 2.9 Generated ice stream network overlayed on RAMP DEM

were used to create the RAMP data and the error associated with each data set is partially propegated into the RAMP dataset. Many improvements have been made, but undoubtedly there are still some errors in the data[14].

### Initial conditions

The process is automated and therefore most error introduced by human ambiguity has been removed. There are, however, two sources of human interpretation, the definition of the areas to be analyzed and the threshold values to be used to generate the basins.

In order to analyze a basin, the total drainage basin must be encompassed within the elevation map. If too small an area is defined (*i.e.,*some of the basin is outside the map) then the entire basin will not be analyzed. If too large an area is defined, there will not enough coverage of the upper threshold scales. The area size should be no larger than 1,000,000 pixel units.

The process has the potential for introducing error as well. The threshold values are entered manually and are not a theoretical best coverage of the spatial scales. This in combination with the area definition issue could lead to a small area not being covered by an adequate number of spatial scales.

The resolution at the present time is to ensure than only drainage basins with more than 60,000 and less than 1,000,000 total pixels are analyzed.

The data requirements are not too high. For example it allows basins spanning in size from 200x300 to 1000x1000.

[14]For a full analysis of the error in RAMP see the National Snow and Ice Data Center's website at http://nsidc.org/.

### Monte Carlo selection

At small threshold values, it is possible to generate so many basins so as to weight the aggregate of the data towards basins at that spatial scale. To avoid this weighting, a Monte Carlo selection is performed for threshold values where more than 350 unique basins are generated. Since a machine's randomize function can't be truly random, an argument exists that the data might still be weighted. However, since the basins are mutually exclusive[15], even without random sampling, the only weighting that could occur would be towards basins in a relatively tight geographical sample of the input map.

Providing the selection of basins with a pseudo-random shuffling is good enough to generate a data set that is not weighted too heavily towards a specific spatial scale. In practice, the Monte Carlo sampling only activates if the elevation map is analyzed at a relatively small threshold value. In general, the larger the threshold value, the larger the basins get. Consequently, fewer basins would be required to provide full coverage of the area being analyzed.

### Data interpolation

Data for the river network analysis was obtained from different locations, most notably, the Seamless Data Distribution System[16] maintained by the USGS and Geocommunity[17]. Before performing analysis on these datasets, a nearest neighbor interpolation was run to fill in any missing pixel values. This interpolation might alter small tributaries flow.

In trial runs with and without interpolation the difference was not detectable.

[15] By definition a point belongs to a single drainage basin, and basins cannot overlap.
[16] http://seamless.usgs.gov/website/Seamless/
[17] http://www.geocomm.com/

# CHAPTER 3 RESULTS

The analysis of river basin data includes data from four geographically distinct river drainages. All drainages closely corresponded to the predicted $\frac{3}{2}$ dimensionality. The ice stream drainage analysis breeches the theoretical threshold for a two dimensional system ($\frac{3}{2}$) and so must be thought of as a higher order system.

Some elevation data was obtained from the USGS maintained Seamless Data Distribution System (SDDS) [1]. Although the SDDS offers many options for elevation data, only the data that was produced by the 2000 Shuttle Radar Topography Mission (SRTM) elevation dataset is used. These datasets have a 30-meter resolution, and are horizontally accurate to within 20-meters with a vertically relative accuracy of less than or equal to 10m.

Data was also obtained from Geocomm[2]. These datasets are at 30-meter resolution with a vertical accuracy error of up to 30 meters. Geocomm reports a horizontal accuracy over 90% of the data within 7-meters, with the remaining 10% being accurate between 8-15 meters.

Greenland ice thickness, bed surface elevation and ice surface elevation were obtained from the National Snow and Ice Data Center (NSIDC). The resolution of the data obtained for Greenland is 5 Km per pixel. These data are vertically accurate to within 100-meters (Bamber et al., 2001a) (Bamber et al., 2001b).

[1] http://seamless.usgs.gov
[2] http://www.geocomm.com

Antarctica elevation was obtained from the NSIDC RADARSAT Antarctic Mapping Project Digital Elevation Model Version 2 (RAMP)[3]. This data consists of an amalgam of mapping projects that were combined to produce a continuous surface. The accuracy of the areas of interest are within 15-meters for steeply sloped coastal regions, and within 7.5-meters for the gently sloping interior ice sheet (Liu et al., 1999).

The bedrock surface elevation data of Antarctica was obtained from BEDMAP [4]. BEDMAP contains data from over 100 distinct bed mapping expeditions which used various techniques to gather the data, hence the error estimates for the overall dataset are difficult to determine and widely varying depending on the technique used. The BEDMAP consortium has not yet released accuracy estimates for the entire dataset.

For most ice stream basins, there are many places where there is not a steep enough gradient to cause a channelized flow to form. A fluvial channel network is not necessarily space filling (Montgomery and Dietrich, 1988) and hence many areas are simple hillslope un-channelized flow. These areas are obvious on the basin maps, and many times are indicated by a small linear feature in the graph of the basin. These linear features indicate that many sub-basins at various scales are simply straight flowlines downslope without coalescing into a channelized network.

Using the methods described in Chapter 2, the final analysis of drainage basins was performed and the results are presented below.

## River network analysis results

See Appendix A for all river basin maps.

[3] http://nsidc.org/data/nsidc-0082.html
[4] http://www.antarctica.ac.uk/aedc/bedmap/

### Blue Mountain, MT

Blue Mountain, Montana has many tributaries, none of which maintain water the full year. These channels are subject to yearly runoff erosion. Elevation data was obtained from Geocomm.

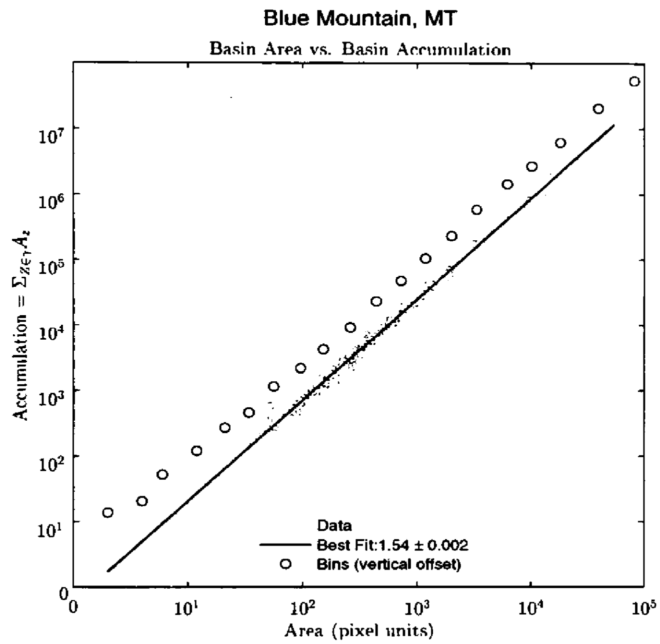


Figure 3.1 Blue Mountain, MT analysis

### Flintrock Drainage Basin, MT

The Flintrock drainage is subject to high runoff and erosion due to the yearly cycle of snowfall. It also has been heavily glaciated. Elevation data was obtained from SDDS.

### Saco River Headwater, NH

The Saco River empties into the Atlantic Ocean and its headwater is in the White Mountains spanning Maine and New Hampshire. Elevation data was obtained from

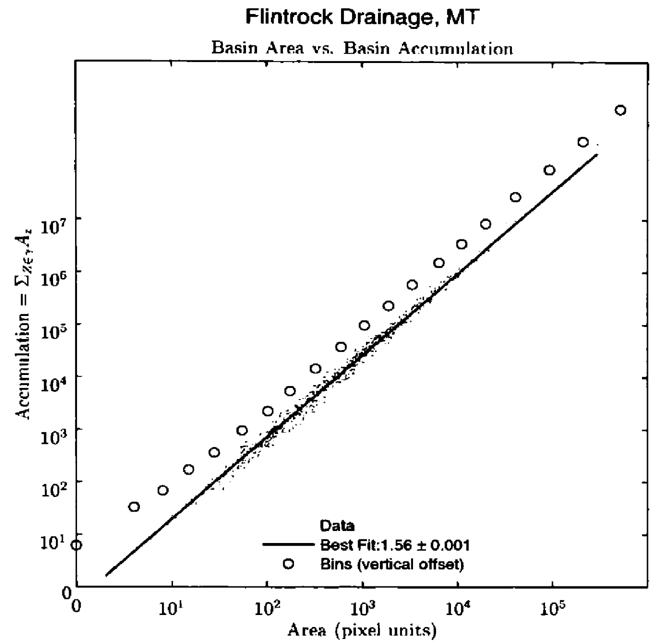


Figure 3.2 Flintrock drainage, MT analysis

SDDS.

### Cow Creek, KY

Cow Creek is a tributary of the Red River in Kentucky. Elevation data was obtained from SDDS.

## Ice stream network analysis results

### Western Antarctic Ice Streams

#### Ross Ice Shelf

The question exists whether the flow direction for the ice streams on the Ross Ice Shelf is dictated by the underlying bed topography. To demonstrably show that flow is not dictated by bed topography, both the ice surface elevation and the bed elevation were used to generate different flow networks. As shown in Figure 3.5 and Figure

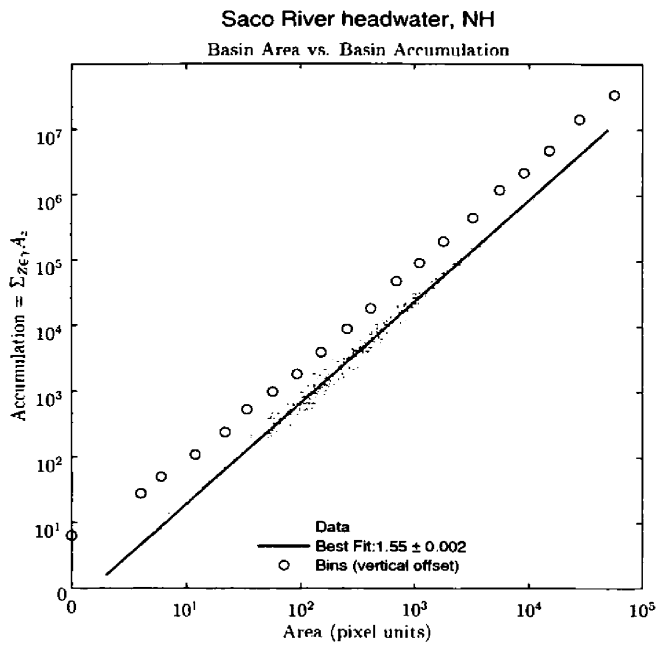


Figure 3.3 Saco River, NH headwater analysis

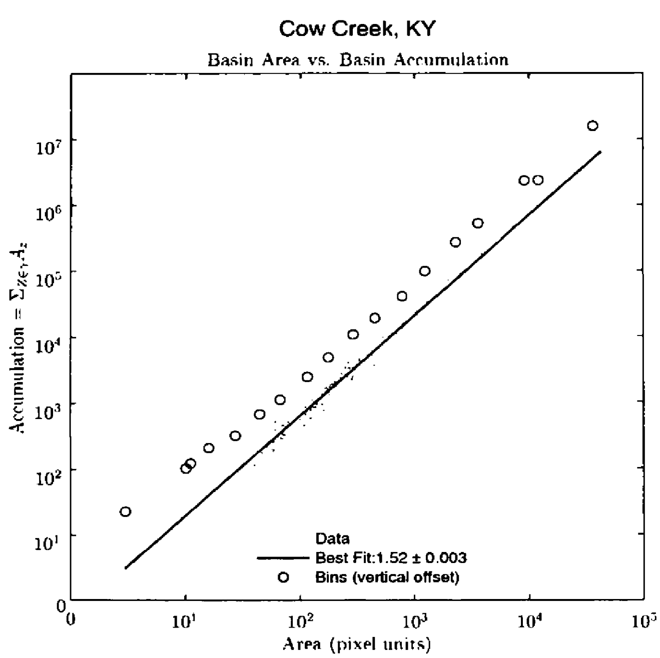


Figure 3.4 Cow Creek, KY analysis

3.6 , they differ in many significant features, the most significant being that the bed elevation network conforms to Hack's Law for river networks, while the ice stream network indicates that a high dimensional process is occurring. Surface elevation data was obtained from RAMP, and bed elevation was obtained from BEDMAP.

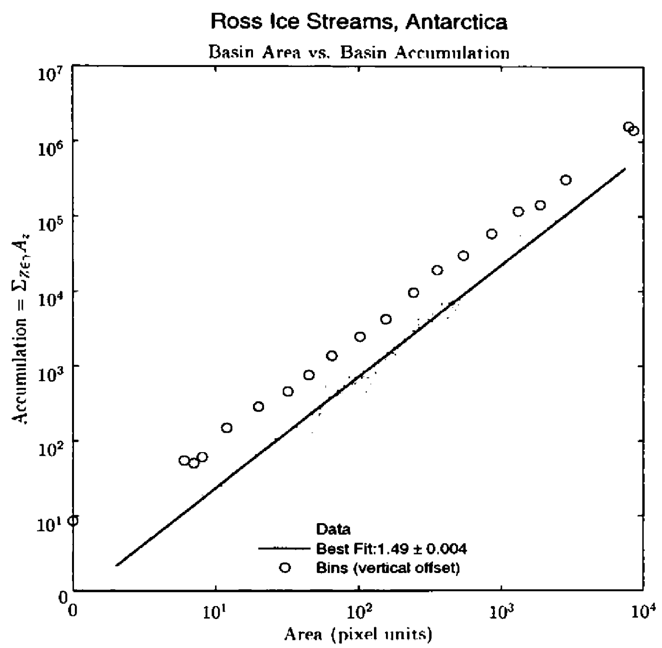


Figure 3.5 WAIS ice streams surface Elevation analysis

An analysis of individual streams in WAIS follows:

**WAIS - Van der Veen Ice Stream (Ice stream B1)** Ice stream B1 is the right fork of Ice stream B as looking up-glacier. Surface elevation data was obtained from RAMP.

**WAIS - Whillans Ice Stream upper (Ice stream B2)** Ice stream B2 is the left fork of Ice stream B as looking up-glacier. The dimensionality exponent for this ice stream is surprising. The results for this stream are within the normal value for river networks. In the case of B2, which might be involved in basal melt water piracy

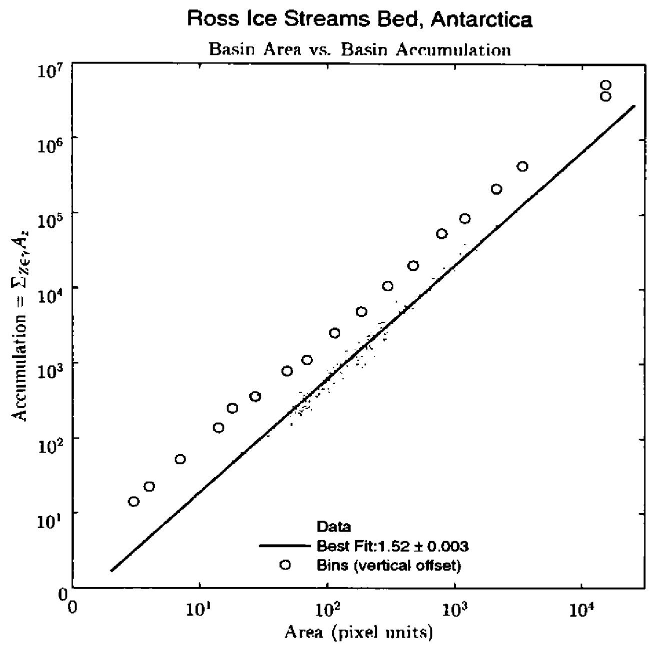


Figure 3.6 WAIS ice streams bed Elevation analysis

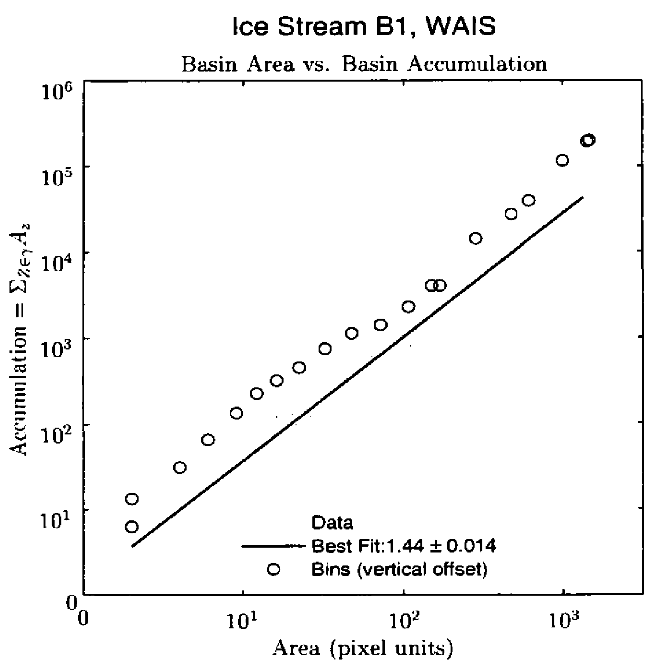


Figure 3.7 Van der Veen Ice Stream basin

from upper Kamb (Ice stream C) (Anandakrishnan and Alley, 1997), the elongation of the drainage network may make the network perform more like a river network. There is a strong linear feature present in this graph dur to the minimal gradient present in large portions of the drainage basin as seen in the bed figure. Surface elevation data was obtained from RAMP.

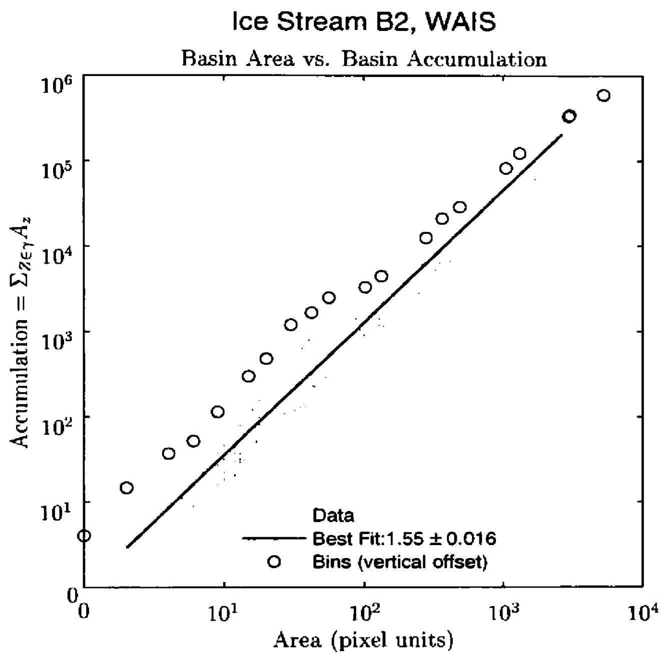


Figure 3.8 Whillans Ice Stream upper basin

**WAIS - Whillans Ice Stream lower (Ice stream B)** The overall character of Ice stream B is heavily influenced by the contribution of B2. Surface elevation data was obtained from RAMP.

**WAIS - Kamb Ice Stream (Ice stream C)** Surface elevation data was obtained from RAMP.

**WAIS - Bindschadler Ice Stream (Ice stream D)** Surface elevation data was obtained from RAMP.

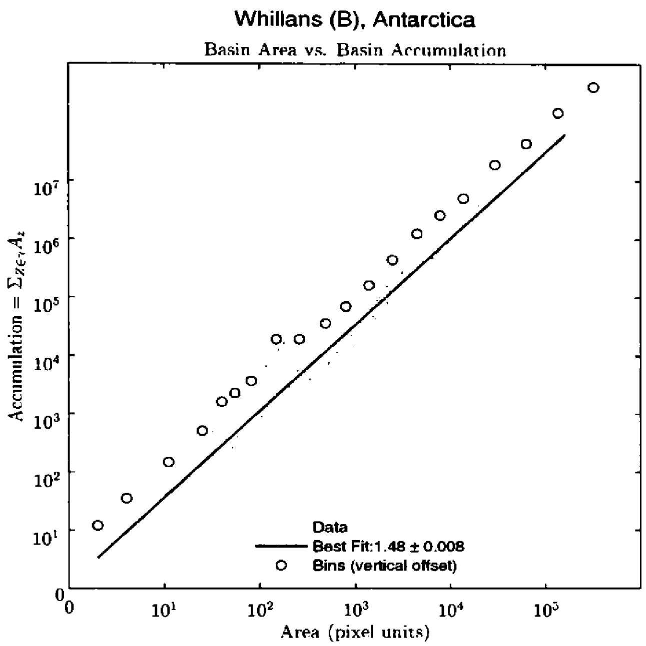


Figure 3.9 Whillans Ice Stream basin

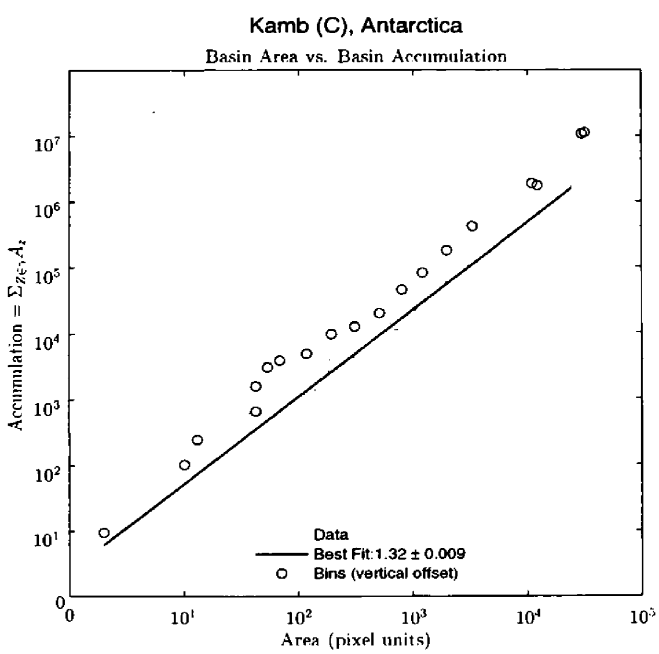


Figure 3.10 Kamb Ice Stream basin

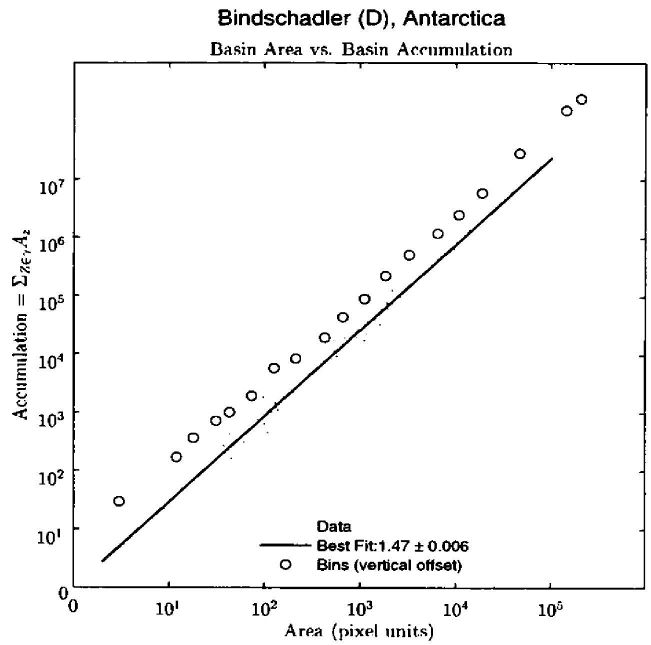


Figure 3.11 Bindschadler Ice Stream basin

**WAIS - MacAyeal Ice Stream (Ice stream E)** Surface elevation data was obtained from RAMP.

### Pine Island Glacier

Similar to the Ross Ice Streams, Pine Island Glacier provides an opportunity to examine the flow network defined by the surface of the ice as compared to a drainage network generated from the bed elevation. It reveals similar differences as the Ross Ice Shelf, the bed topography dictates a network that conforms to Hack's Law, while the actual Pine Island Glacier network does not correspond to Hack's Law, and instead tends towards a system of higher dimension. Surface elevation data was obtained from RAMP, bed elevation from BEDMAP.

The channels generated are very similar to previous research (Vaughan et al., 2001).

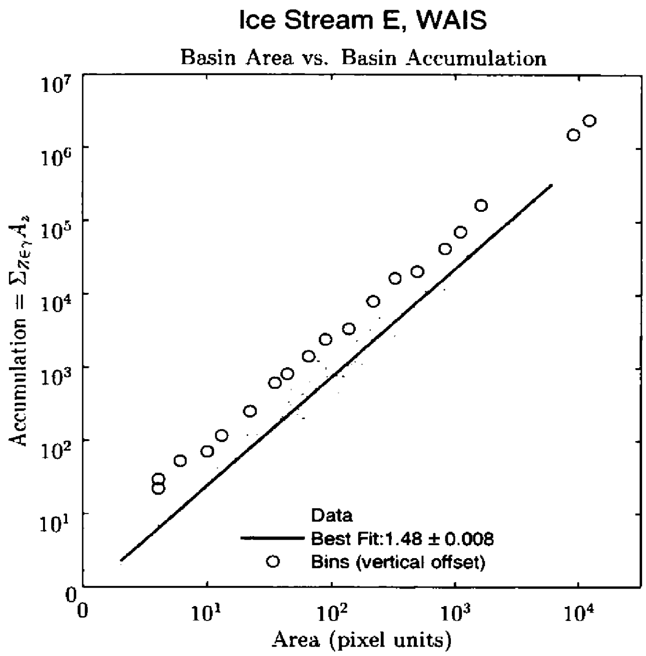


Figure 3.12 MacAyeal Ice Stream basin

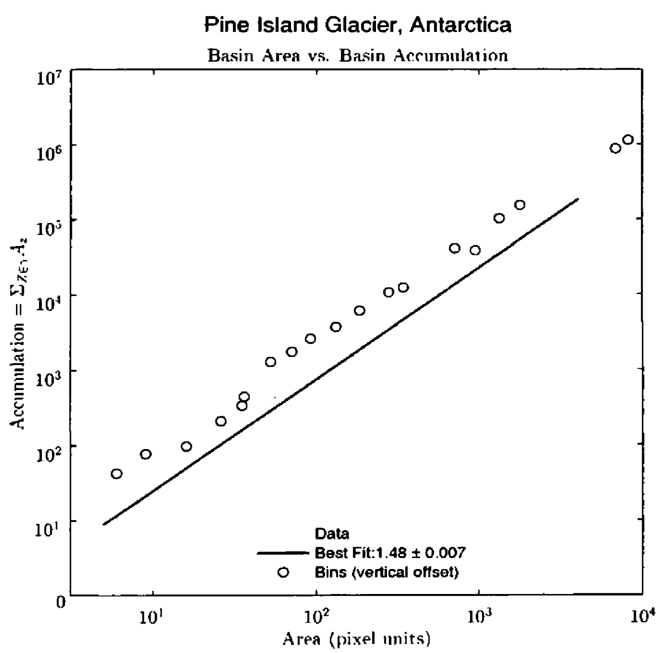


Figure 3.13 Pine Island Glacier analysis

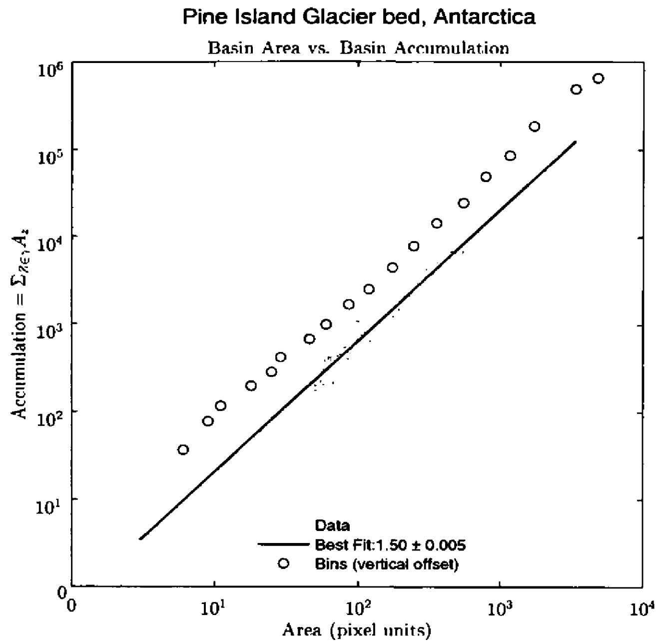


Figure 3.14 Pine Island Glacier bed analysis

### Thwaites Glacier

Along with the Pine Island Glacier, Thwaites glacier is one of the fastest moving ice streams in Antarctica, so examining the flow of these two glaciers is warrented as they are general indicators of the overall status of ice flow. Surface elevation data was obtained from RAMP, bed elevation from BEDMAP.

### Amery Ice Shelf

Surface elevation data was obtained from RAMP.

### Ronne-Filchner

Surface elevation data was obtained from RAMP.

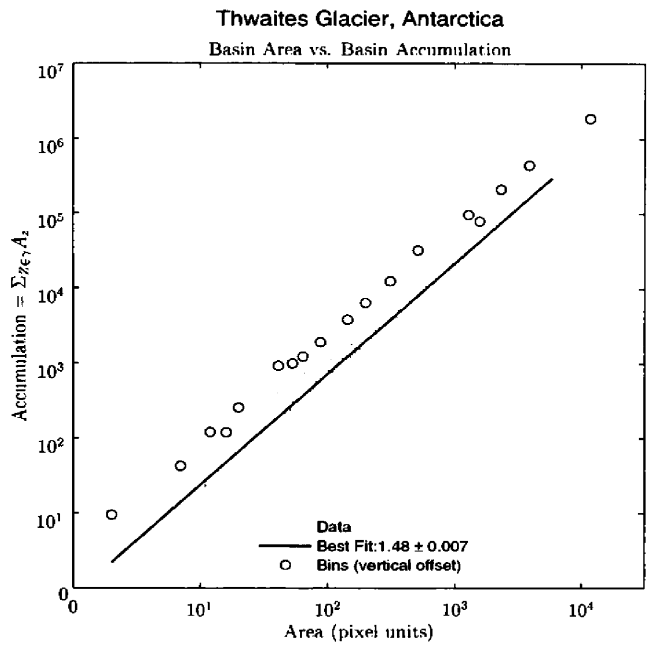


Figure 3.15 Thwaites Glacier analysis

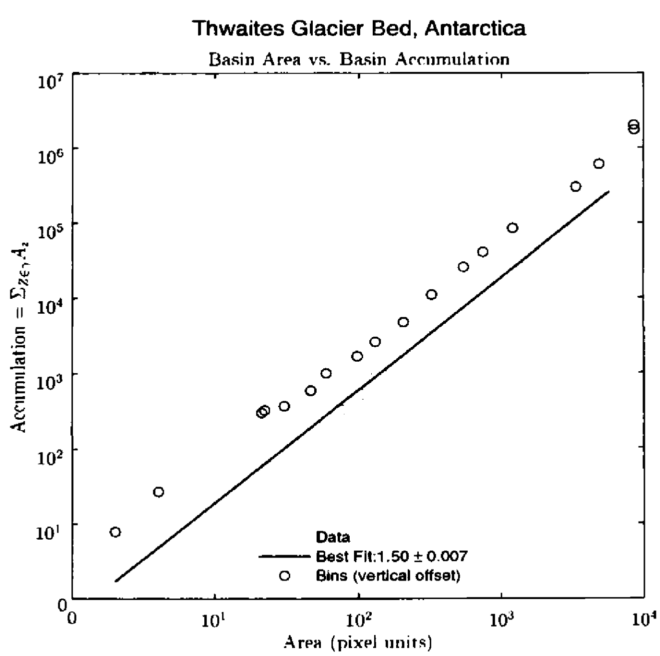


Figure 3.16 Thwaites Glacier bed analysis

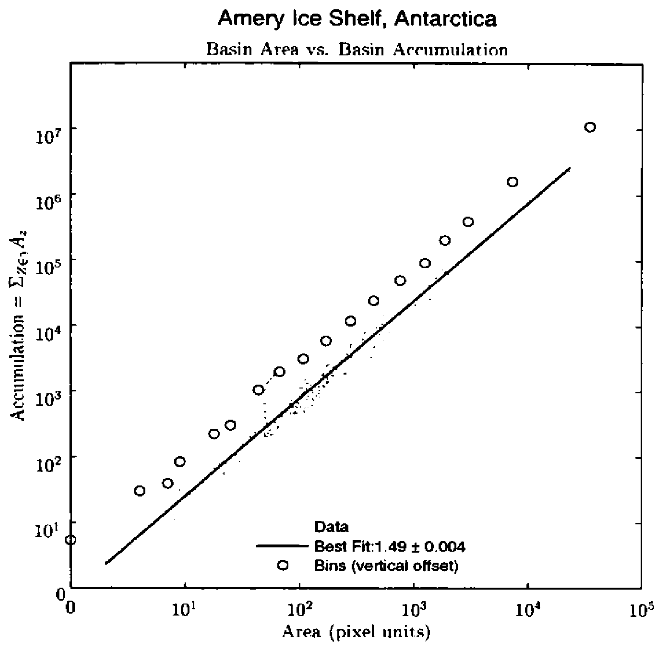


Figure 3.17 Amery Ice Shelf (Lambert Glacier), Antarctica analysis

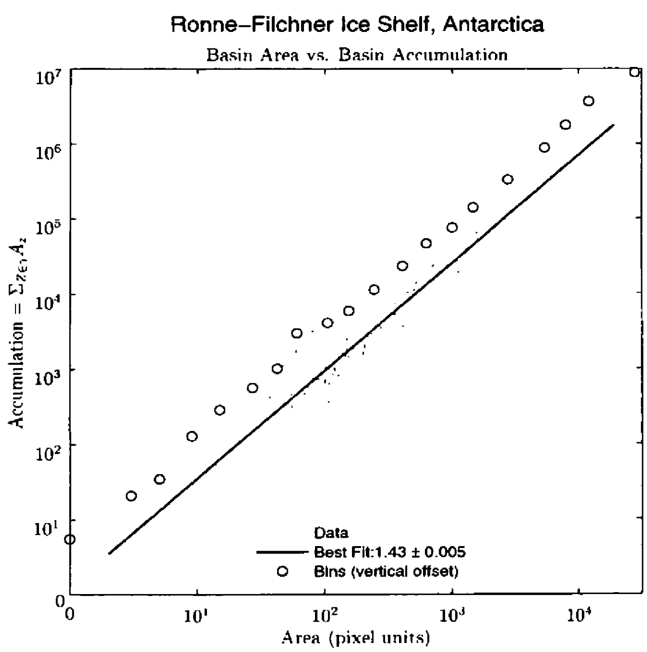


Figure 3.18 Ronne-Filchner Ice Shelf, Antarctica analysis

### Greenland

Greenland provides another opportunity to examine the effects of the bed topography on surface flow. The surface slope shows divergence from the bed topography. Surface and bed elevation data was obtained from NSIDC.

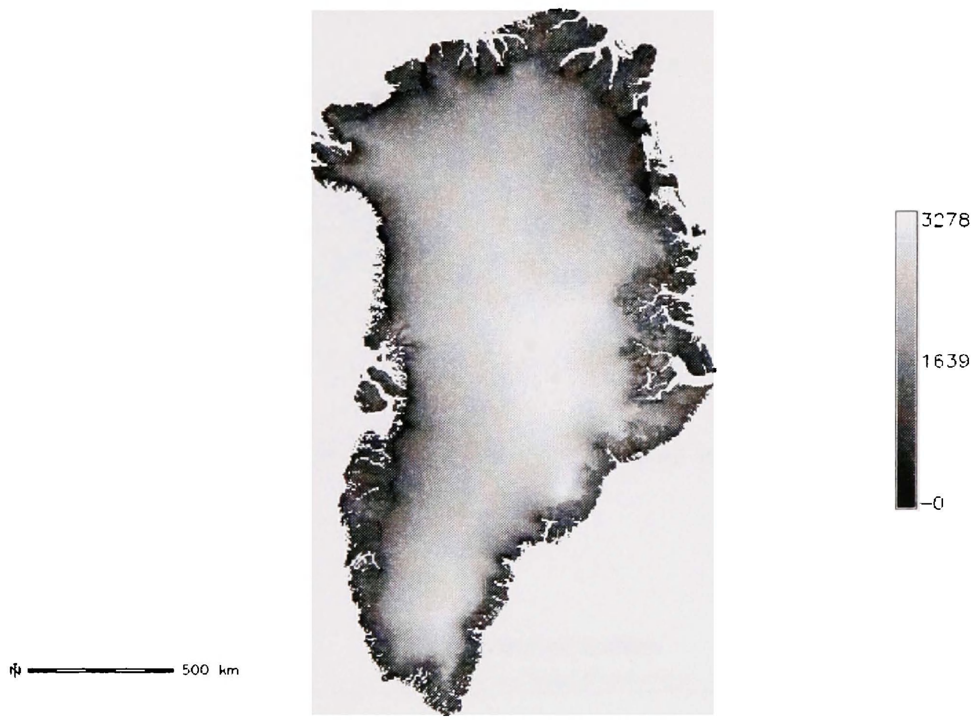


Figure 3.19 Greenland surface

## Mars channel analysis

### Mars channel network

NASA has recently made available elevation data for the entire surface of Mars as a series of images. A section of the Martian surface provided a unique opportunity to analyze the character of a channel network that was formed under different conditions than on Earth. Over most of the surface, craters seemingly dominate the analysis,

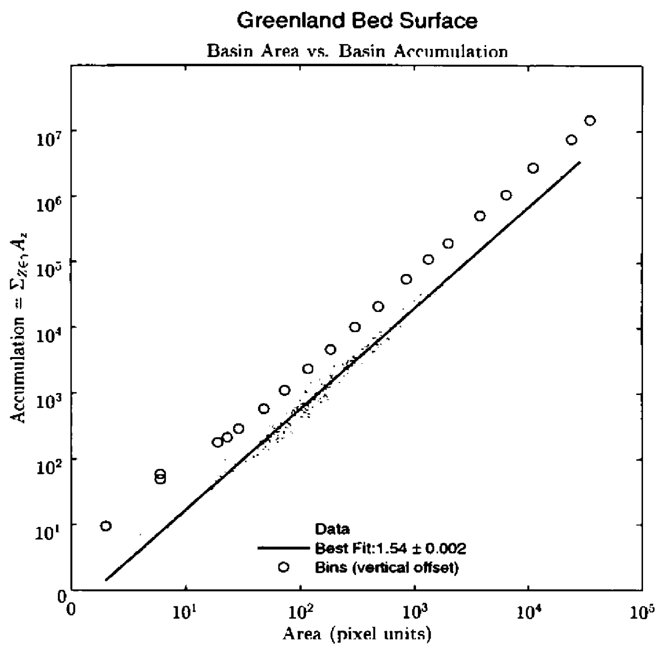


Figure 3.20 Greenland bed analysis

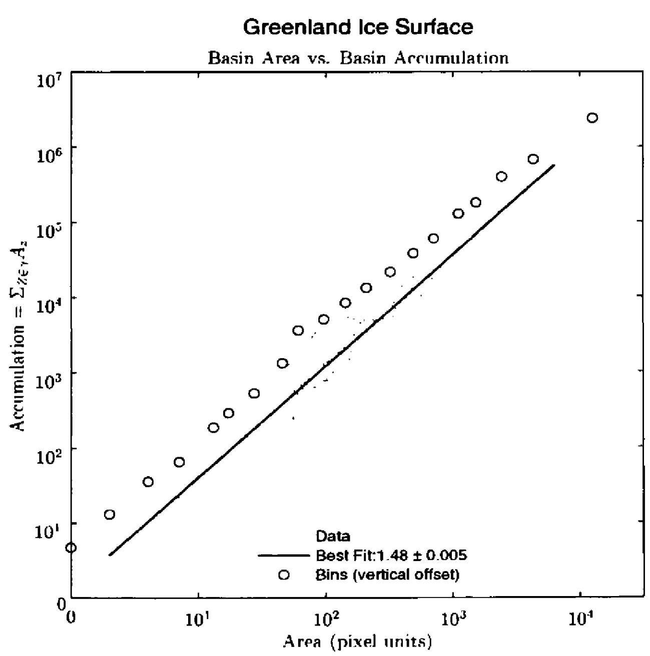


Figure 3.21 Greenland surface analysis

but a few sections of surface that are not completely cratered yield what appear to be the remains of transport networks.

Using the processes developed in this thesis, a drainage analysis for a Martian channel network was performed. Surface elevation data is derived from the digital images that NASA has made available via the Planetary Data System Geosciences Node[5].

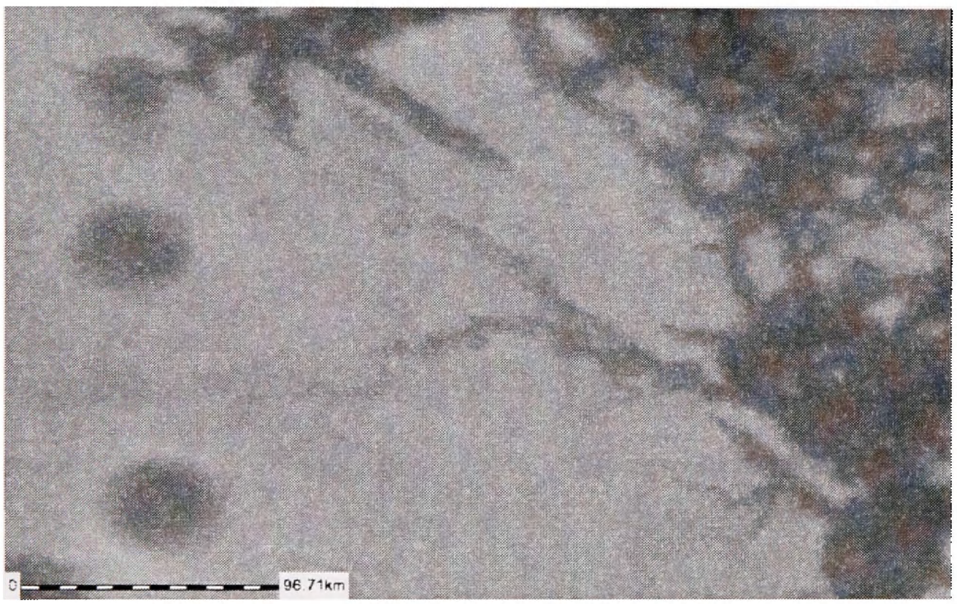


Figure 3.22 Martian network surface image

## DEM resolution

The Digital Elevation Model (DEM) resolution determines with what accuracy the model resembles actual elevation. The 30-meter DEM refers to a single square pixel representing 30-meters on a side. A flowing network may not be adequately generated if the resolution is too low. Hence, the slope of the dimensionality exponent line might

[5]http://pds-geosciences.wustl.edu/missions/mgs/megdr.html

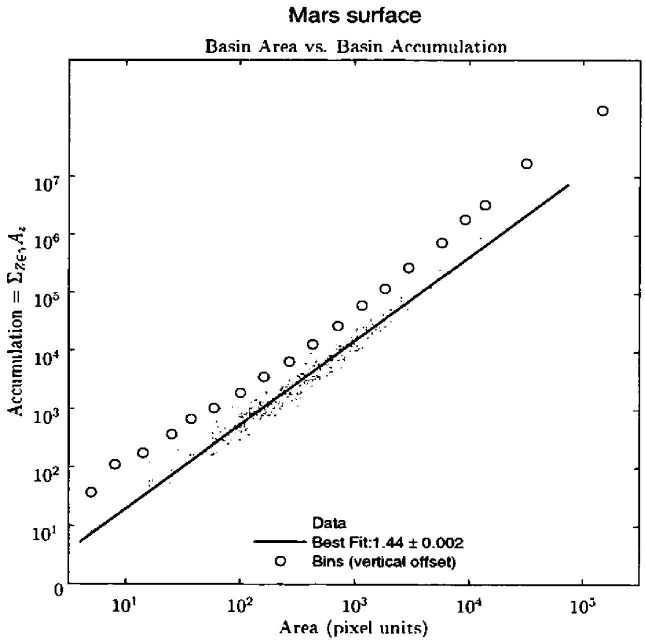


Figure 3.23 Martian network analysis

be effected. To show that this effect does not influence the determined dimensionality of ice streams, an analysis of a river network and an ice stream at different resolutions follows.

### River network

The cow creek network was analyzed at 30-meter, 60-meter and 120-meter resolutions to determine the effect on the dimensionality exponent. The results indicate that as the resolution increases the exponent tends to increase, supporting the claim that river networks exist in two dimensions - the exponent diverges from the theoretical minimum of 1.5.

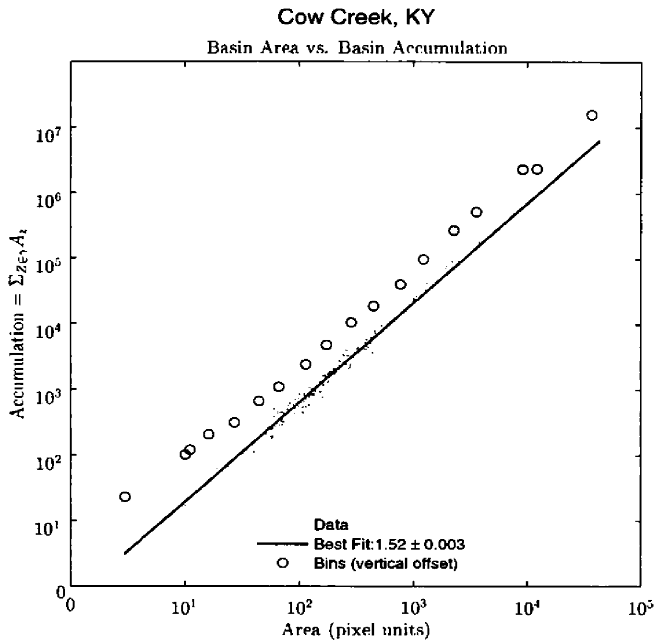


Figure 3.24 Cow Creek at 30-meter, KY

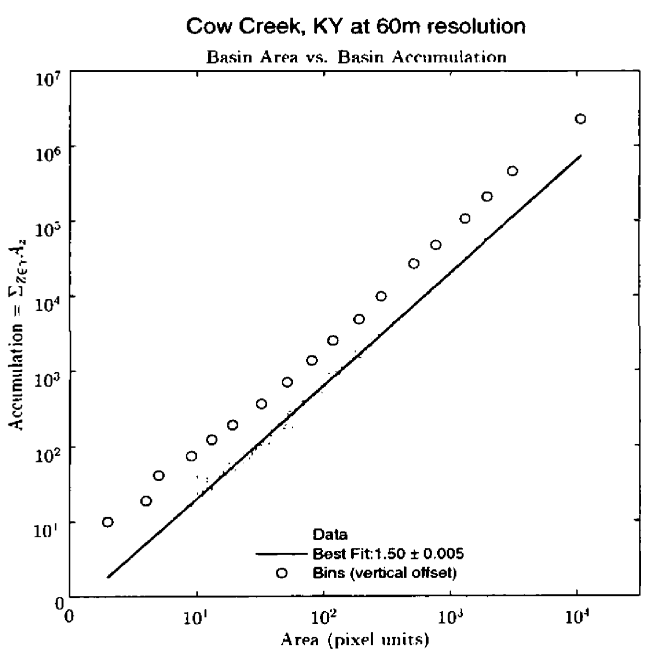


Figure 3.25 Cow Creek at 60-meter, KY

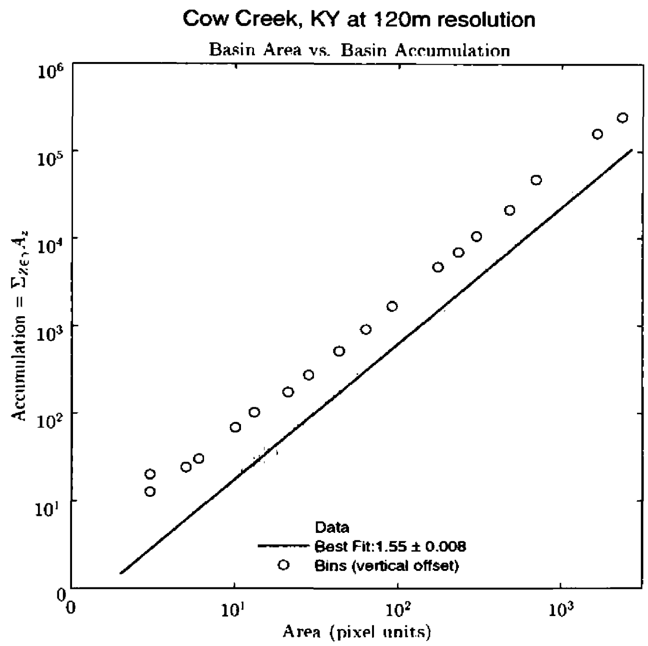


Figure 3.26 Cow Creek at 120-meter, KY

### Ice stream network

Performing similar analysis on an ice stream shows that even at a higher resolution, these networks remain consistently below the threshold value. This indicates that the result of the dimensionality calculation is not an accident of resolution.

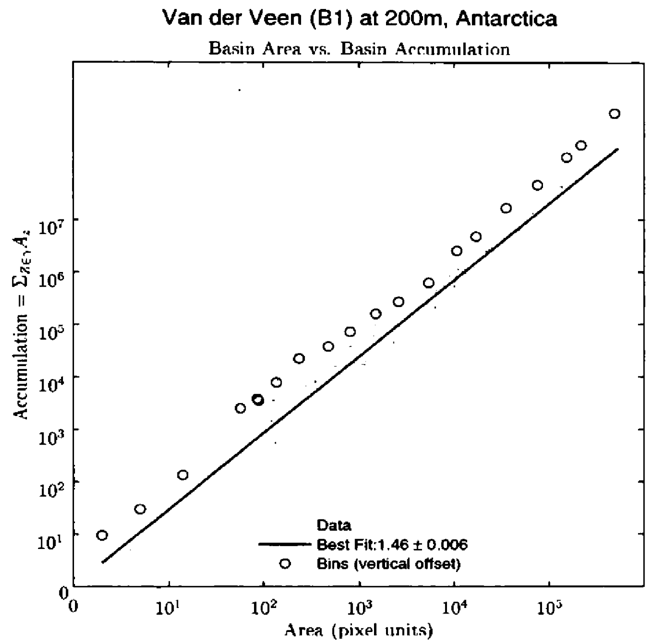


Figure 3.27 Van der Veen ice stream at 200-meters, Antarctica

## Statistical significance

Performing a standard one-tailed t-test reveals the statistical significance of these results. This test shows that the differences in the dimensionality between river networks and ice streams are strongly significant. There are eight observations of rivers (including the bed topography results) and twelve observations for ice streams. Using these observations, the resulting data required for the test can be calculated.

There are 18 degrees of freedom. The result is a t-value of 3.41 which exceeds the 1% required value of 2.55 (Triola, 1986). This places the result in the extreme

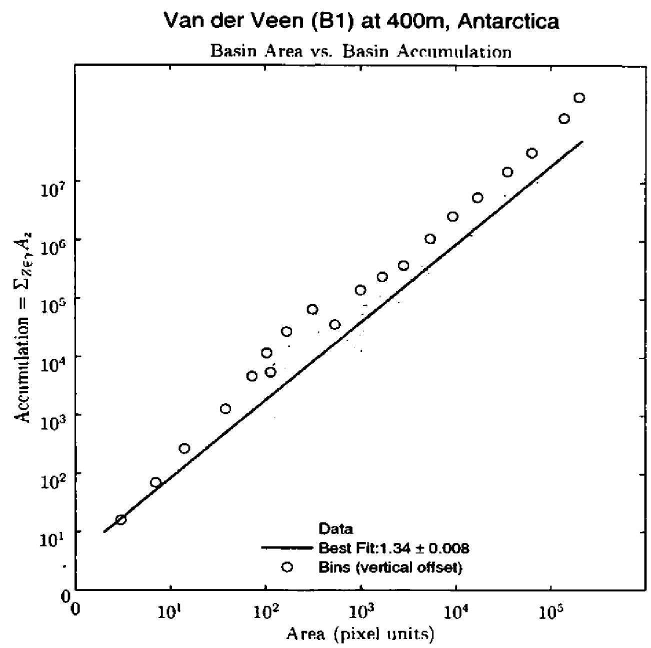


Figure 3.28 Van der Veen ice stream at 400-meters, Antarctica

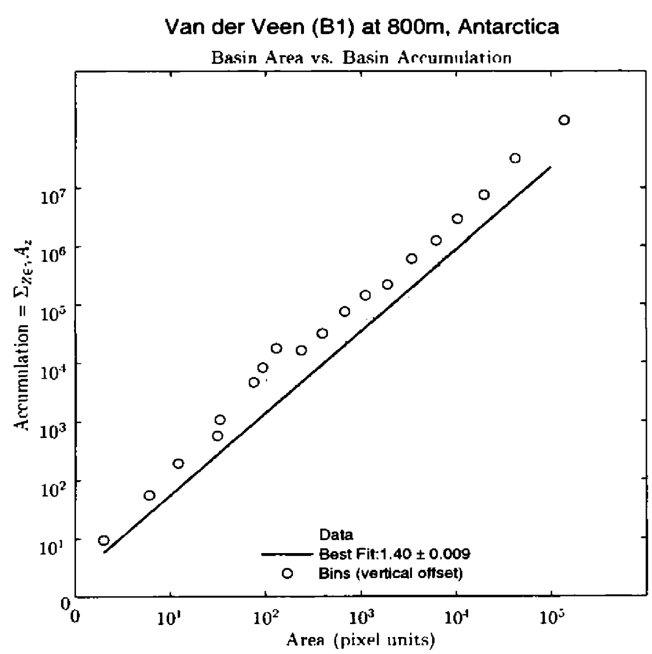


Figure 3.29 Van der Veen ice stream at 800-meters, Antarctica

Table 3.1 Statistical data

| *Measurement* | *Rivers* | *Ice Streams* |
|---|---|---|
| Number of observations | 8 | 12 |
| Mean | 1.53 | 1.47 |
| Standard deviation | 0.02 | 0.05 |
| Variance | 0.0005 | 0.003 |

1% of the distribution indicating statistical significance (*i.e.*,the result is not due to sampling error).

## All drainages and dimensions

Averaging all exponents for rivers yeilds a dimensionality exponent of 1.53. Averaging all exponents for ice streams yeilds a dimensionality exponent of 1.47. Provided below is a summary of all river networks and ice stream networks that were analyzed, the corresponding dimensionality exponent and Hack's exponent for the system.

Table 3.2 Analysis of river drainage basins for dimensionality

| *River* | *Dimension* $\alpha$ | *Hack's Exponent h* |
|---|---|---|
| Blue Mountain, MT | $1.54 \pm 0.002$ | 0.54 |
| Saco River headwater, NH | $1.55 \pm 0.002$ | 0.55 |
| Cow Creek, KY | $1.52 \pm 0.003$ | 0.52 |
| Flintrock drainage, MT | $1.56 \pm 0.001$ | 0.56 |
| Greenland bed | $1.54 \pm 0.002$ | 0.54 |
| Ross bed | $1.52 \pm 0.003$ | 0.52 |
| Pine Island Glacier bed | $1.50 \pm 0.005$ | 0.50 |
| Thwaites Glacier bed | $1.50 \pm 0.007$ | 0.50 |

Table 3.3 Analysis of ice stream drainage basins for dimensionality

| *Ice stream* | *Dimension* $\alpha$ | *Hack's Exponent h* |
|---|---|---|
| All Ross ice streams | $1.49 \pm 0.004$ | 0.49 |
| Van der Veen (B1) | $1.44 \pm 0.014$ | 0.44 |
| Whillans upper (B2) | $1.55 \pm 0.016$ | 0.55 |
| Whillans (B) | $1.48 \pm 0.008$ | 0.48 |
| Kamb (C) | $1.32 \pm 0.009$ | 0.32 |
| Bindschadler (D) | $1.47 \pm 0.006$ | 0.47 |
| MacAyeal (E) | $1.48 \pm 0.008$ | 0.48 |
| Pine Island Glacier | $1.48 \pm 0.007$ | 0.48 |
| Thwaites Glacier | $1.48 \pm 0.007$ | 0.48 |
| Amery | $1.49 \pm 0.004$ | 0.49 |
| Ronne-Filchner | $1.43 \pm 0.005$ | 0.43 |
| Greenland | $1.48 \pm 0.005$ | 0.48 |
| Mars | $1.42 \pm 0.002$ | 0.42 |

# CHAPTER 4 INTERPRETATION AND CONCLUSION

Channel networks are abundant in nature and provide an interesting and useful method of categorization. For example, if a system exhibits an allometry that is similar to river networks, then domain knowledge may be applied to bring existing analysis techniques to bear on the new system. Similarly, if a system's dimensionality exponent falls below the theoretical lower limit of a dimensionality class, it is then known that the system must be characterized by higher order functions. The dimensionality exponent provides a probe into the structure of a network (Maritan et al., 2002) which can be used to indicate the techniques required to examine the system.

Rivers have been shown to exhibit a two dimensional character (Rodriguez-Iturbe and Rinaldo, 2001). Hack's Law is a prime example of this characterization. For natural river networks, Hack's Law is always a number that is greater than 0.5. Performing dimensionality analysis on river networks and comparing the results to the results gained by performing similar analysis on ice streams indicates that ice streams belong to a different class of channel network than river systems.

## Conclusion

Channel networks are an excellent device to use when dealing with a natural system. Most times, the channel network can be represented as a power law and hence can be elegantly characterized by the determination of its characteristic exponent.

This process is generally robust to statistical anomalies and is not prone to wild fluctuations. The exponent might even be thought of as a probe to the internal structure of the network. Given the ubiquity of channel networks, and the data we can determine based on their exponent, it is worthwhile to find methods with which to define them.

Applying the recently discovered properties of channel networks to ice streams reveals an interesting fact about their character - that their physical existence is fundamentally more complex than river networks. The results produced in chapter three conclusively demonstrate that river networks and most ice stream networks have dissimilar allometry, and that all relevant physical characteristics of ice streams clearly cannot be captured in two dimensions.

Perhaps the most promising aspect of this result applies to modelers - it is prudent to determine the dimensionality of a system before attempting to build a model. The results of that determination might indicate that a simpler model of the system will not be descriptive enough to capture all the relevant behavior. The so-called "shallow ice approximation" ice stream models that are reviewed in (Payne et al., 2000) cannot fully capture the behavior of ice streams. A fully 3D model such as the one described by Pattyn, 2003 is more realistic.

## Limitations

Using a first order method to query the ice stream network (like the method used in this process) was adequate for determining dimensionality, but it would be better to use a higher order method for finding these data. As shown in the results, it is indicated that river analysis tools are unable to capture all the relevant characteristics of ice streams. Ice streams form more complex networks than rivers and might have more in common with a three dimensional biological system than previously suspected. A three dimensional system has as theoretical minimum dimensionality exponent of $\frac{4}{3}$ or $1.3\bar{3}$, and none of the ice streams examined approach this theoretical minimum.

Currently, an issue that exists with the process define here (which could be resolved if this work were continued), is the manual determination of threshold values. It would benefit the process to have an adaptive threshold value that always maintained a strict four (or more) orders of magnitude of spatial scaling for basin sizes. As it is, areas smaller than 50,000 pixels have a weight inherently applied to the smaller spatial scales simply because there are no basins at the larger spatial scales. Similarly, basins of extremely large area (greater than ten million pixels) have spatial scales that will be wholly unaccounted for since the maximum basin size for the current process is hardcoded to 50,000.

A potential method for solving this issue would be to precompute good threshold values based on the size and configuration of the data set being analyzed. Comparing networks in this way would add noise to the inter-network scaling exponent[1] (Maritan et al., 2002) comparisons, but the results might be more realistic.

## Future work

It would be interesting to expand this work to include a universality class component as in Dodds and Rothman, 2000, where one might be able to say that a certain system exhibiting a dimensionality exponent D is more or less mature/efficient than another system exhibiting an exponent that was smaller or larger.

This could possibly be done by adding a time component to the procedure, which

[1](Maritan et al., 2002) refers to comparisons between mutually exclusive basins as inter-network species and comparisons between sub-basins in the same watershed as intra-network species.

would be the next logical step to answer: How does a network's dimension change over time? A method of charting Hack's Exponent changing over time is given in (Leheny and Nagel, 1993) and shows that over time, an artificially evolving river network system has a tendency to minimize Hack's Exponent. This result would be interesting to compare with a similar evolutionary model of an ice sheet.

Another interesting extension would be to extrapolate networks from the glacial record to determine if glaciers of the past exhibited more river-like behavior. Performing analysis on heavily glaciated areas (such as Montana and Alaska) provides insight to this, but that was not the focus of this research, hence no comparisons were made to areas that were not heavily glaciated.

Finally, it would be interesting to compare Hack's Exponents produced by two dimensional ice sheet models with those produced by three dimensional models. This could be compared to actual ice stream exponents. The results from this thesis seem to indicate that the three dimensional models would produce exponents closer to those seen by examining actual ice stream networks.

# APPENDIX A EXTRA TABLES AND FIGURES

All basin graphs are included here. The analysis of the basins exists in the text of chapter three.

## River basins

These basins depict the flow accumulation for river networks as determined by `r.watershed`. The bed basins for the Ross Ice Shelf, Pine Island glacier, Thwaites glacier and Greenland are all somewhat hypothetical, since there is a large uncertainty associated with the elevation data under the ice.

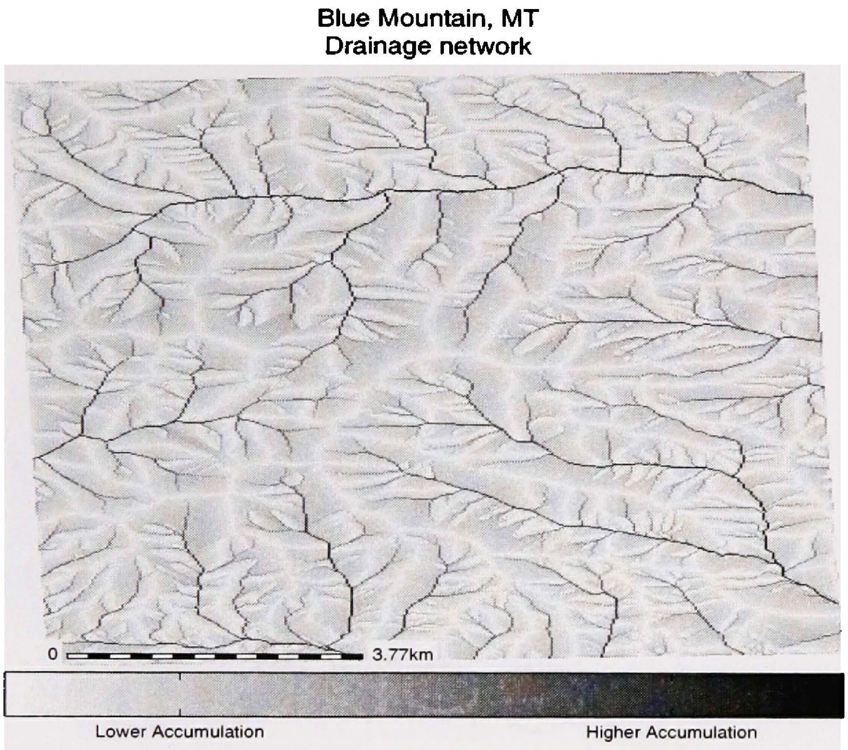


Figure A.1 Blue Mountain, MT basin

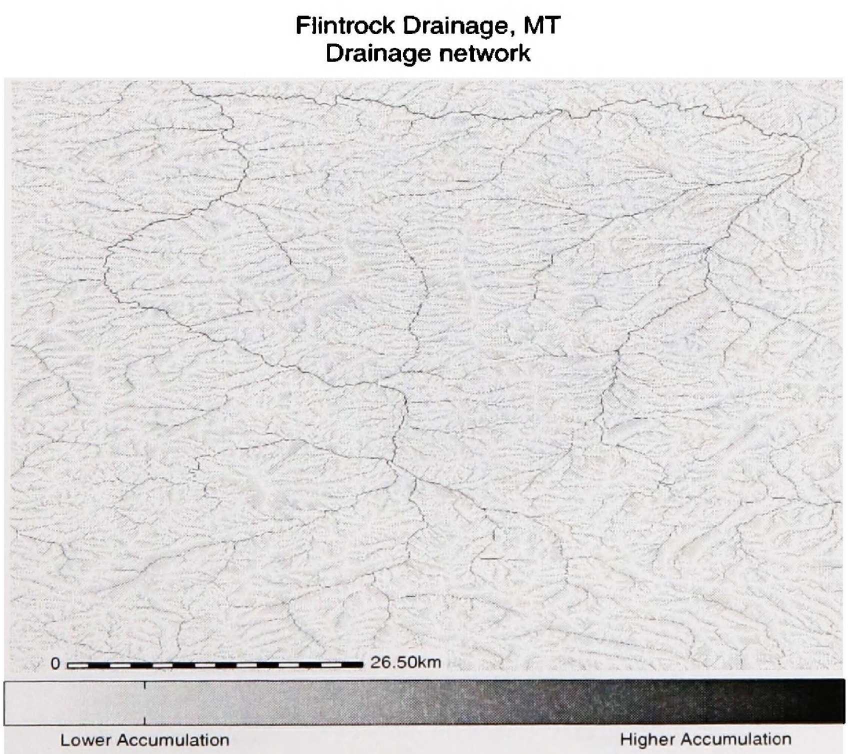


Figure A.2 Flintrock drainage, MT basin

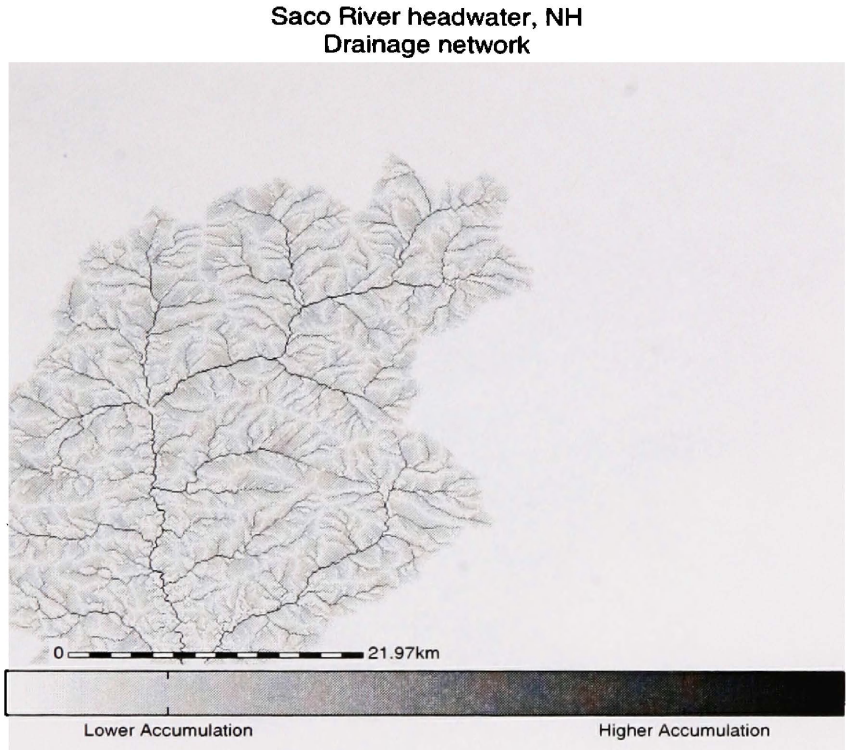


Figure A.3 Saco River, NH headwater basin

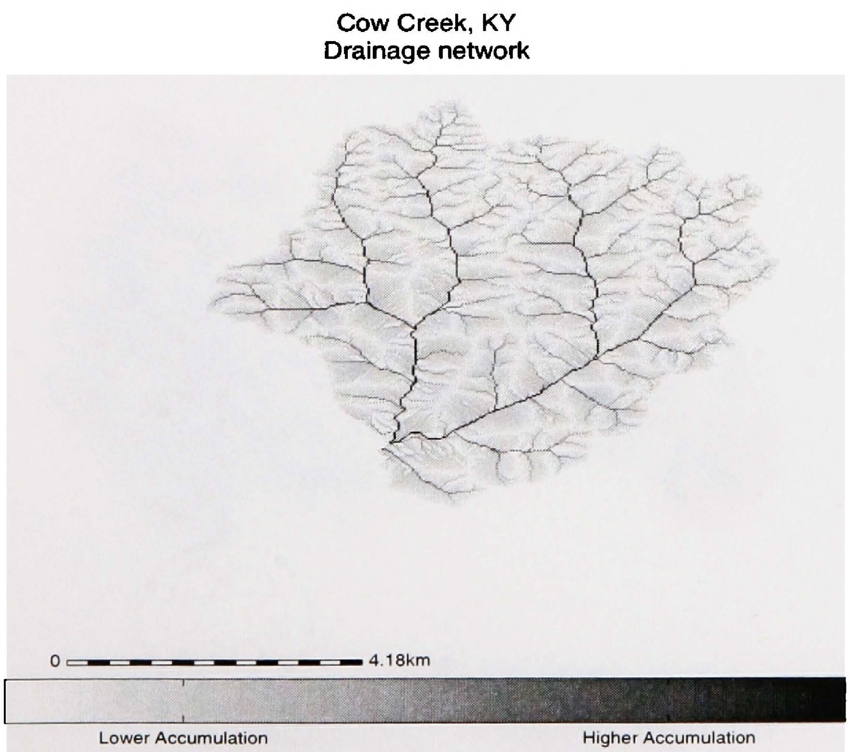


Figure A.4 Cow Creek, KY basin

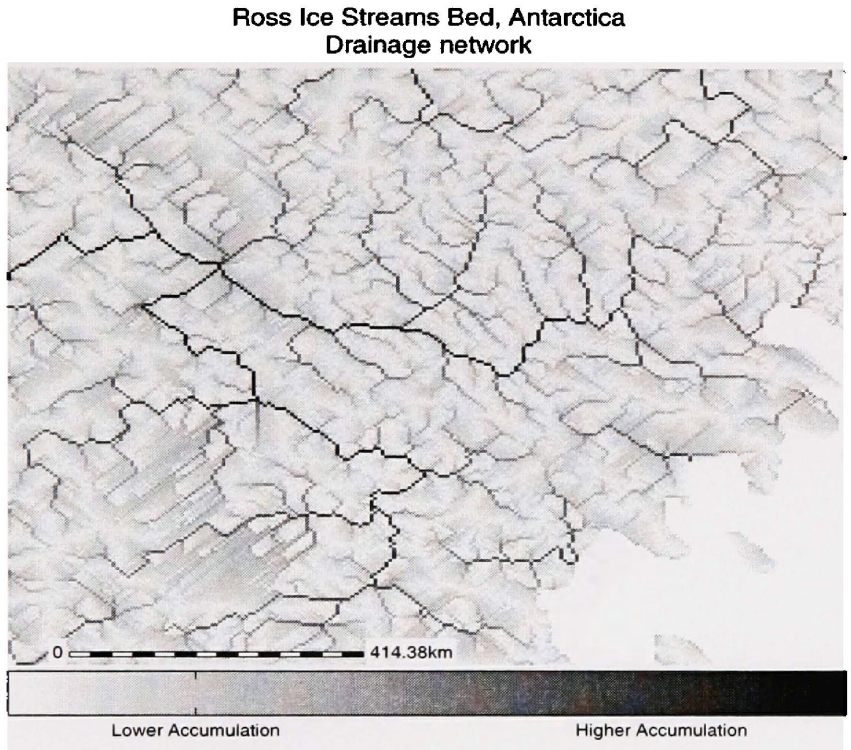


Figure A.5 WAIS ice streams bed Elevation drainage network

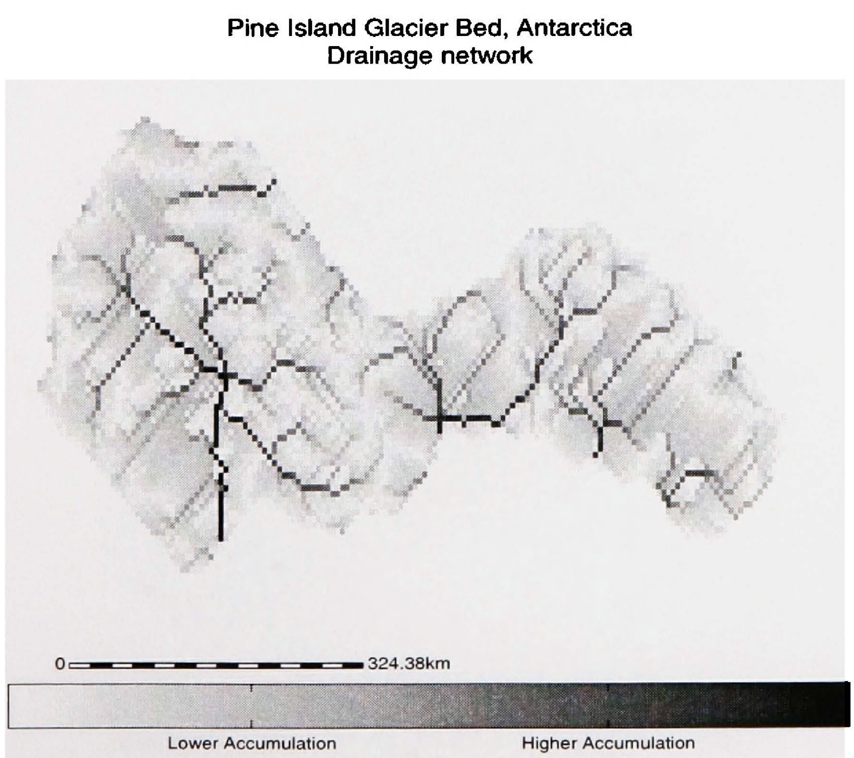


Figure A.6 Pine Island Glacier bed drainage network

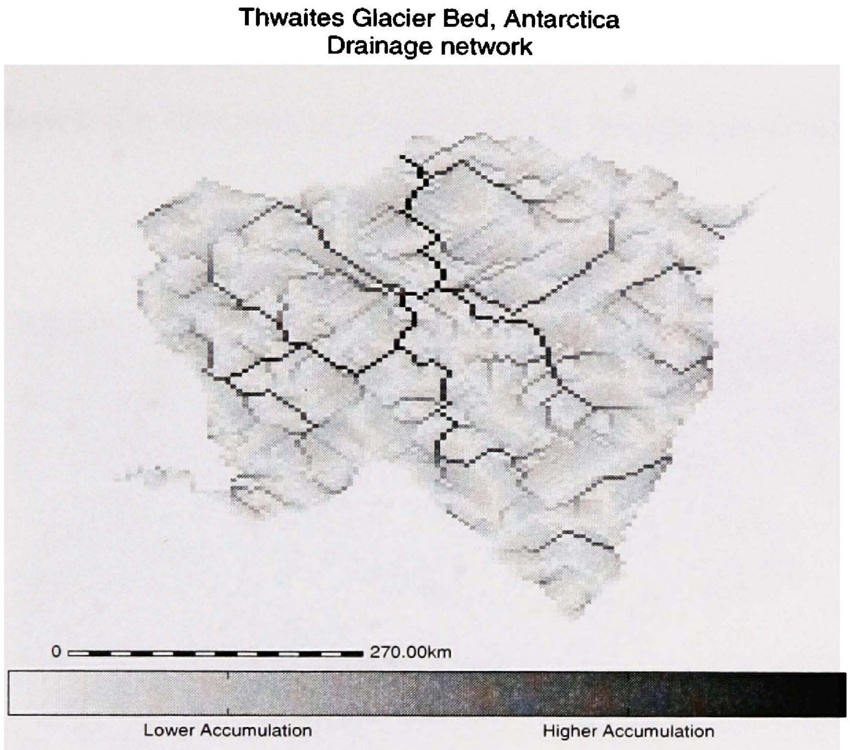


Figure A.7 Thwaites Glacier bed drainage network

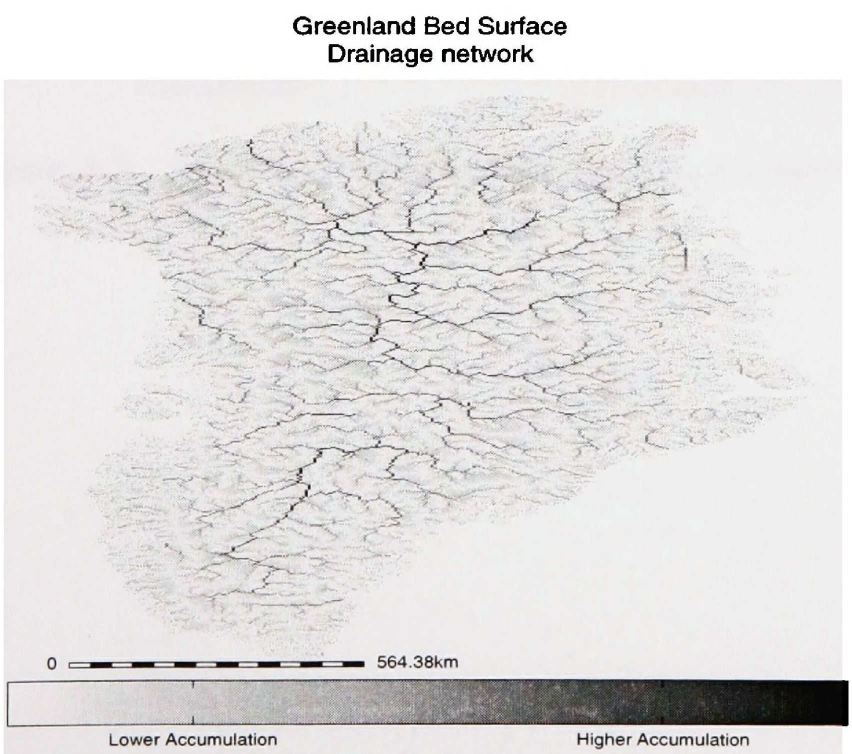


Figure A.8 Greenland bed drainage network

## Ice stream basins

These basins depict the flow accumulation for ice stream networks as determined by `r.watershed`.

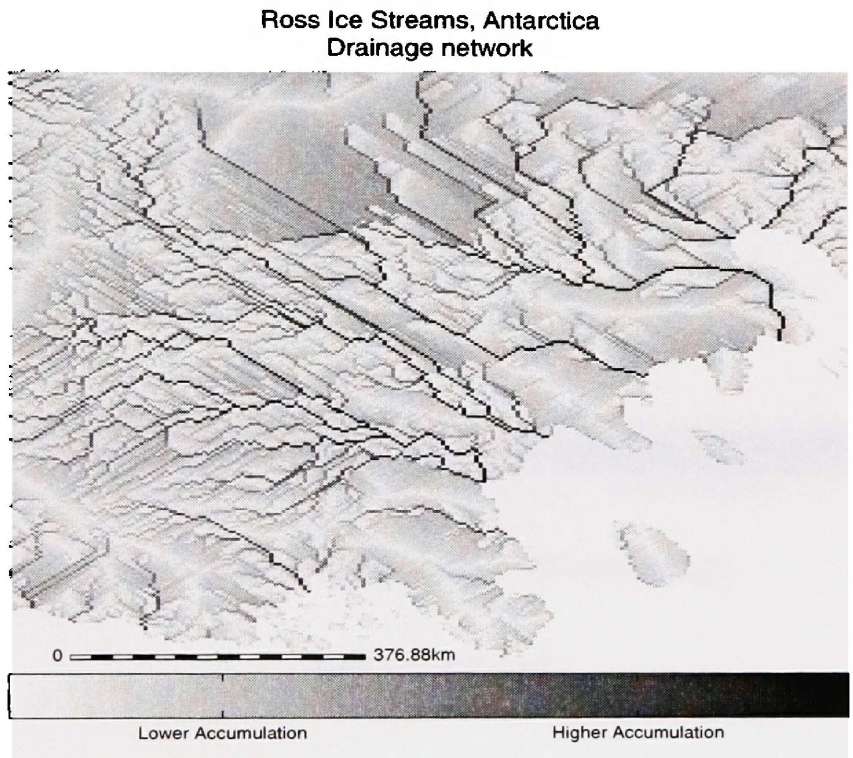


Figure A.9 WAIS ice streams surface drainage network

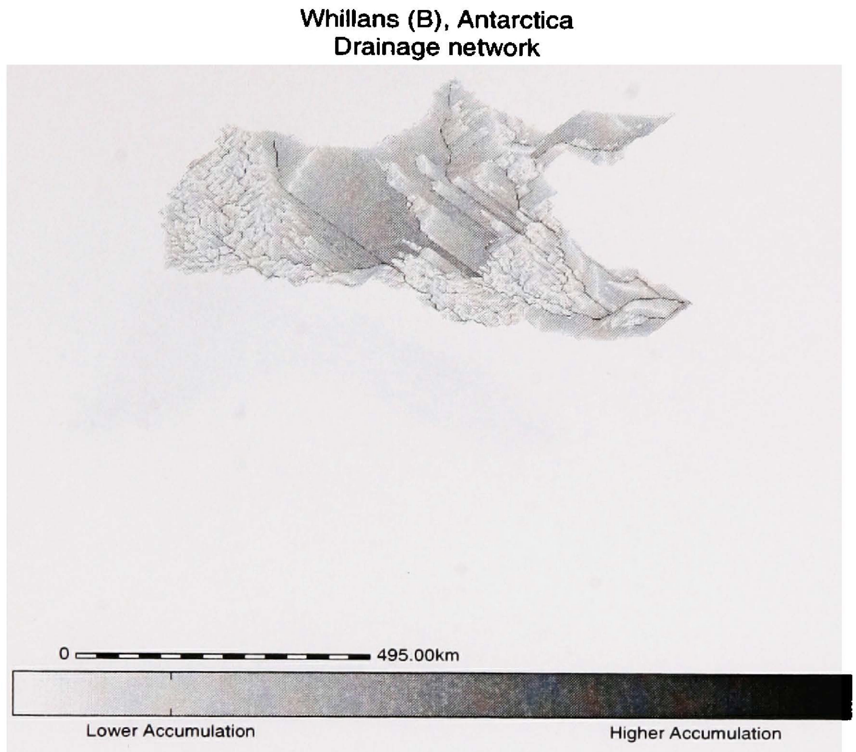


Figure A.10 Whillans Ice Stream graph

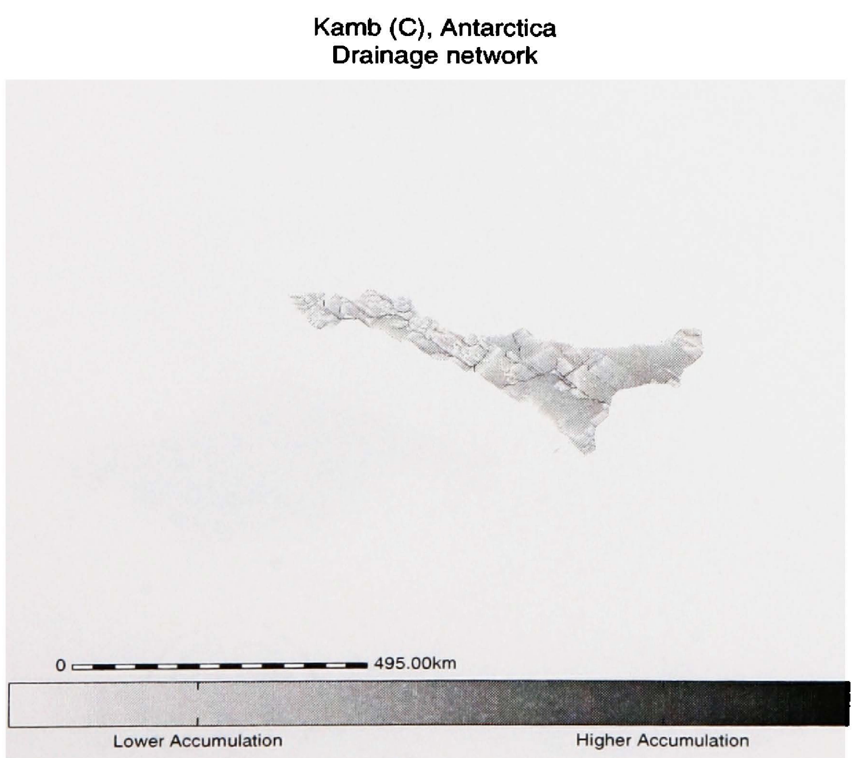


Figure A.11 Kamb Ice Stream graph

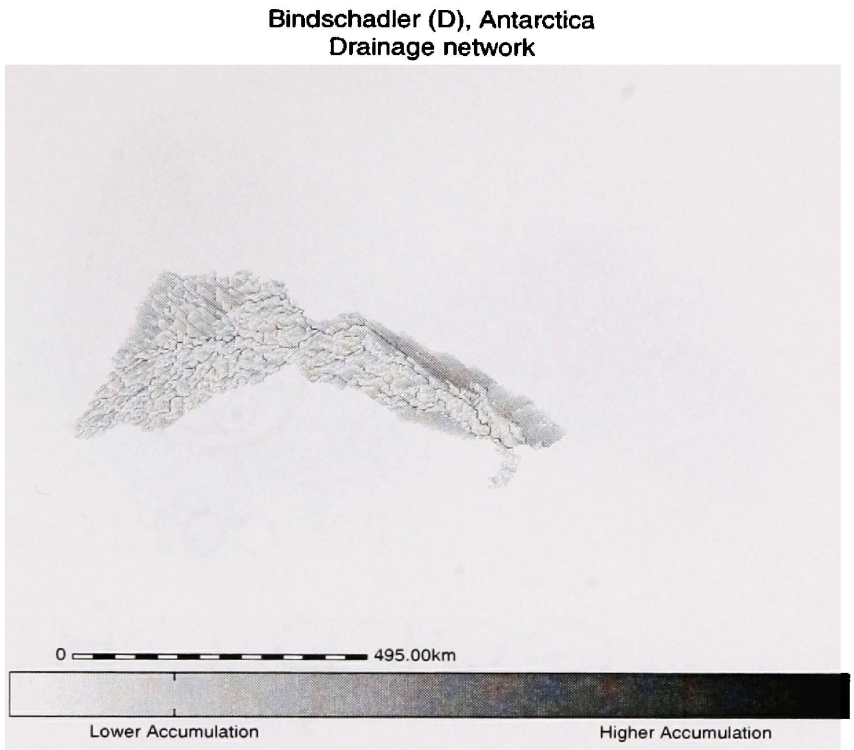


Figure A.12 Bindschadler Ice Stream graph

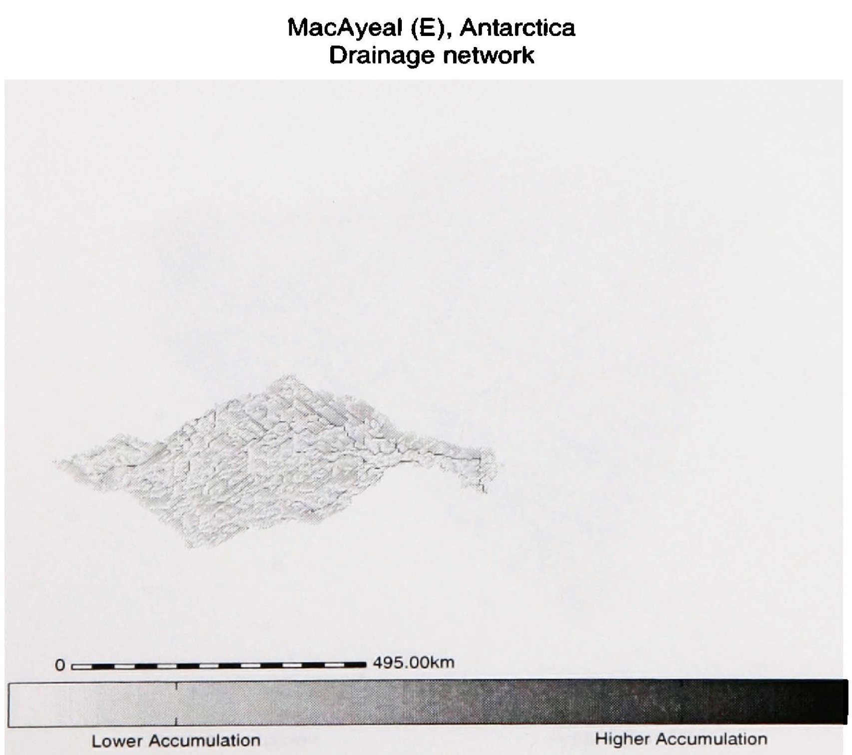


Figure A.13 MacAyeal Ice Stream graph

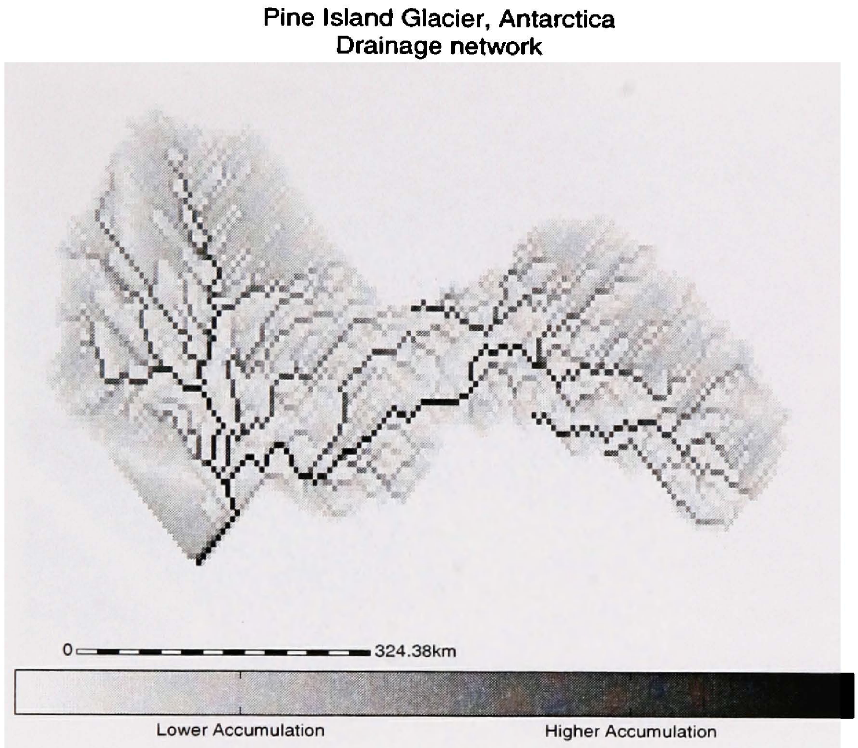


Figure A.14 Pine Island Glacier surface drainage network

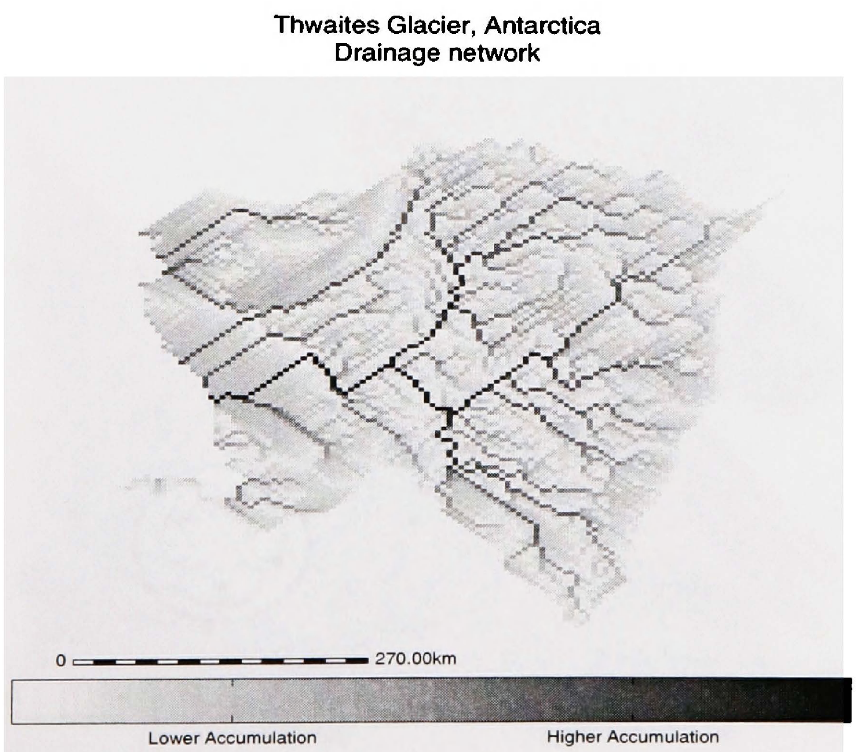


Figure A.15 Thwaites Glacier surface drainage network

Amery Ice Shelf, Antarctica
Drainage network

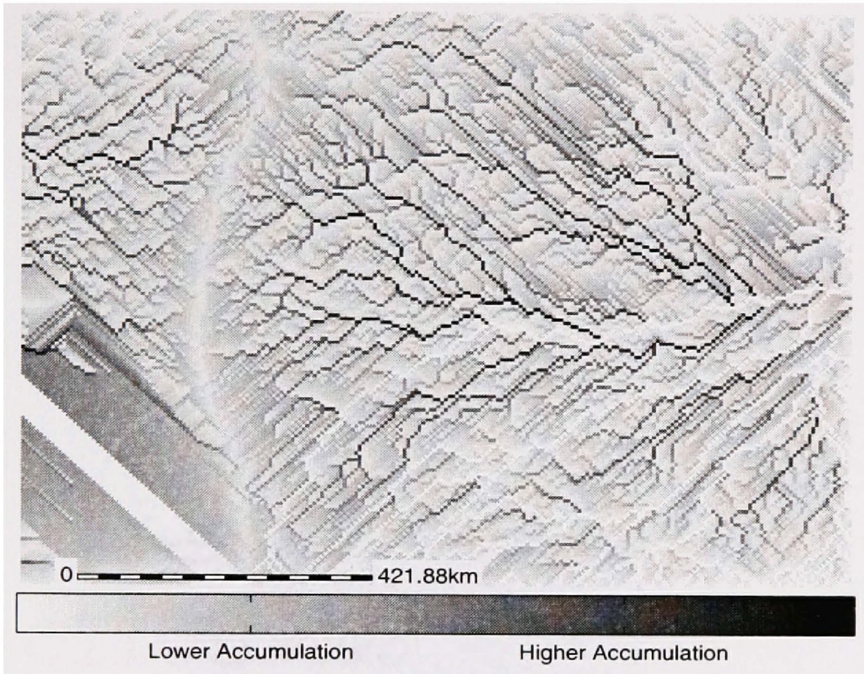


Figure A.16 Amery Ice Shelf (Lambert Glacier), Antarctica basin

Ronne–Filchner Ice Shelf, Antarctica
Drainage network

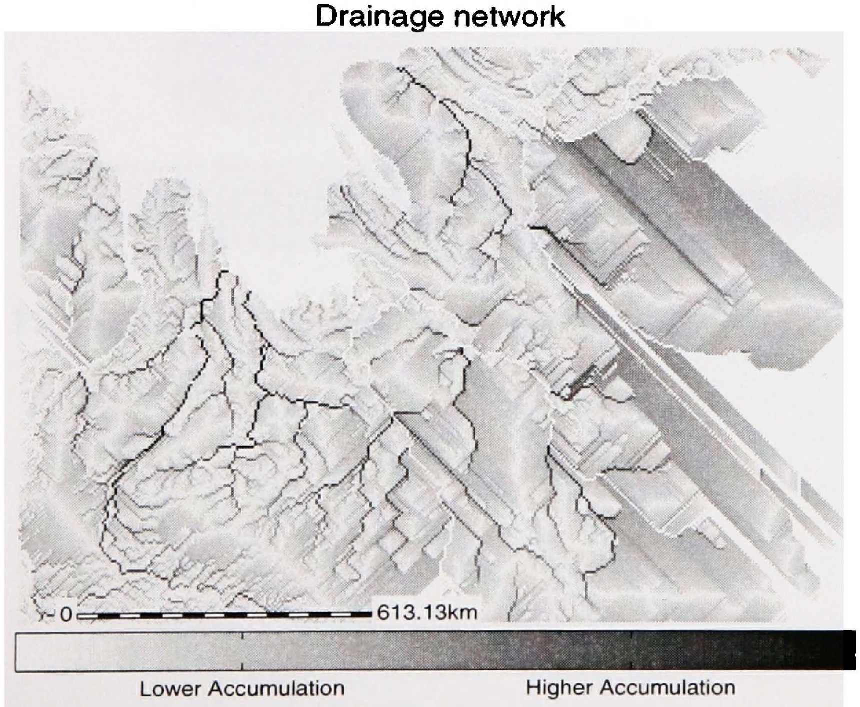


Figure A.17 Ronne-Filchner Ice Shelf, Antarctica basin

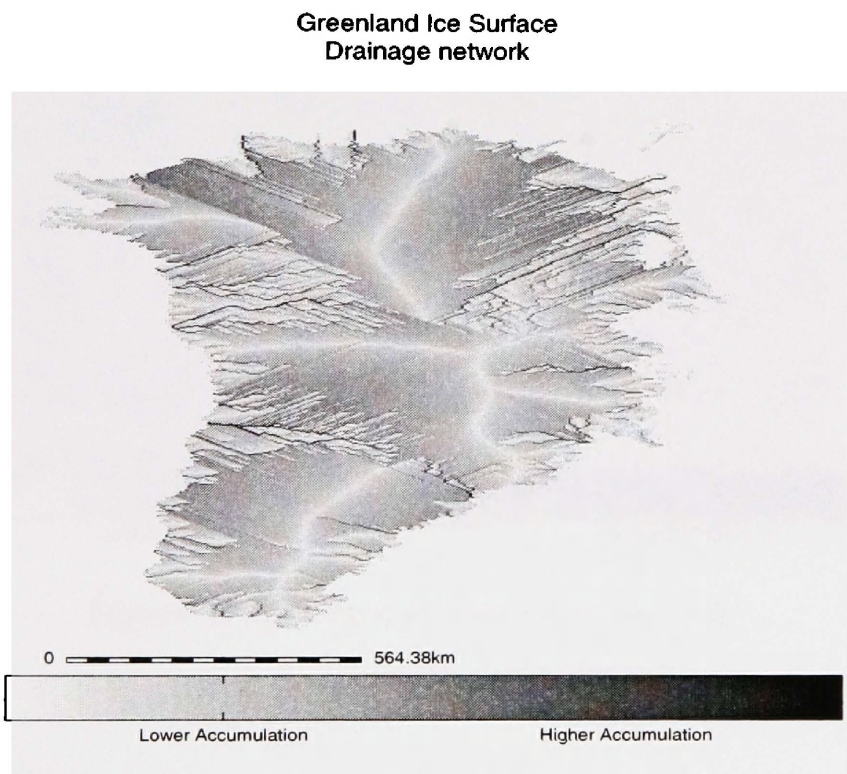


Figure A.18 Greenland surface drainage network

## Mars basins

This basin depicts the hypothetical flow accumulation for a network on the surface of Mars as determined by `r.watershed`.

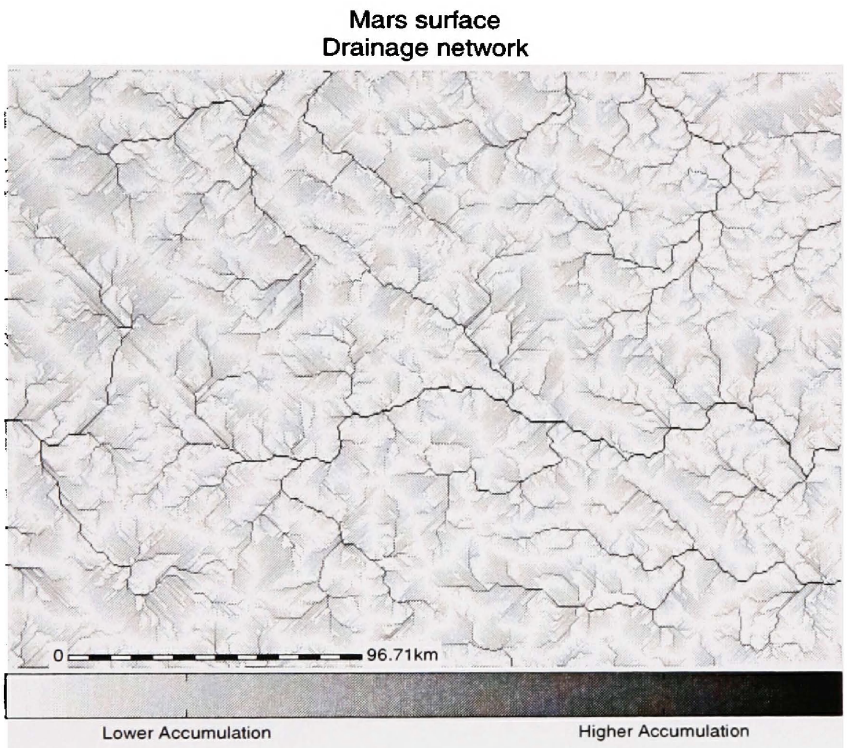


Figure A.19 Martian drainage network

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