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A STUDY
of
PARETO'S INDIFFERENCE CURVES APPROACH AND RECENT DEVELOPMENTS

by

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Diplome des Ecoles Superieures de Commerce

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FOREWORD

When I came to Montana State University I was planning to take graduate work in Business Administration. The Business School does not prepare its students to a Master's degree, so I turned to Economics and really I am glad to have done so. This for several reasons. Having first studied the working of a firm from the inside and in a practical way, it has been interesting to study the working of the whole economic system, and the theories which pertain to that matter.

Second, I think that studies in economics are more interesting for a foreigner than studies in business if he is interested in getting acquainted with the American economy. The business field is more technical and practical, but economics gives a more general view of the economic set up.

Third, I have become interested in economics, and particularly the subject I have written about, more than I have ever been interested in anything before. Perhaps it is because economics is a science which is still in formation, or perhaps it is more interesting to study controversies than to learn dogmas or rules set up by a long series of masters.

At the beginning this thesis was to be mainly based on a translation of Pareto's work, Manuel d'Economie Politique. It would have included a biography of the author, the translation of the four main chapters of the book (the others being related to general methodology or sociology) and a concise appraisal of the work.

I wrote the translation, but the discussion of Pareto's ideas and of their relations to modern theories appeared to offer a much more interesting field of study than a mere translation which was more an English major's work. The result of this shift in my interests is this thesis. It still includes a biography of Pareto and a general survey of his works, but the part of the translation is much reduced. Only the translation of some paragraphs will be given in order for the reader to realize exactly what Pareto's ideas were. And besides the appraisal of Pareto's theories, I try to show briefly in this paper the evolution of theories based on the indifference curve approach--from the Lausanne School, through Marshall, Hicks, Scitovsky, Leontieff and the present day articles by Samuelson and other students of the subject.

I cannot begin this thesis without expressing my gratitude to the whole Faculty of the Economics Department of Montana State University and particularly to Dr. Ely, Chairman of the Department, and to Mr. Thweatt, my thesis advisor. These professors gave me many encouragements and much help for the writing of this paper; they have been for me the best representatives of American friendliness.

INTRODUCTION

WHY A THESIS ON PARETO?

One of the inconveniences which prevent the international diffusion of economics is that each economist writes in his own language. It is difficult to be both an economist and a linguist. I mean that it is difficult to be enough of an economist to understand the theories, and enough of a linguist to understand the language (not always very good) in which the theories are written. Some economists are both; for instance, Schumpeter was both a linguist and a very great economist, and his works are of much help for those who want to get acquainted with the ideas developed by foreign schools. For other economists there is the solution of reading translations in their own language. Sometimes, however, these translations are not very good. I have experienced this myself with the translation of the Manuel into French. If there is no translation, and if some economists have some knowledge of the foreign language, they try to read the originals. This sometimes leads to misinterpretations (this will be noticed for Hicks' and Pareto's theory of superior goods), because the work and the training of an economist are primarily in the field of economic science and not in the field of languages. Finally, if there is no translation and if they do not know anything of the language, they pick up, here and there, in other books, the information they need. But such a method prevents extensive work to be done on the theories of foreign authors.

For Pareto's economic writings there was no translation. The best text is published in what I shall call without hesitation, bad French. Moreover, any American theorist interested in consumption, value theory and general equilibrium should read something about Pareto, go back to the very source. As a matter of fact, Pareto is often referred to in many articles and books written in English. When I first met Professor Thwaitt he told me that he would be interested in the theories which may be found in the main work of Pareto, Le Manuel d'Economie Politique.

This fact and all those I cited previously were the reasons for which I chose this topic for my thesis. I am only a student in the economic field, but I have the big advantage of reading my native language. I think that this is important, especially when one reads a translation as bad as the French edition of the Manuel.

I think that the importance of the subject and its interest will appear in the remaining pages of this thesis. All of Pareto is not studied in this work--neither is only Pareto studied. My purpose in writing this thesis is to present Pareto's graphic approach to the problems of consumption, production and general equilibrium; to explain some points which are not clear in the text (maybe because of a bad translation into French); to compare Pareto's work with modern studies of the same subject, and to show how the Paretian approach often has been the source of modern theories. The accomplishment of this purpose will give us a fairly complete view of these portions of Pareto's writings that are interesting to modern economists, and, at the same time, to show the evolution of the indifference curve approach.

The plan of the thesis is simple. The first part deals with Pareto's biography and includes a general survey of his contributions. The second part has two chapters: one on Pareto's theory of value and consumption, the other on modern theories in the same field. The third part deals with production theories and is built on the same framework. In the fourth part I shall study Pareto's idea of the general equilibrium and we shall conclude our work by studying the most recent attempts in the graphic representation of economic equilibrium.

If this paper gives the reader a better understanding of Pareto's theories and the reasons for which he is so often referred to, my purpose will be accomplished--so answering, at least partly, the desire expressed by Hicks: "may I advocate...in favour of a new edition of Pareto where diagrams would be at least drawn again, enlarged and correctly marked!"¹

¹Hicks, "L'Economie de Bien Etre,"
L'Economie Appliquée, N. 4, (Oct-Dec 1948), p. 436,
 (Presses Universitaires de France).

PART I

PARETO'S BIOGRAPHY

HIS WORKS

HIS PLACE AMONG THE MAIN ECONOMISTS

CHAPTER I

PARETO'S BIOGRAPHY

HIS PLACE IN THE HISTORY OF ECONOMIC THOUGHT

A. HIS WORKS:

Vilfredo Pareto was born in Paris in 1848. His father was an Italian marchese, from Genoa, exiled in Paris for political reasons. His mother was French. His writings show both influences; the French influence can be noticed in his line of thought and the Italian influence can be noticed in the form of his style.

He went to Italy in 1875 and later studied engineering. He graduated in 1869. First he worked in industrial management. Later he was appointed president of the Italian Iron Works. He became interested in economic matters and in 1892-1893 he wrote some articles in the Giornale degli Economisti. In 1893 he was appointed successor of Walras at Lausanne University. The second part of his life was dedicated to economic research until 1912. An annual income received by inheritance permitted him to retire from his chair of economics. His home was then, from 1906 on, at Celigny on the Lake of Geneva. Schumpeter has said that "this interesting fact deserves to be noticed, that so great an influence could have been exerted by a man who lived in resolute though hospitable seclusion in a shabby house full of cats (hence Villa Angora) that was then not convenient to visit."¹ From

¹J. Schumpeter, History of Economic Analysis, (New York Oxford University Press 1954), p. 860.

1912 on, his entire work was dedicated to sociology. The Manual which was published in 1909 includes several chapters on sociology. Vilfredo Pareto died in 1923.

What kind of a man was Pareto? The first thing to stress is the importance of his mathematical studies in engineering. His knowledge of mathematics was not only practical but also theoretical. This helped him to emphasize the use of mathematics in pure economics, after his predecessor Walras. The period when he was in industrial practice is also very important. During this time he obtained the large experience that economists need even in the theoretical field. During that period Pareto became interested in economics through the writings of Francesco Ferrara.¹ He had a deep and personal knowledge of Greek and Roman classics which accounted for a great part in the formation of Pareto's personality which appears to have been both strong and temperamental. In fact, in his line, Pareto seems to stand alone in the field of Italian and European politics. Schumpeter, in the chapter of Ten Great Economists which is dedicated to Pareto says, "Pareto cannot be pigeonholed. He paid court to no ism."² During the second part of his life he showed a great independence of spirit. Before the call to Lausanne he had already decided to emigrate to Switzerland. This desire for isolation arose from his participation in the politics of his country: Pareto did not agree with the Italian government, which he thought to be incapable and corrupted. In fact, he seems to have fought against many policies of his times and always with an

¹Francesco Ferrara (1810-1900) Italian politician and economist. See J. Schumpeter History of Economic Analysis, p. 512.

²J. Schumpeter, Ten Great Economists (New York Oxford University Press 1951), p. 111.

extreme honesty. For instance, he attacked also the persecution of the French clergy but not because he was a Catholic. It was by political honesty rather than by economic conviction that he fought the parliamentary government. So he appears as an ultraliberalist but not of a liberalism similar to the English Classical liberalism. This eclecticism in his culture, permitted many schools or groups to find a basis for their ideas in his books and especially in the articles he wrote in different papers. But, in general, the claims of these schools do not reach the real ideas that moved Pareto.

Pareto seems to have been a man of strong and honest independence which he preserved by his emigration to Switzerland and then by his life of the coast of the Lake of Geneva. Perhaps we could say that Pareto represents the Latin tradition of thought among the economists of his time. Schumpeter said, "there is no point, whatever in judging his action--or indeed, any action or sentiment of his--from the standpoint of Anglo-American tradition."¹ We shall have to remember this when, later, we study the theory of production and other approaches which are difficult to understand for an economist of Anglo-American background.

At what moment did Pareto arrive in the development of economic theory? What is his place among other great economists? We are going to try to answer these questions.

At that time economics on the continent consisted mainly of historical interpretations. The factors emphasized were historical

¹J. Schumpeter, Ten Great Economists, p. 118.

and a fundamental character was claimed for each of them. The method was arbitrary and intuitive rather than logical and scientific. There were also some economists who based their theories on qualitative or hypothetical postulates. Thus hedonism and the theories of some classical economists were based on the assumption of personal interest which was considered as the first cause of economic activity. Economics was at a stage at which each doctrine was particular, each problem was treated separately and then integrated with the others in an arbitrary way. Most of the doctrines gave too much importance to a few economic phenomena, whereas other factors were not used. With such theories it was very difficult to build an approach to the study of general equilibrium. Only a necessarily incomplete juxtaposition was possible. The construction which compares and assembles partial truths is necessarily artificial. A restriction often made to the theories was the condition "ceteris paribus" (other elements staying constant). One conceives that, as far as a study of general equilibrium is concerned, such an assumption cannot give anything very valid.

But Jevons, Cournot and Edgeworth, each one in his particular field of interest, started serious studies. The theory of value was progressively improved and the first real jump toward a concept of general equilibrium was accomplished by Walras who was the predecessor of Pareto in Lausanne.

Who was Walras? Walras was a Frenchman born in 1834, and Schumpeter said that Walras' very way of thinking was French in the same sense in which Racine's writings and Poincaré's mathematics are

French. He filled the economics chair in Lausanne in 1870 and kept it until 1892. He did not form any school but had an early influence on Pantaleoni and through him on Pareto. Here is what Schumpeter says about him: "...so far as pure economics is concerned, Walras is, in my opinion, the greatest of all economists. His system of economic equilibrium, uniting, as it does the quality of revolutionary creativeness with the quality of classic synthesis is the only work by an economist that will stand comparison with the achievements of theoretical physics."¹ Several concepts were precised by Walras: pure economics, general economic equilibrium, general interdependence. He began by separating pure economics from applied economics. Then he defined the concept of marginal utility and most of all realized the importance of general interdependence. He was among the first economists who wanted economics to be a science like physics and because of that he emphasized the use of mathematics in economic science.

We saw that Pareto was the successor of Walras in Lausanne. His theories show a very strong Walrasian influence. The theoretical work of Pareto has a sociological, philosophical, methodological background. It was very different, and even diametrically opposed to Walras'. But as far as pure theory is concerned, Pareto is really Walrasian. Pareto himself did not think of denying it. When Pareto came to Lausanne he discovered that Walras had developed a system of equations which he was using to represent general equilibrium. On these equations Pareto based his theory. During the first years of

¹Schumpeter, History of Economic Analysis, p. 826.

his teaching and writing in Lausanne Pareto never went beyond the limits of Walras' work. His articles in the Giornale degli Economisti in 1896-1897 and his Cours cannot be called, as some economists claim, his masterpieces. Pareto realized this when he did not want to publish a new edition of his Cours. His Cours is only "a brilliant Walrasian treatise"¹ said Schumpeter. In 1897 with some publications in the Giornale degli Economisti and the resume of his Cours in Paris he started working on his own. Later he left the Walrasian theory of value and introduced the one we are going to study, namely one based on indifference curves. He also transformed Walras' theories of production, capital, money, etc. These new theories appeared in the Manuale di Economia Politica, published in 1906. The Manuel d' Economie Politique, published in 1909, which is the French version of the previous work and is famous by the improvements brought to the mathematical Appendix. This work marks the peak of his economic thinking.

But even in that book:

...it is not more than Walras' work done over...with so much force and brilliance as to grow into something that deserves to be called a new creation...there are not unimportant points in which Walras' system remained superior. Recognition of the quality of his creation does not excuse Pareto's less than generous attitude toward the teaching of Walras from which he put himself at a greater distance than was really necessary.²

Although Pareto was a follower of Walras yet he nevertheless is considered as an important economist in his own right. I think that

¹Op. cit., p. 860.

²Ibid., p. 861.

in some fields Pareto brought something really new. An examination of the main characteristics of his work follows.

After Walras, Pareto reached for a higher generalization of the study of economic equilibrium. "With Walras it (pure economics) became...the theory of a special case of economic equilibrium, namely of free competition."¹ Later Pareto says "In our Cours it (pure economics) became the general theory of economic equilibrium, and we go still farther in that way in this work."²

Triffin notices also that Pareto tended to generalize more and more the theory of general equilibrium:

Pareto, however, in opposition to Marshall does not isolate the monopolist from the rest of the economic system. On the contrary he admits the monopolist into his general system of equations side by side with all the other sellers in the economy, no matter whether these be also monopolist or simply competitors.³

The main contribution of the "Lausanne School" was to emphasize the interdependence of economic phenomena, the study of which required a knowledge of mathematics. Here once more we only have to quote Pareto:

The principal merit of this scientist, (Walras), his very great merit is based on the study...of a general case of economic equilibrium. General economic equilibrium...casts light

¹Vilfredo Pareto, Manuel d'Economie Politique (2nd edition, Paris: Giard, 1927) p. 151.

²Ibid.

³R. Triffin, Monopolistic Competition and General Equilibrium Theory, (Cambridge, Massachusetts, Harvard University Press, 1947), p. 54.

upon the great principle of mutual dependence which requires the use...of mathematical logic.¹

To Pareto the subject of interdependence was based on and arose out of subjective considerations. That is to say that the economic problem was shaped by the activities and value judgments of individuals and not of the mass. It was the highest aim of Pareto's work to define relations and correlations among the economic facts and to show the development of the real economic process. He thought that the excessive synthetic simplifications used by the economists could not lead economic science as far as it was possible. He knew, as we just saw, that Walras' work opened the way to this precision. But, as a matter of fact, from a certain point, he discovered that Walras' studies became merely descriptive and based on observation of the mass.

Here is what Pareto thought about the famous assertion "ceteris paribus":

Such a method conforms to the dispositions of human minds; but it is a mistake nevertheless because it promotes the search for a simple expression of phenomena which are so highly complex that one cannot represent them with the help of a curve.²

Methodologically, the work of Pareto also appears as a reaction against the a priori hypothesis used by the economists of historical schools. Pareto's hypothesis are the most universal possible, based on the elementary activity of individuals, and free of historical postulates. According to Pareto, after having determined his theory, the economist must look for the differences between the reality and his conclusions.

¹Cited by Demaris in The Development of Economic Thought, ed. H. N. Spiegel (New York: Wiley, 1952) p. 635.

²"La Teoria dei prezzi dei signori Anspitz Lieben e le osservazioni del professore Walras," Giornale degli Economisti, 1892.

And then if there is any difference he must correct his theory. But the theory itself, at the beginning must be free of any historical hypothesis.

For Demaria the stress on the absolute and fundamental character of the individual element, which Pareto places at the basis of his general construction of equilibrium, is the major contribution of Pareto to scientific speculation.¹

The real importance of Pareto among the economists arises from his theory of value. We shall examine it in detail when we deal with consumption. Here we are only going to outline the topic.

In modern theory, J. R. Hicks, a British economist, is famous for his value theory. This economist and Mr. R. D. G. Allen published in Economica in February and May, 1934, an article, "A Reconsideration of the Theory of Value." This article has two parts. The second one is, in fact, a mathematical demonstration by Allen: "A Mathematical Theory of Individual Demand Functions." We are interested in the first part, written by Hicks, which deals with general economics and which can be found with some development in Hicks' book, Value and Capital. In this article Hicks exposes his value theory which can now be considered as the most advanced. He bases his own theory on Pareto's theory. But here let Hicks himself speak:

...there has been only one major achievement in this field (value) since 1900... (it) was the work of Pareto, whose Manuel (and particularly its mathematical appendix)

¹"Demaria on Pareto," in H. W. Spiegel (ed.), The Development of Economic Thought.

contains the most complete theory of value which economic science has hitherto been able to produce.¹

And Hicks continues:

Of all Pareto's contributions there is probably none that exceeds in importance his demonstration of the immeasurability of utility. To most earlier writers, to Marshall, to Walras, to Edgeworth, utility had been a quantity theoretically measurable; that is to say, a quantity which would be measurable if we had enough facts. Pareto definitely abandoned this and replaced the concept of utility by the concept of a scale of preferences.²

The first reactions against the old conceptions of utility came from Fisher in his Mathematical Investigations into the Theory of Prices. He realized that the theory of equilibrium could be based on considerations of indifference. But he believed that if we consider more than two commodities it is impossible to deduce any utility function from these considerations.

More simply, Pareto said that even if it was possible to deduce a utility function from a scale of indifference, it would be rather indeterminate.³ The big improvement brought by Pareto was to avoid measurement of utility and to accept only a concept of ordinal utility with a system of indexes. Yet later in his article Hicks says "As it happens, this task was not by any means completely carried through by Pareto himself. Much of his theory had already been constructed before he realized the immeasurability of utility, and he never really

¹Hicks, "A Reconsideration of the Theory of Value," Economica, N. S. Vol. 1 (February, 1934), p. 52.

²Ibid.

³V. Pareto, Manuel d'Economie Politique, p. 139.

undertook the labour of reconstruction which his discovery had made necessary.¹ We shall have the opportunity to verify this and to see how Hicks completed this theory.

Anyway, I think that without exaggeration we can, with Schumpeter, call Pareto "the patron saint of the modern theory of value."²

Another contribution of Pareto's to the domain of economic theory has been the generalised use of indifference curves for the study of general equilibrium. This theoretical tool had been previously used by Cournot and by Edgeworth. The latter had used it for the study of international problems and as an approach to consumption in his book Mathematical Psychics. But Edgeworth used indifference curves in a way contrary to the way followed by Pareto. Let us quote a footnote in the Appendix to the Manual.

The notions of indifference curves and of preference curves have been introduced by the professor F. Y. Edgeworth. He deduced the definition of these curves from the concept of utility which he assumed to be a known quantity. We have reversed the problem. We showed that it is possible to deduce the determination of the economic equilibria from the idea of indifference curves which is directly given by experiment.³

B. PARETO AND DYNAMIC ECONOMICS:

All Pareto's work I have examined thus far is essentially in the field of static economics. At the beginning of the third chapter of

¹Hicks, "A Reconsideration of the Theory of Value," Economica, N. S. Vol. 1 (February, 1934), p. 54.

²J. Schumpeter, Ten Great Economists, p. 130.

³Pareto, Manuel d'Economie Politique, p. 540.

the Manuel, Pareto says that he is going to work in the realm of statics. He barely touched upon the question of how to elaborate a theoretical apparatus which would approach closely the concrete phenomena which are essentially dynamic.

However, in two articles he gave some equations of dynamic equilibrium.¹ Nevertheless, according to Demaria, these equations are written in such a way that they are no more than quasi-static. Pareto knew that an indeterminateness arose from exogenous factors. If one does not examine those factors, "time" has no significance at all.² I think the reason Pareto turned to sociology during the last decade of his life was because he wanted to study those exogenous factors. We also know that, in his correspondence, Pareto often mentioned his intention of writing a treatise in dynamic economics. He was stopped by the lack of statistical material. Pareto was very conscious of the limits of his work and he says himself in the Manuel that any theory is more or less inexact, but is a basis for prospective improvements.³

We have just given an outline of Pareto's contributions. We are not going to study these contributions in which we are particularly interested.

¹Giornale degli Economisti, 1891 and 1901.

²Ideas taken from Demaria, Op. Cit., p. 644.

³Manuel d'Economie Politique, p. 158.

PART II

CONSTITUTION

CHAPTER II

PARETO'S THEORY OF CONSUMPTION

At the beginning of chapter III Pareto proposes a plan for studying economic equilibrium. There are two ways, one of which may be selected. First, he would study every subject and then put all the parts together in order to obtain the complete theory. Second, he would first pose the general idea of the phenomenon, then study the details of the subject previously dealt with in a general way. He would then reach a complete and definitive idea of the problem and of its solution. The first method, analysis and synthesis, is the most rational, but the second is the one which best fits the study of economics because the details have been more often studied than the general problem. The second method may be summarized by: synthesis, analysis, synthesis.

Pareto considers that economic equilibrium is the result of the opposition of tastes and obstacles so his study will include:

1. a general part which will deal with tastes, obstacles and equilibrium;
2. a special part for tastes;
3. a general part for obstacles and
4. a part in which he will examine how those elements act when there is equilibrium.

These four parts are chapters III, IV, V and VI of the Manuel d'Economie Politique. For our study, which has mainly the indifference curve approach for its object, chapter III is the main one. However, we shall mention parts of the other three which may be of

some interest for us.

A. PARETO'S IDEA OF ECONOMIC EQUILIBRIUM:

Before starting the study of indifference curves it may be of interest to examine Pareto's concept of economic equilibrium of which he gives this definition: "One can say that economic equilibrium is the position which would last indefinitely if there was no change in the conditions under which it is observed."¹ A stable equilibrium, if it is only slightly modified, will immediately come back to its first position. If only tastes were to be considered the equilibrium would occur when the individual would have everything he wants. But we must take the obstacles into account. Because of obstacles the individual will never be allowed to have plenty of everything. Pareto gives a definition of obstacles "...they prevent movements, they prevent the use of certain means and they prevent some variations to occur."² To these two kinds of elements he adds a third one: "the real conditions which determine the position of individuals and of transformations of goods."³

So Pareto has set down the basis for his study of economic equilibrium in the first twenty-nine paragraphs of chapter III. For almost the whole study he made use of indifference curves. In using them for the study of tastes, he made an important contribution to

¹Manuel d'Economie Politique, p. 154.

²Ibid.

³Ibid., p. 155.

economic theory. He will also use them for the study of obstacles. But his work in this connection did not have the same importance as in the study of tastes. In fact we shall see that the explanation he gives about this question is rather short and not very helpful for an understanding of his reasoning. In this part much interpretation has to be done. In order to determine the economic equilibrium he will use the curves which are the results of his indifference-curves approach to the problems of tastes and obstacles. He also made a peculiar use of those curves; for instance, when the same curve is considered as two different curves on the same diagram in two different cases. We shall try to explain this use of indifference curve, but it will be without any help from Pareto. But nevertheless I think we must consider with respect one of the first endeavours to study economic equilibrium in a general way.

B. CONSUMER AND VALUE ANALYSIS:

At the beginning of his study of indifference curves, Pareto starts studying the concept of utility.¹ We know that this part of the Manuel is very important in the history of economic thought because it is one of the first attempts to clarify this concept of utility.

In fact, this concept appears to be the basis for the study of consumption, production and, therefore, general equilibrium. But when

¹Manuel d'Economie Politique, p. 156.

one looks at the problem closely a difficulty appears at once: is the concept of utility measurable? If one realizes that utility is mainly a relation of an individual to a thing, and not the property of an object, one sees that it is very difficult to measure. It would be difficult to find a unit which would correspond to something concrete, which would be something other than an abstraction only created in order to facilitate reasoning. And even if a means existed which could be used to measure utility it has not been found. Perhaps this is not very important because economists can get along very well without it.

The fact is that a theory entirely based on a concept of utility suffers from the fact that utility is not measurable. Pareto says: "We admitted that the thing called 'pleasure, value of use, economic utility, ophelimity' was a quantity but no demonstration has been given..."¹

Pareto, as a matter of fact, does not pretend to be the first economist to be conscious of this problem. He says that Jevons, Walras, Cournot, Marshall, Edgeworth and Fisher understood the fact that the use value of a commodity depends also on the quantity already possessed. Pure economics was born when the concept of marginal utility was discovered, and Pareto, conscious of the difficulties which exist in this concept, pushed the study of economic equilibrium further without using that dangerous concept as a basis of his work.

¹Manuel d'Economie Politique, p. 139.

This is important because one may be amazed that right at the beginning of his work the first objective of the author is to find a new word to express that concept. He does not want to use "utility" because the same word has several different meanings. The new word will be "ophelimity" as it was in the Cours.¹

Then the question to be asked is: Since Pareto does not want to use this concept, why does he think it necessary to invent a new name to describe it? And for the reader who keeps reading the Manuel the question is: Why does Pareto keep using the concept of ophelimity throughout his work when he was so careful to show that it was not necessary and even dangerous? This is the question which is asked by Hicks in his book, Value and Capital.² To one who reads the Manuel carefully it appears that Pareto was conscious of that difficulty. I think the reason that he kept using the concept of ophelimity is obvious. In the text of chapter III he tried "to explain without using algebraic symbols, the results reached by mathematical economists"³ in order to allow non-mathematical economists to read his work. The algebraic symbols and reckonings are in the appendix. We may easily understand that, in order to explain thoroughly something which has a mathematical basis without using mathematical expressions, he had

¹See Part I Infra, on Pareto's general works. The Cours was published in 1896-1897.

²Hicks, Value and Capital, 2nd edition, (Oxford: Clarendon Press, 1946), p. 18.

³Manuel d'Economie Politique, p. 161.

to use some means which are not quite intellectually perfect. In order to make his reasoning understandable Pareto had to use the word "ophelimity" or "utility" for the simple reason that anybody knows what it means even if it is not scientifically advisable. It seems that the only thing to be done was what Pareto did. Namely, he pointed out that the basis of his study was "a necessary determination of combinations indifferent for the individual" which is a fact of experiment, as it is shown in the Appendix where the theory is demonstrated rigorously.¹ Then he points out that "when we talk about ophelimity, one must always understand that we want only to indicate one of the systems of ophelimity indexes."² I do not think that, when ophelimity is used as an ordinal concept, the fact that it is not measurable is an inconvenience. However, ophelimity is not always used as an ordinal concept. Pareto also says that the concepts of value of use, utility, ophelimity, ophelimity indexes, etc. facilitate very much the exposition of the economic equilibrium theory but they are not necessary to build that theory. Hicks found a way of exposing his value theory without using the concept of utility. He replaced it by "marginal rate of substitution". The meaning of diminishing marginal rate of substitution is given by Hicks: "The more X is substituted for Y the less will be the marginal rate of substitution of X for Y."³ Since X and Y are two commodities which can be measured, their marginal rate of

¹Ibid., p. 160.

²Ibid., p. 159.

³Value and Capital, p. 20.

substitution can be measured, and the difficulty of utility is overcome. As we shall see later Hicks uses this concept for the theory of complementary goods.

As for Pareto, like many economists of his time, he was confronted with the problem of cardinal and ordinal utility. If we examine the theories of utility before the Manuel we see that they dealt mainly with cardinal utility. Menger and Böhm-Bawerk thought that utility was a directly measurable quantity.¹ For Marshall pleasure and utility were not directly measurable. He was more prudent, and Schumpeter says, "this was no doubt a step in advance."² Edgeworth introduced the fact that the utility of a commodity was not only a function of the quantity of that commodity, but also of the quantities of the other commodities that enter the individual's budget. This was an idea which brought a complication by introducing a number of new equations.

Pareto made the next step forward when he showed that we could do without that concept of measurable utility and introduced the concept of ordinal utility. This is based on the fact that everybody knows that some things give him more pleasure than others. That is what he means when he talks of "ophelimity indexes" and "system of indexes". I shall show later that sometimes he comes back to cardinal utility. This, according to Schumpeter, might be because of his Walrasian background.

¹J. Schumpeter, History of Economic Analysis, p. 1060.

²Ibid.

As it has just been seen the final job was done by Hicks and Allen in a way which is both simple and exact. I shall give a theory of the "rate of substitution" later.

C. THREE "MODELS" OF PHENOMENA:

The next step of Pareto is to define three "models" of phenomena which are, in fact, a simplification of the types of markets used in modern theory. Model (1) exists when the individual acts only in order to reach directly his personal advantage. He seeks "exclusively to satisfy his tastes, a certain position of the market being given."¹ When he acts according to model (2) he seeks "to modify the conditions of the market in order to get an advantage or for another aim."² Later he points out that the people who act according to model (1) are in competition and that those who act according to model (2) are monopolistic competitors. Model (3) corresponds to the collectivist organization of society.

Obviously this classification is rather general and actually too simplified. Many improvements have been made since the appearance of the Manuel, such concepts as pure competition, perfect competition, and monopolistic competition have been introduced in order to facilitate and to make more precise the study of equilibrium. Here Pareto only gives a general framework. Hicks says in Value and Capital that the work of Pareto has been very influential but that it is only a beginning.³

¹Manuel d'Economie Politique, p. 163.

²Ibid.

³Hicks, Value and Capital, p. 5.

D. INDIFFERENCE CURVES OF TASTES:

Pareto's reasonings appear to be the prototype of all constructions of indifference curves. In a footnote Pareto points out that he took the word indifference curve from the work of Edgeworth, who, as we know, deduced them from the existence of "utility". I shall now summarize Pareto's analysis, and in particular examine some points about which Pareto's ideas differ from modern theories. Some of his ideas will be examined as he explains them in chapter IV which deals with tastes.

Pareto's reasoning about indifference curves of tastes are in paragraphs 52 to 59 and 93 to 99. Their study will be short since they are very similar to modern indifference curve analysis.

Pareto begins by setting up an indifference schedule between two commodities: bread and wine. The individual is indifferent about having any one of the combinations of bread and wine since in each one what he loses in terms of bread he gets in terms of wine or vice versa. If he was confronted with all the combinations he would not know which one to choose.

The next step of Pareto is to build the indifference curve which arises from the consideration of the indifference schedule. He does it by representing each combination by a point between two axes representing bread and wine.

Then he defines the index: two combinations which are equivalent have the same index; if one combination is better than the other

It has a higher index. These indexes are called ophelimity indexes. So on the same indifference curve all the combinations have the same index.

If one has other indifference schedules on other pleasures given by other initial combinations, one will have indifference curves with different indexes. Some indexes are higher and better than the first one that we considered; some others are lower and inferior. The axes will be covered with a complete representation of the tastes of an individual, between bread and wine. If the indexes of ophelimity are considered as representing the height of a curve we may look at the curves as representing a hill on a map. This will be the pleasure hill. Therefore an individual who enjoys a certain combination of bread and wine may be represented by a point on the hill. His pleasure will be represented by the index in an ordinal way (for instance, index 6 is better than index 5). So the individual will always prefer the combination which has the highest index.

E. THE EQUILIBRIUM OF THE CONSUMER: (FIG. 1)

Pareto considers an individual who is trying several combinations of bread and wine. He will stop when his tastes are satisfied in the best way. This point is the highest on the part of the pleasure hill the individual may run over. Here Pareto considers that the individual moves along a straight line because this case is generally the most frequent. If the individual has every week a quantity om of A, let us suppose that in order to transform A into B he follows

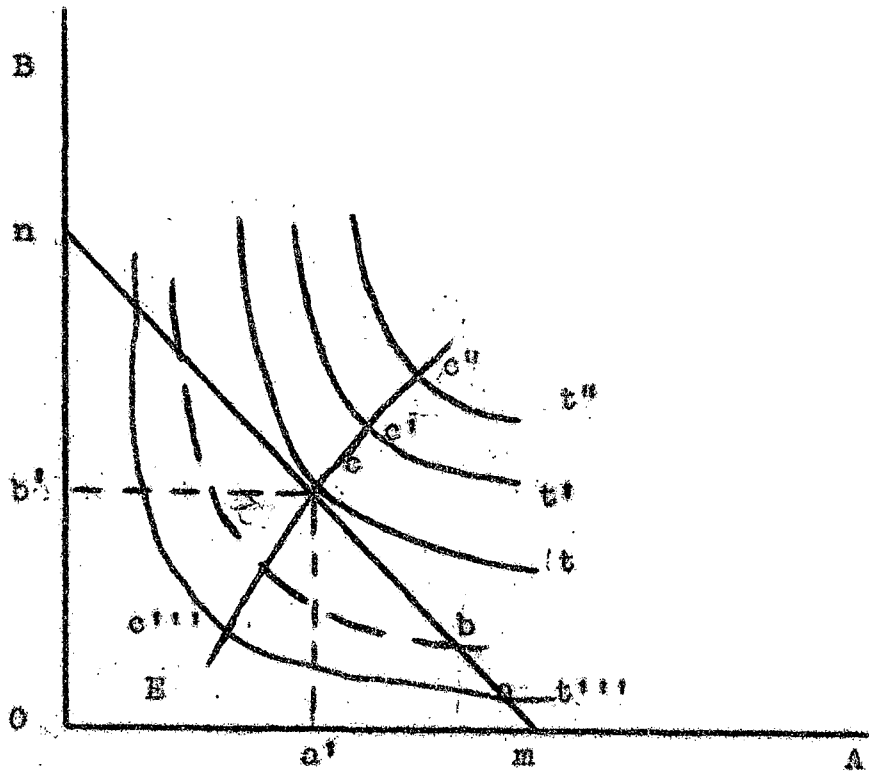


Fig.1

the straight path mn (Fig. 1). At a there is not equilibrium since the individual prefers to go to b where the index is higher on curve ti . The same move occurs at b . But at c if the individual keeps going he will arrive at a lower indifference curve. So at c there is equilibrium. On the other indifference curves c , c' , c'' , d'' are other equilibrium points on other paths which are supposed to be run over. If we join all those points we shall have the exchange curve $E E'$.

Pareto says that terminal points between m and c may be equilibrium points. We shall examine this later.

The modern theory of indifference curves, as it is currently taught, is very similar to this. However, there are some differences. First, Pareto does not consider what is the shape of the consumer's indifference curve in the main text. Hicks and the modern theorists deduced the shape of the indifference curve only by deductive reasoning. Pareto did something like that in his chapter IV, but he is much less clear than the modern theorists. Nevertheless, in chapter IV which is entirely dedicated to tastes he understood ophelimity decreases when the quantity increases. The modern theory makes the considerations on the shape of indifference curve without so much trouble.

Second, Pareto's study omits any use of the concept of price or income; this complicates, and at times makes his analysis erroneous.

But we must acknowledge that a study of both theories shows that Pareto was really the one who started demand theory and the indifference curve approach in their modern form.

F. THE PATHS:

By 'path' Pareto seems to mean any line which is run over by the individual on an indifference map. Such a concept exists also in the modern indifference curve approach to demand theory. But in modern analysis what Pareto calls a 'path' could be a price line, or at least one may think so at first thought. In reading Pareto's analysis on this question one is amazed not to find anything about prices. When, much later, Pareto talks about prices (paragraph 152) he says that he pursued his entire study without directly taking prices into account in order to show that pure economics does not especially consider a market with prices, but more generally, an equilibrium arising from the opposition of tastes and obstacles. Scientifically this is an idea right in the line of thought of Pareto, but practically it gives very odd results. On a price line the individual moves in a way which is imposed by the price ratio of two commodities. This may be easily understood and, by nature, a price line is a straight line since the price-ratio is constant.

When one looks at the paths of Pareto one sees that he did not draw all of them straight. On the contrary, at least at the beginning of his analysis, he drew a curved path.¹ In the text Pareto does not give any reason for the movements of the individual on the indifference map (it would be difficult to give any without using the words "income" or "prices"). One only knows that the individual went from one point to another and the author does not give any reason for this.

¹Pareto, Manuel d'Economie Politique, p. 172. See also Fig. 3 on page 33 of this thesis.

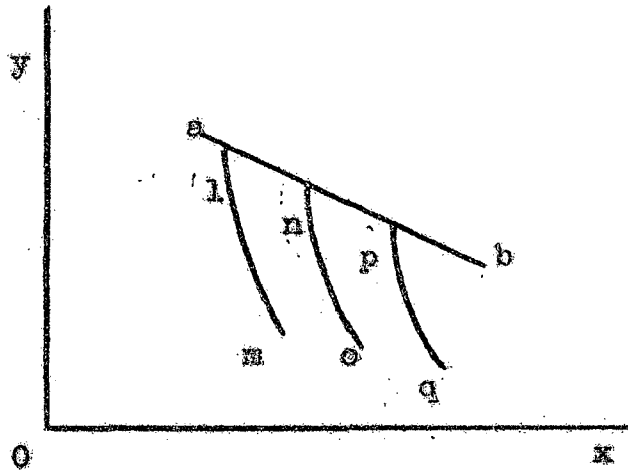


Fig. 2

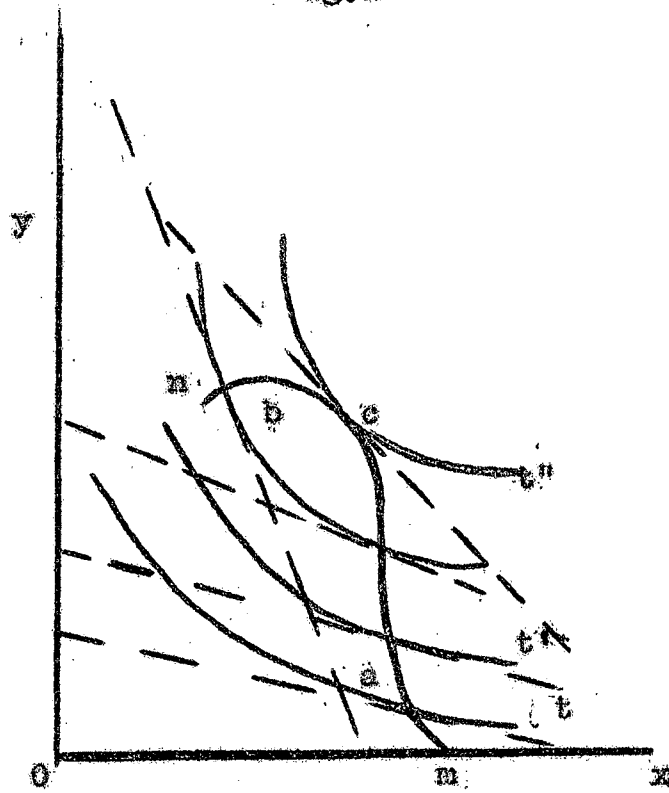


Fig. 3

If one tries to explain the strange shape of the path mn in Fig. 3 one has only one explanation available: a succession of changes in prices and in income occurring either at the same time or at different moments. But then the path is no longer similar to a price line but rather to a price-consumption curve or an income-consumption curve or both at the same time.

Later, when he studies the equilibrium of tastes (paragraph 97), Pareto uses a path which is straight and obviously a price line, the slope of which measures the ratio of the prices of two commodities (Fig. 1, mn in page 30). He supposes similar paths (or price lines) which may be run over by the individual. Since he does not want to use "income" he cannot say why there is a movement of that sort. And so he gets an "exchange curve" which is nothing but an "income-consumption line". The nature of the "exchange line" is very different from that of a price line. Nevertheless, Pareto calls it a 'path'. If we turn to Fig. 3, where the curve mn is called a path, we shall see that mn is, in fact, similar (though not in shape) to the exchange curve of Fig. 1.

Confusion arises from the fact that Pareto calls two very different things by the same name: "path". Because he did not want to use prices and income, Pareto made a mistake which obscured his meaning. In chapter III Pareto shows that he knew about the price lines of which the slope measures the price ratio of two commodities. "the price of B in terms of A equals the slope of the straight line in n on axis OB and the price of A in terms of B is expressed by the slope of the same line on axis OA (see Fig. 1)"¹

¹Pareto, Manuel d'Economie Politique, p. 208.

Another matter worthy to be studied is the question of the points at which the individual stops. First there is the tangency point of a path to an indifference curve and this is clearly explained in paragraph 97 of chapter III. In that case the path is a price line.

But earlier in his study¹ Pareto talks about terminal points at which the individual who runs over the path mn (Fig. 3) must stop because of "obstacles". One may think of many obstacles but any kind of obstacle in a market may be translated into terms of prices and income. Prices and income are the manifestation of "obstacles" rather than the best known obstacles as Schultz says.² Anyway they are the only ones which can be represented on a diagram. Thus on the path mn the individual will stop at the point where the price line (representing the obstacles) will cross the path representing a price consumption curve). At that same point the price line is tangent to an indifference curve. On the price consumption curve this will be a terminal point.

What is the nature of the tangency point on such a price consumption curve? We can see why changes in orientation occur because of combined changes in the price ratio and in real income (See Fig. 3). But we cannot see how the individual would voluntarily stop at the tangency point on path mn (on Fig. 3). Rather, he would follow the movement of his real income and of the price ratio; in other words the

¹Ibid., p. 172.

²Schultz, The Theory and Measurement of Demand, (Chicago: University of Chicago Press, 1938), p. 24.

movement of "obstacles" which do not depend on him. This movement is represented by the movement of the price line. The individual has to follow the movements of the price line which represents the changes in his income and in the price ratio of the two commodities. So it is obvious that the tangency point on path *mn* (Fig. 3) (which is a price-income consumption curve) is not similar to the tangency point on path *mn* (Fig. 1) (which is a price line). The price-line represents a choice the consumer has when he disposes of a certain income with prices being given. On that line the consumer moves until he reaches the maximum "opholimity" position. He is free to do so and as long as neither prices nor income do not change, there are no obstacles which could stop him. Thus there is no terminal point.

On the price-income-consumption curve the individual is not free to move. (Who is able to determine entirely his own income and the prices of the commodities consumed?) The movements of the individual on that line are conditioned by the "obstacles" over which he has no, or very little, power (at least in model 1). I think then that we can say that all the points at which the individual stops on that curve are terminal points. If there happens to be a tangency point, it is not similar to the tangency point of the price line and it is itself a terminal point.

G. SOME OTHER POINTS NOTED BY HICKS:

In his book Value and Capital Hicks wrote a footnote about a mistake in Pareto's reasoning:

Instead of relying solely upon the true principle of diminishing marginal rate of substitution (that the rate will diminish when X is substituted for Y along an indifference curve) he put forward also what we may now justly regard as a false principle--that the marginal rate of substitution of X for Y will diminish when the supply of Y is reduced without any increase in the supply of X. If this were always true it would exclude the possibility of X being an inferior good.¹

Pareto begins by saying: "this characteristic seems to belong surely to the goods the consumption of which is independent. If one has 5 of X and 5 of Y and that, of one passes to a higher indifference curve, one has still 5 of Y and 10 of X it seems...that at that second position one will give more X for 1 of Y than at the first position (diagram 4)...But this is doubtful for the goods which have a dependence of second type (if Y is an inferior commodity and X a superior commodity). When X and Y are consumed at the same time by an individual, one conceives that he may exchange a certain quantity of X for a certain quantity of Y; for instance 1 of X for 5 of Y. But when the individual has plenty of X and Y disappears from his consumption, he may refuse to give 1 of X for even a very large quantity of Y."²

We can represent this reasoning on two diagrams which are shown in figures 4 and 5.

If we take a look back to the footnote of Hicks we see that he transformed the reasoning of Pareto by assuming a reduction in Y when X is constant. According to Hicks, Pareto neglected the fact that X could be an inferior good.

Pareto assumed in fact an increase in X when Y is kept constant (this is the other side of Hicks' interpretation), but he does consider the possibility of having to deal with an inferior good. He says explicitly: "If Y is an inferior good and X a superior good"³ If X

¹Hicks, Value and Capital, p. 29. This is related to page 573 of the Appendix of the Manual where Pareto deals with this problem.

²Pareto, Manuel d'Economie Politique, p. 573.

³See quotation of Pareto from page 573 of the Manual.

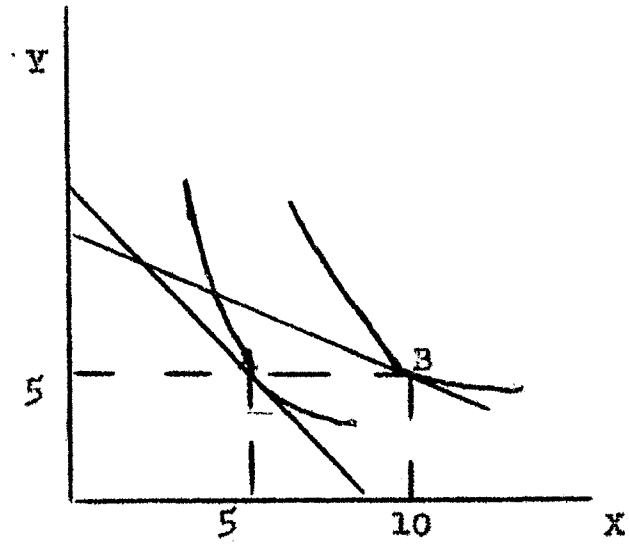


Fig. 4

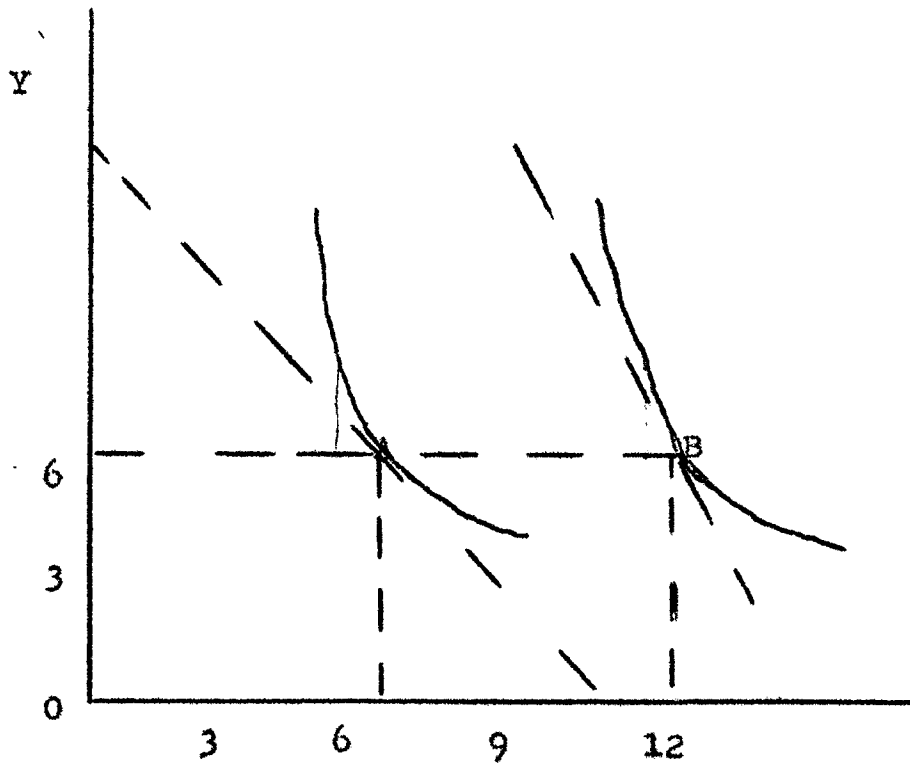


Fig. 5

was an inferior good the slope of the indifference curves would be steeper on axis OY (Fig. 5). They would be much less steep on the X axis. But basically nothing would be changed. This is developed, less neatly though, in the text of chapter IV.¹

We may also notice that figure 5 is the one which is used to study the so-called Giffen paradox. Pareto, moreover, studies that problem in paragraph 48 and 49 called "Relation between opheimity or indifference curves and supply and demand". He explains clearly that when the price of an inferior good decreases its consumption may start increasing but will soon fall because superior goods will come and replace it. However, progress has been made in this question with diagrams which permit us to distinguish an income effect and a price effect in that movement. But here also Pareto has been a predecessor to the modern economists.

Let us now consider the modern theory of this phenomenon. Let us consider figure 6. Y is money and X is the inferior good (it was Y in figure 5).

On figure 6 we see that when we apply some price-lines representing a decrease in price ($P_1 > P_2 > P_3$) the quantity increases at first: $q_1 < q_2$ and, when the price keeps decreasing, it decreases $q_2 > q_3$. The quantity of money spent on X keeps decreasing $O'M_1 > O'M_2 > O'M_3$ because X is an inferior good.

On the next diagram (Fig. 7) we distinguish the price and income effect. We see that when the price decreases from P_1 to P_2 the quantity $Oq_2 < Oq_1$.

¹Pareto, Manuel d'Economie Politique, p. 252 and 267-268.

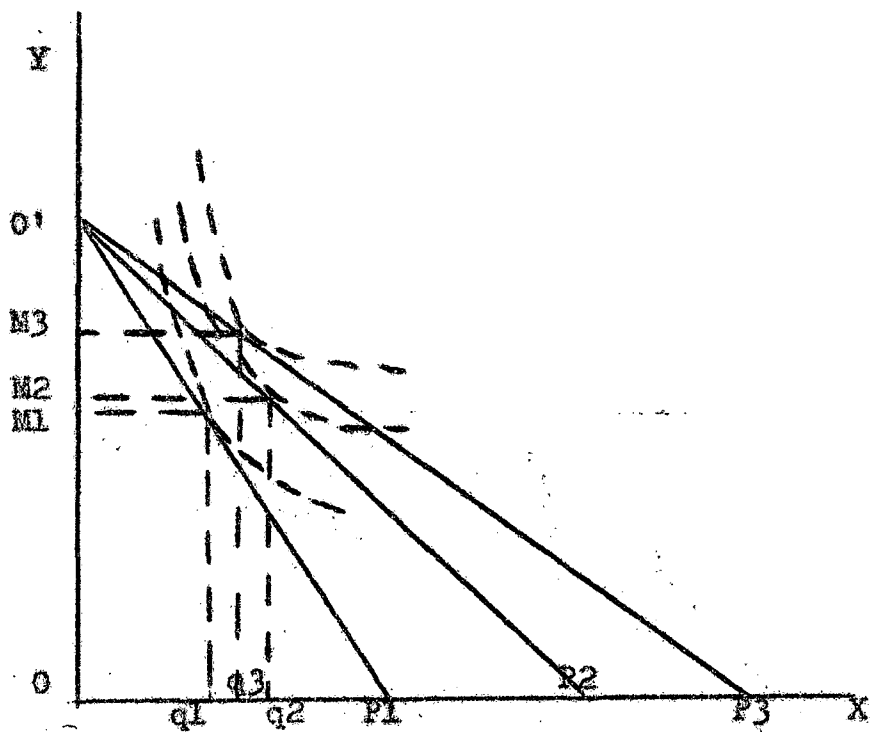


Fig.6

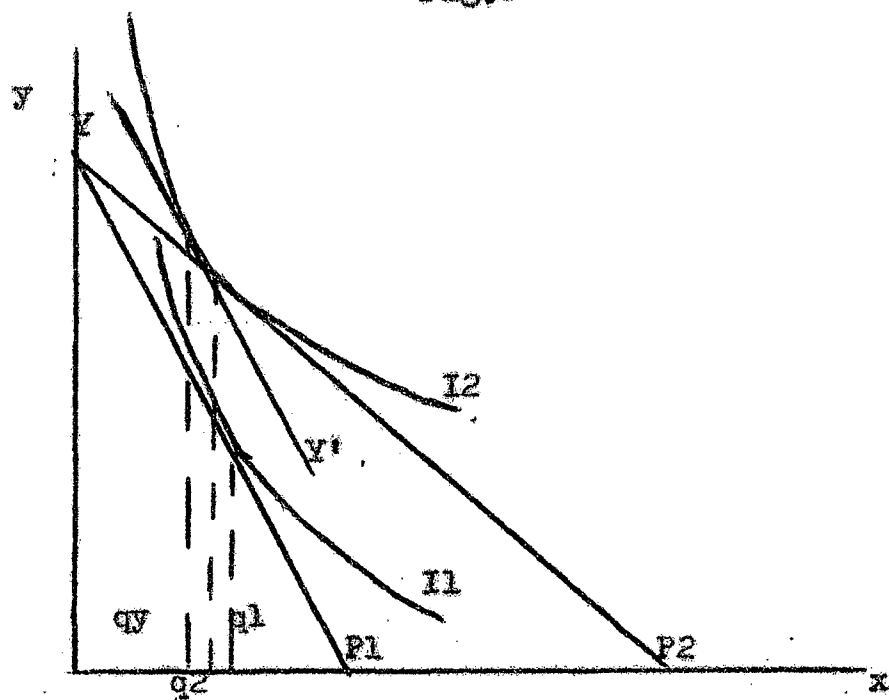


Fig.7

If we had only a change in income the quantity Oq_1 would be determined by a parallel XY' to the first price line, tangent to the second indifference curve I_2 . The difference between Oy_1 and Oq_1 is the income effect, but there is a price-effect which is contrary to the income effect but smaller; it is the difference between the income effect and the real decrease $q_1 \rightarrow q_2$.

H. COMPLEMENTARITY:

Pareto, with Edgeworth, was one of the predecessors of the modern theories of complementarity as it is expressed by Hicks in his book Value and Capital. In paragraph 39 of chapter IV of his Manuel, Pareto defines complementary goods thanks to the concept of "elementary ophelimity" which is very close to the modern idea of marginal utility: "the pleasure given by a small quantity of A added to the consumed quantity is ordinarily larger when we suffer less from the need of other commodities; consequently, the elementary ophelimity of A increases when the quantities of B, C increase."¹ And we see that here Pareto did not keep his promise about the use of ophelimity that he made in chapter III, paragraph 36. In that paragraph he said that by ophelimity he meant one of the systems of ophelimity indexes. Here, ophelimity must be taken as something measurable since it is the basis of the definition of complementary goods. Furthermore, in paragraph 35, chapter III he points out that even if we found a unit in order to measure utility it would remain undetermined if the utility depends also on the consumption of other goods.

¹Pareto, Manuel d'Economie Politique, p. 268.

So Hicks is quite right when he notices this at the beginning of his chapter on complementarity.¹ In order to overcome this difficulty he uses in his own definition the term "marginal rate of substitution for money" which, as a matter of fact, is the "marginal utility in terms of money". Money in that case is the third and stable element to which two complementary goods are compared. It is almost impossible to compare them to each other since in the definition the quantities change at the same time.

But another point made by Hicks must be rectified. He says "when he tried to translate his definition into terms of indifference curves he got into difficulties."² It does not appear to me, however, that he got into difficulty for real complementary goods. For Pareto the indifference curve of two complementary goods is formed by two perpendicular straight lines, each of them parallel to one of the axes and meeting at the point where there is enough of the two commodities to use them. When two commodities are absolutely complementary there is a ratio in which they must be used, let us say one to one as for gloves (Fig. 8). So the crossing of the two straight lines which form the indifference curve of index i will be at A (one left glove and one right glove). If we are at B we have one left glove and 3 right gloves, at C we have one right glove and 3 left gloves but these positions are indifferent to us because we cannot use the extra gloves. So the curve really represents an indifferent

¹Hicks, Value and Capital, p. 43.

²Ibid.

choice. And here, ophefinity really means a system of indexes.

I think there is more than "some" parallelism between the case when X and Y are complementary, and that in which the indifference curves between X and Y are very bent.¹ But it is true that the matter becomes more difficult when Pareto deals with goods which are "only approximately complementary". Here is the reasoning of Pareto: If an individual has the smallest quantity of bread necessary not to die by starvation, say one slice, and the smallest quantity of water necessary not to die by thirst, say one glass, it is evident that he would not give up any quantity of bread for any quantity of water: therefore, the indifference curve will be with a 45° angle. For larger quantities of bread and water there will not be an angle any more but a curve between two straight lines; but when the individual has plenty of water and bread so as to be completely satisfied there will be a 45° angle again.

If we consider the first indifference curve drawn by Pareto, with bread and wine as examples, we see that something is wrong. At point B the individual will have one glass of water and 2 slices of bread and in that case anybody would prefer to be at B than to be at A (on Fig. 9). So the index of ophefinity, as Pareto says, is not the same all over the so-called indifference curve, so that curve cannot be called an indifference curve. This difficulty arises, I think, from the fact that bread and wine are not really two commodities complementary to each other. Bread and water are both necessary to an individual; a minimum quantity of both is needed in order to stay

¹ Op. cit., p. 45.

alive; that is why no substitution is possible between them at the minimum point. But there is not any fixed ratio which rules their use when one increases the quantity of either bread or water. The case of bread and water cannot be compared to the case of right and left gloves.

The relation of the two former commodities is not at all similar to the relation between the two later ones. That is why the curve which was in "indifference curve" for the gloves is not any more an indifference curve for bread and water. For combinations of bread and water, on a certain part around the angle, there is no indifference on that curve. Of course if we gave a tremendous number of slices of bread to the individual, from a certain point on there would be indifference.

When he wanted to treat another degree of the problem, Pareto, as a matter of fact, changed the nature of the problem. This problem, at least when we consider quantities which appear to be absolutely necessary, cannot be conveniently treated by the indifference curve analysis.

If the quantities of both bread and water increase the angle is going to become a curve. The point at which the curve becomes a straight line is the one where there is such a quantity of bread of such a quantity of water that even if one increases the quantity of bread without decreasing the quantity of water the index of opportunity is not increased. This point is similar to the one at which, on the first curve, indifference started. But then there could not

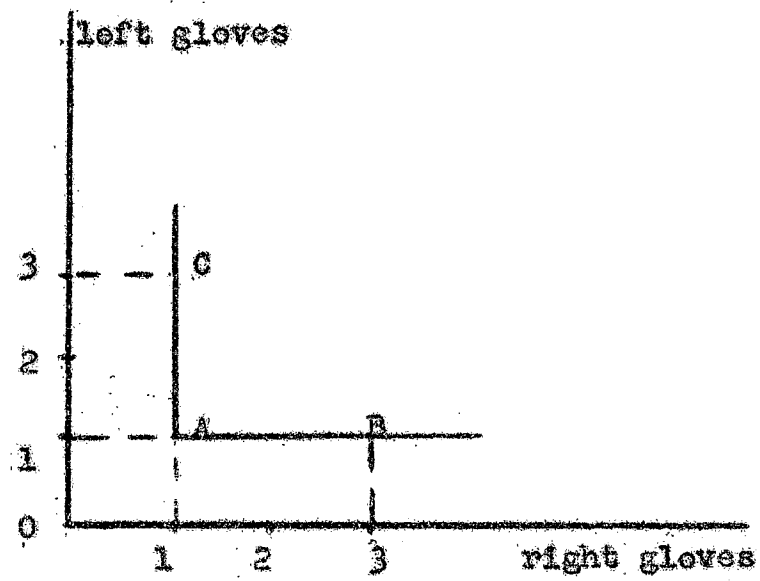


Fig. 8

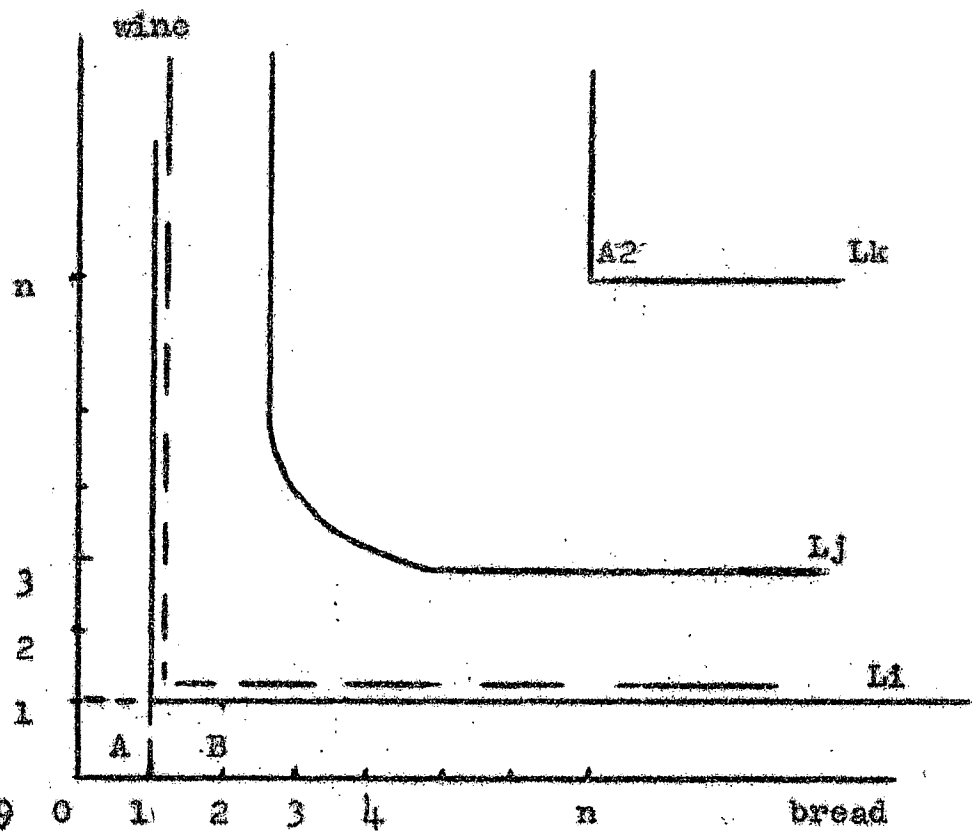


Fig. 9

be any curve to the left of that point because no commodity could suffer a decrease in its quantity. In this second case both quantities of bread and wine, to a certain extent can be decreased. This is represented by the curve.

The third curve is built on the assumption that the individual has plenty of bread and water, enough to be quite satisfied (this happens most of the time in civilized countries for water). The point determined by those quantities is A_2 on Fig 9, and there the individual does not desire more water nor more bread; so he is indifferent about being at any place on the curve since at any place he has plenty of both commodities. Since no more water or bread is needed by him we could say that their marginal utility is equal to zero as it was for the extra left or right gloves. This curve is really an indifference curve. The central part of the curve I is not an indifference curve; the indifference parts of that curve begin somewhere at the right of A and above A_1 .

CHAPTER III

MODERN INDIFFERENCE CURVES APPROACH

FOR CONSUMPTION

Nowadays the leading theorist of the indifference curves of tastes is Hicks, who published his, Value and Capital, in 1939. In the first part of his book, Hicks deals with indifference curves. In the first chapter we can observe that when he wants to develop the indifference curve approach he decides that the best way to do so is to take over the Paretian analysis. The changes brought to those reasonings are mainly changes in form. Now prices and income are directly considered and so the ambiguities of Pareto's reasoning are avoided. For the same reason the different sorts of paths are distinguished in price-line and income-consumption curves and price-consumption curves. These transformations brought more clearness to the theory which became more concrete. But as far as the indifference curve approach is concerned, we see that the basic sources are found in the Manual.

We also saw that Hicks' works are important because he brought to an end the evolution of the utility concept. In that evolution we saw how Pareto had been "the beginning of the end" when he avoided the difficulties presented by cardinal utility. However, he did not perform the job completely by finding something which could conveniently take the place of cardinal utility. So he did not stick to

his resolutions for he still used the concept of marginal utility (for instance, in his works on complementarity).

The big merit of Hicks in that matter was to introduce the concept of "marginal rate of substitution" which advantageously took the place of "marginal utility" in all the cases where it had to be used. So utility became only an ordinal concept, a "system of indexes," as Pareto wanted it to be. This was the big improvement brought to Pareto's theory of value.

Considering directly prices and income, Hicks applied the price lines to his indifference map with more perspicacity than Pareto. He distinguished clearly the income effect and substitution effect of a change in price. In Pareto's writings that distinction was implicitly made since we saw that we very clearly understood the so-called "Griffen paradox" and the case of inferior goods. The job of the modern theorists was to put down those ideas on diagrams in order to make them more easily understandable, as we saw in the modern study of the Griffen case. But here also Pareto had the idea which was used as a source. Pareto's theory of value was also an important source for modern theorists because of the statement of interdependence which it includes. We know that Pareto wanted to study an equilibrium which was really general. He claims that it is by this that his work brings something new. Demaria also insists on that point.¹ This care of interdependence permitted Pareto to make a very interesting study of complementarity even though, on some points, he made some mistakes.

¹"Demaria on Pareto," in H. W. Spiegel (ed.), The Development of Economic Thought, pp. 623-631.

Briefly, the main improvements which have been brought to the indifference-curve approach to the value theory and to the consumer's-demand theory have been the introduction of marginal rate of substitution, the distinction of price line, price-consumption curves, and income-consumption curves, more clearness in the study of changes in prices and some ameliorations in the study of complementarity. The considerations on elasticity have been introduced too, but like many other ideas they were implicit in Pareto's works. But now elasticity is studied in an interesting way as a function of the direction of the price consumption curve.

For the sake of clearness we are going to see how modern theorists treat the consumer's indifference curve analysis; the reader will be able to realize the differences between Pareto's work and the modern theory and to see that basically there are not many differences.

The construction of a system of indifference curves is based on the tastes of an individual for two goods. If an individual has, for instance, 10 units of y and 10 units of x he may be indifferent about having less x , if he gets more y , or about having less y if he gets more x . That is to say, if we do not take into account the prices of the commodities, he will be indifferent to having 10 of x and 10 of y , or 8 of x and 15 of y , or 7 of y and 15 of x . Thus we can find an infinite number of combinations of two goods which will have the same importance for an individual (Fig. 1, page 54). If one measures on two axes x and y one can define by their coordinates the points which correspond to 10 x and 10 y , 8 x and 15 y , 7 y and 15 x . It

is always possible to join these points by a curve (and also all others which can be found by a further examination; as a matter of fact, this further examination would be very difficult and of no interest to us for it is possible to define the general shape of this curve by a simple reasoning and that is enough for us).

If one starts from a larger quantity of y than of x and if y decreases it is evident that the amount of x will have to increase. Therefore, we see that the curve will slope downwards, toward the southeast side of the diagram, from left to right. Now, more precisely, will this curve be a straight line? We can see that if it is so, when y decreases by one unit, x increases by one unit, whatever the quantities of y and x are at that moment. Then if the curve is straight y and x are perfect substitutes for one another.¹ It is easy to see that, as a matter of fact, we cannot find two really different commodities which will be perfect substitutes. Therefore, in most of the cases the substitution cannot be represented by a straight line. In fact, when one has a large quantity of y and a small quantity of x if one wants the quantity of x to decrease, the quantity of y will have to increase by an always larger amount for each unit of x lost. If the quantity of y decreases, at the beginning, when the quantity of y is still much larger than the amount of x , a little increase of x will be enough to compensate for a large decrease in y . But as the quantity of y is getting smaller and smaller, the increase of x will have to get

¹ Pareto deals with this problem in Manuel d'Economie Politique, p. 250.

larger and larger in order to compensate for a decrease taken on a smaller and smaller quantity of y. If we examine how this can be represented on the diagram we shall see that the indifference curve will be convex to the origin of the axes.

This convexity of indifference curves of consumers brings the concept of rate of substitution. The rate of substitution of x for y is the quantity of y necessary to compensate for a loss of one unit of x at each point of the curve. We can see that this quantity of y, necessary to compensate for a loss of x, increases as the quantity of x decreases and as the quantity of y increases. That is the principle of diminishing rate of substitution. In fact, the slope of this curve represents the ratio of the quantity of x lost to the quantity of y gained. In this ratio the quantity of y gained always increases when the amount of x lost is constant, therefore, the ratio decreases. The slope of the curve can also be expressed by the ratio of the marginal utility of y to the marginal utility of x. This ratio varies in the same sense as the previous one. For, when the amount of y increases, the marginal utility of y decreases; therefore, the ratio

$$\left(\frac{m.u.y}{m.u.x} \right).$$

This is what is expressed by Hicks when $\frac{\text{marginal utility of y}}{\text{marginal utility of x}}$
 slope of the curve $\frac{\text{amount of x lost}}{\text{amount of y gained}} = \frac{\text{marginal utility of y}}{\text{marginal utility of x}}$.

As a matter of fact, this can be more clearly explained in the following way. The amount of x lost and the amount of y gained must be equivalent not as to their quantities but as to the utility they

give at a certain moment. This condition is necessary in order to remain on the same indifference curve. In other words, as Hicks explains it, the amount of x lost multiplied by the marginal utility of x must be equal to the amount of y gained multiplied by the marginal utility of y . We notice that for this reasoning we started from one indifference curve based on a combination of 10 of x and 10 of y ; but we may imagine other indifference curves based on combinations of 9 of x and 9 of y and 11 of x and 11 of y ; 12 of x and 12 of y , etc. Thus the space between the axes will be filled with a whole pattern of indifference curves for which the previous reasoning would have the same value. Then we should obtain a map of indifference which would represent the tastes of an individual as for two commodities x and y .

Until now we have considered two commodities in terms of their quantities. On the market another element exists: the price. In order to get an idea of the consumer's demand we have to take this item into account. The quantities of commodities x and y which can be consumed by an individual with a fixed income are determined by the price of these commodities. We have considered the ratio of the amounts of x and y ; we are now going to consider the ratio of the prices of the commodities. With a fixed income and given prices of x and y an individual can purchase a whole series of different quantities of x and y . When this individual purchases more x he will have to purchase less y , and inversely. On two axes (Fig. 2) OX and OY let us measure the amounts of x and y . We could draw a line of the quantities of x

and y which can be purchased with a fixed income. This curve will slope from the northwest of the graph to the southeast from the left to the right, since when x increases y must decrease. This happens regularly because the ratio of the prices is constant. Therefore, the curve will be straight and will meet the axis at points x and y showing the quantities of x and y which can be purchased if the whole income is spent for one commodity. Therefore, OX and OY are proportional to the income of the individual, and if this income increases (the price ratio remaining the same) we shall have a new line $x' y'$ with OX' and OY' , longer than OX and OY and parallel to the first one. So on the same graph (since the axes are the same) we have a system of curves showing the possibilities of an individual for two commodities as for the quantities, and a curve showing, with a given income, the possibilities as for the prices.

The point at which the price line is tangent to an indifference curve indicates the quantities that will be elected in order to be purchased. In fact, if this point was at a or b , before or after the point of tangency (Fig. 2) the individual could still purchase a larger total amount of x and y by going upwards or downwards on the price line and finding indifference curves which represent quantities which bring to him a larger utility than the ones represented by the indifference curve on which he was staying before. At this point of tangency the price ratio is equal to the slope of the indifference curve or to the ratios we have previously defined. When he is on it,

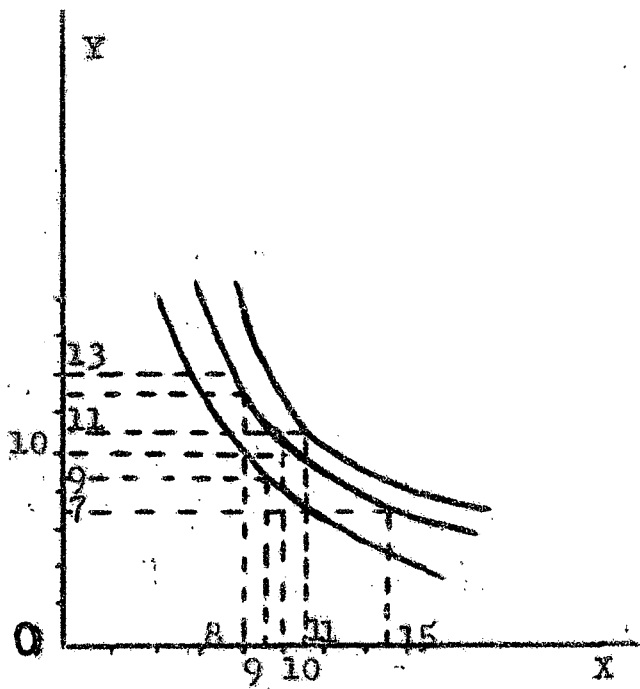


Fig. 1

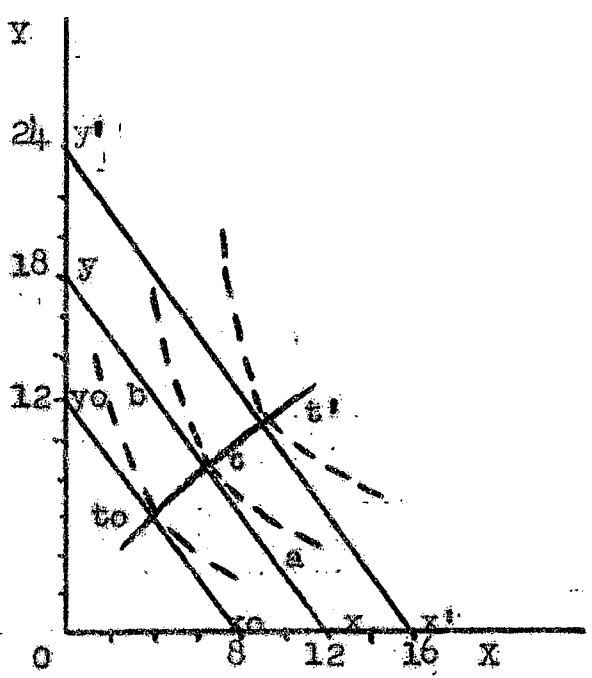


Fig. 2

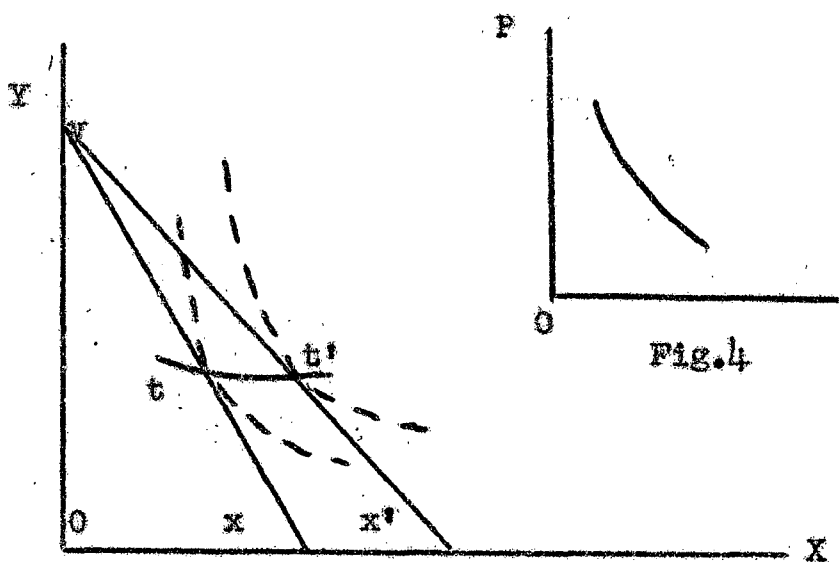


Fig. 3

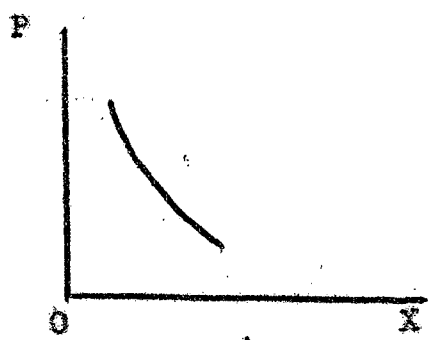


Fig. 4

the individual, if he goes upwards or downwards, must get on a lower indifference curve; there, if is a point of equilibrium.

Now if the income increases or decreases there will be new curves $x^1 y^1$ or $x^0 y^0$ parallel to the first one. If there is a decrease of income they will be closer to the origin, farther if there is an increase. All these curves will have points of tangency with indifference curves. If we join all these points (Fig. 2) we can obtain an income-consumption curve showing the way in which consumption changes when income varies.

If now the price ratio changes, the income remaining the same, the slope of the line must change. If the price of X decreases and if one spends one's whole income on X, more units of X will be purchased; but for Y the point y will remain the same (Fig. 3). The inclination of the price line will decrease on OX (it would increase for a decrease in the price of Y). What happens to the point of tangency? It goes in the sense of an increase in X. That is to say, the price line is tangent to a higher indifference curve, but at t' a larger proportion of X will be purchased than at the first point of tangency t . So with various prices for X we can draw a series of price lines, each one having a point of tangency with a different indifference curve. By joining all these points we get a price-consumption curve, which shows that as the price of a commodity decreases the quantity purchased increases. Now on a separate diagram (Fig. 4) we can illustrate this statement. Of OX measure quantities of X demanded and OP the prices, we shall have a curve sloping downward from the left to the right (quantity increases when price decreases).

As a matter of fact, the increase of the demand is the result of two tendencies: the individual will buy more X since its price is lower (this is similar to an increase of the income of the individual); on the other hand, he will substitute X for other commodities since the price ratio is changed.

The class of exceptions to this rule is formed by the inferior goods. When the decrease of the price of an inferior good occurs, it frees a part of income which will be used by the individual for the purchase of better goods.

PART III

PRODUCTION

CHAPTER IV

PARETO'S THEORY OF PRODUCTION¹

Pareto's theory of production as a source for modern theories is not as important as his theory of consumption. His approach to the problem of production is based on the indifference curve and will be the topic of this chapter; Pareto calls the indifference curves of production "indifference curves of obstacles". This definition of obstacles has been given at the beginning of chapter II. The difference between the two types of obstacles that he distinguishes is rather difficult to see. But it will become clearer later on in the course of this study.

A. INDEPENDENCE CURVES OF OBSTACLES

Pareto supposes that a commodity A is transformed into another one B, and he proposes to measure the quantities of A and of B on two axes. If, for each quantity of A put in the process, he indicates the quantity of B produced he obtains a curve that he calls "indifference curve of obstacles". This curve has an index of B because on it transformations are operated without any residua. Then he builds an indifference curve of the same type, but with one more unit of A as a basis. To this curve he gives the index 1. He explains this in the following way. (Fig. 1) If one produces a^0b^0 of B with oa^0 of A, one is on the curve of index 0. If one produces the same quantity

¹This part of the paper is related to chapters III and V of the Revue d'Economie Politique.

a^1c^1 (equals a^1b^1) of B but operating on curve of index 1 one uses one more unit of A. So when one operates on curve of index 1 there is one unit of A left (since only $1a^1$ of A is necessary to get a^1c^1 of B). If another curve is built with one more unit of A it will have index 2 and so forth.

Then Pareto builds another indifference curve with one less unit of A than for the first curve. This is, he says, the curve of index -1 "because in the transformation of oa of A into ab of B, one gets only $ca^{1/2}$ of A."¹

This does not explain much. However, at the beginning of chapter IV he takes an example and his reasoning is clearer (Fig. 2). If A is wine, and B is vinegar, and if one unit of wine gives one unit of vinegar the indifference curves will be straight and their slope will be 45° . If one has 2 of wine on the indifference curve of index +1, one produces 3 of vinegar. To produce 3 of vinegar one should have one more unit of wine. So, since one unit of wine is missing, the curve has an index -1. So the curve of index 0, on which transformations are operated without any residuum, will be called the complete transformations curve.

These curves are called indifference curves because on each one of them the producer gets the same residuum or profit. So the producer will try to pass from a lower indifference curve to a higher one in order to get a higher profit. But certain "circumstances" often prevent the producer from going up on the profit hill determined by the indifference curves. So he has to stay on the complete transformations curve. Pareto seems to assume that on the complete

¹V. Pareto, Manuel d'Economie Politique, p. 173.

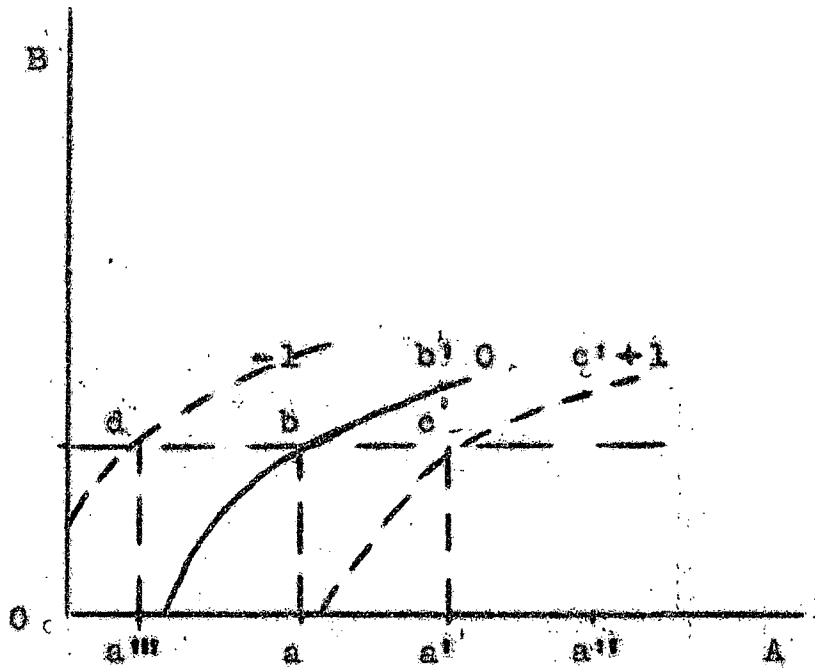


Fig. 1

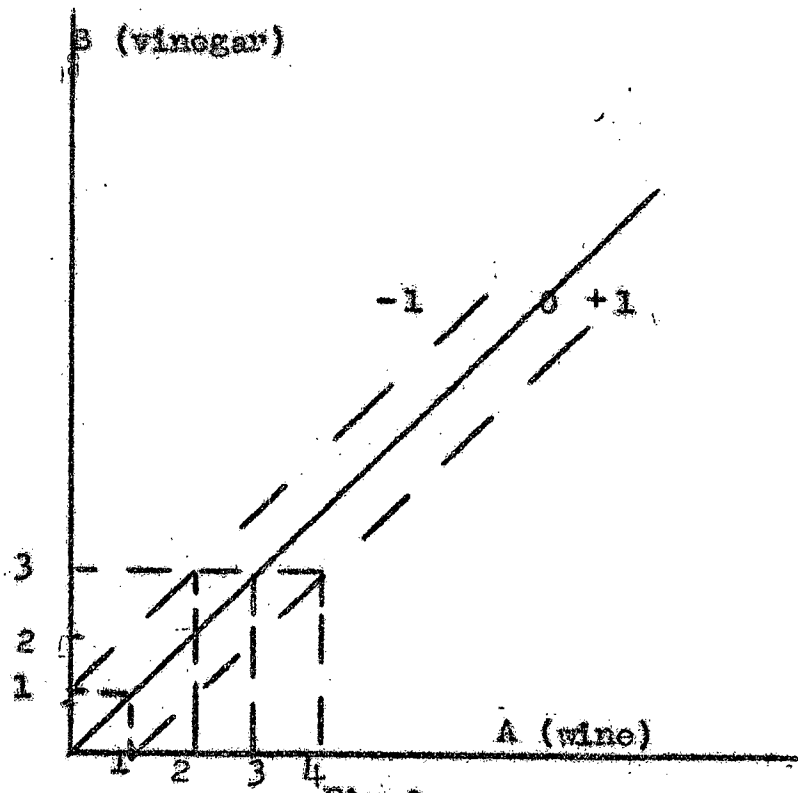


Fig. 2

transformations curve the producer already receives his salary. Pareto, besides, notices that the indifference curves of the producer have a shape different from the indifference curve of the consumer. This is obvious since for the consumer's curve there is indifference between two commodities. For the producer's indifference curve there is indifference because the profit remains constant on the same curve. In fact, these two types of indifference curves have nothing in common.

The use that is made of the indifference curves by Pareto appears to be very different from the one which is made of them by modern theorists like Scitovsky.¹ We can realize this if we compare the study which has just been made and the modern indifference curve approach which is exposed at the end of this part. And when we go over the modern theory we find something which is exactly like the indifference curve of Pareto: the productivity curve, drawn for a factor of production and a product of output. In Pareto's analysis commodity A is nothing but a factor and commodity B, which is produced, is the output. We shall see that really the modern indifference curve analysis has been changed greatly since Pareto. As a matter of fact, Pareto never thought of using the indifference curves for the study of factors and of products. However, Pareto did study the factors of Production; he saw that some factors are limitational, others are substitutional.² Anyway, he could have used, with profit, the indif-

¹ I. Scitovsky uses the indifference curve analysis in his book, Welfare and Competition, (Chicago: R. D. Irwin, 1951).

² V. Pareto, Manuel d'Economie Politique, p. 290.

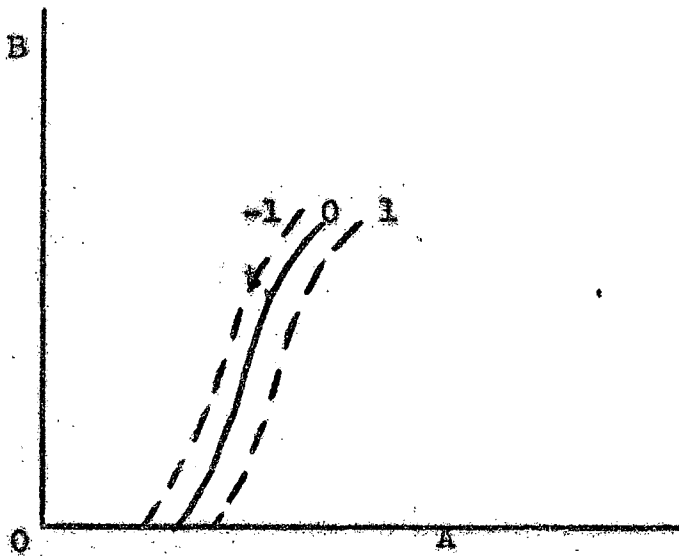


Fig. 3

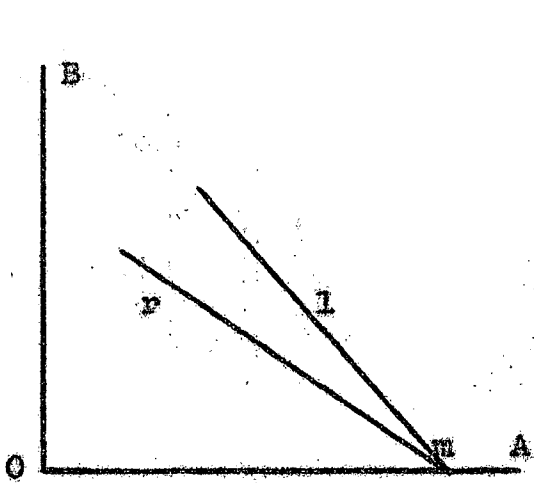


Fig. 4a

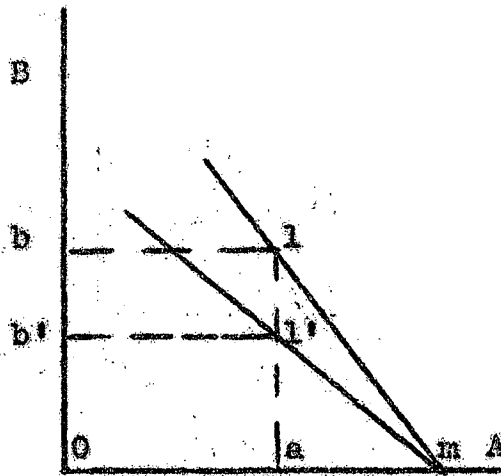


Fig. 4b

ference curve approach. In fact, paragraph 76 of chapter V contains the elements necessary to draw an indifference curve for two factors, and so an indifference curve approach to the study of factors could have led to the profit-indifference curve of Pareto, which is in fact a productivity curve. Used in such a way, the graphic analysis only led to the study of one factor and of one product.

Pareto did not say much about the shape of these productivity curves; we shall call them from now on "Paretian indifference curves." As a matter of fact, he did not dedicate much of the Manual to the particular study of these curves. For him they seem to be mainly a means to study equilibrium.

However, in paragraph 102 of chapter III he distinguishes two types of goods. The first one includes goods of type B of which the cost of production decreases when the quantity produced increases, and he gives a graphic example on Figure 3. The second kind includes the goods of which the cost of production increases. Later, in paragraph 105, he notes that for the first kind of goods the cost of production will finally increase sooner or later. Then the shape of the curve will change. This is about all that he says about the shape of the curve. He did not use it to study the concept of productivity of commodity A, which, in his analysis, is the factor.

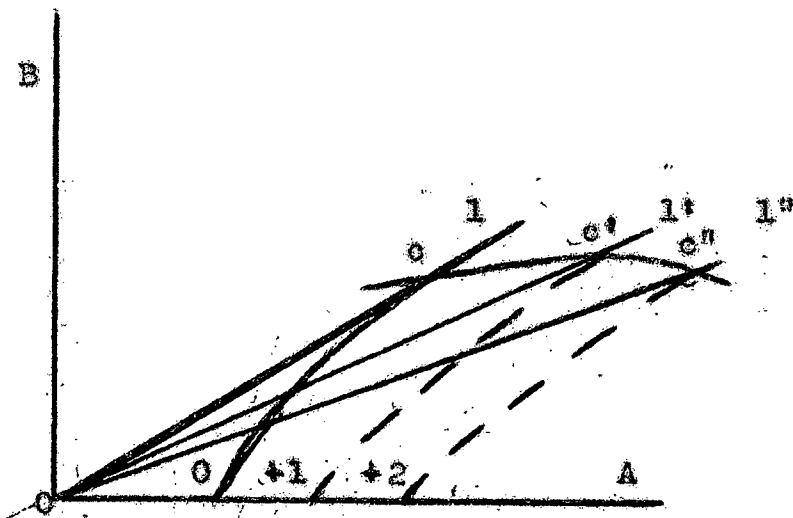


Fig. 5

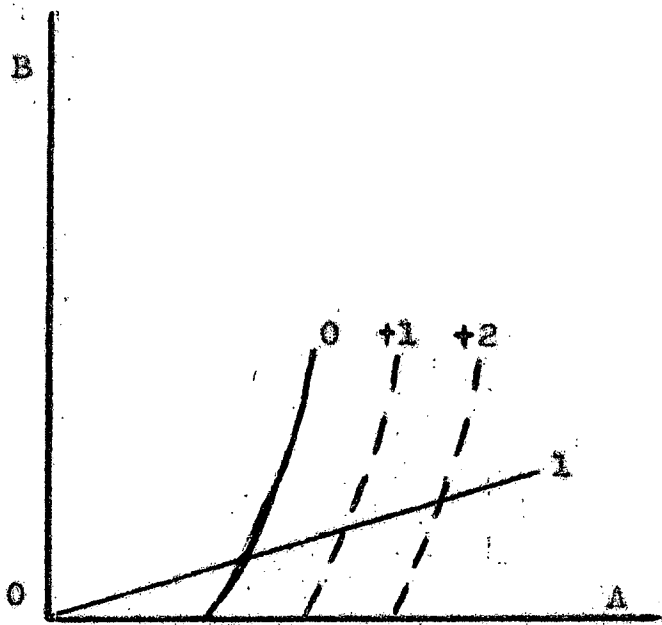


Fig. 6

B. THE COMPETITION OF PRODUCERS AND THE COMPETITION OF THOSE WHO EXCHANGE

Pareto distinguishes the competition of those who exchange and of those who produce. Those who exchange try to go as high as possible on the pleasure hill. In order to explain the movements of the individuals, Pareto gives a diagram which we reproduce in Fig. 4a.

Pareto says that if an individual has a surplus of A he is going to try to get more B and in order to do so he is going to give up a larger quantity of A for the same quantity of B. All that this individual is doing is to offer a higher price for B in terms of A. So we can see that when he is at e, the individual will decrease the slope of the "path" ml on oa. As a matter of fact, this path is nothing but a price line. We know that a change in the slope means a change in the price-ratio. But since Pareto did not use the concept of prices, if we want to follow him here we have to complete his diagram (Fig. 4b). If the individual has a surplus of B he will offer less A for the same quantity of B, that is to say, he will decrease the price of B in terms of A. This will be translated on the diagram by an upward movement of the path mR. Normally an individual keeps moving that way until he is satisfied with his position. Competition will prevent him from comparing the positions on two different paths. So he will select positions very close to each other. Those who produce will try to reach the highest possible point of this profit hill; in fact, they will try to get the largest surplus of A, so their movements will occur always in the same way. Competition tends to prevent the producer from going on the profit hill.

G. EQUILIBRIUM OF THE PRODUCER: INCOMPLETE AND COMPLETE COMPETITION

In this part of our study I have been helped by the research made by R. Triffin of Harvard in his book, Monopolistic Competition and General Equilibrium Theory.

I shall first explain Pareto's theory. Afterward I shall try to see in what ways it corresponds to the modern theories. I shall have the opportunity to continue our considerations on competition and on different markets.

The equilibrium with which Pareto deals here is an individual equilibrium. The goal of the producer is to go as high as possible on the profit hill. So on Fig. 5 if the producer follows a path ol he will stop at point e where ol is tangent to an indifference curve. As a matter of fact, on this path the point e has the highest index of profit. At any other point on ol the index would be lower since the producer would be on a lower profit-indifference curve. Here, Pareto says, competition is "incomplete"; the meaning of that term will be examined later.

If one considers other paths ol' , ol'' , tangent to other indifference curves, and if one joins these tangency points one will get a "maximum profit curve". The part of this curve where the profit index is positive, will be the part where equilibrium is possible.

I shall now consider the case in which competition is "complete". This case is represented in Fig. 6. The path ol cannot be tangent to any of the indifference curves which are represented. On the part of ol which is in the negative region of the diagram, there will not be

any equilibrium possible since the producer operates at a loss. Equilibrium will only be possible on the positive part of cl . On this line the producer will try to go as far as possible in the direction of positive indices, that is to say as high as possible on the profit hill. The equilibrium will occur at a "terminal point" which will be determined by obstacles. In that case competition is "complete".

This was the theory of Pareto. Several questions come up after the reading of such a theory. What is the meaning of "complete and incomplete competition"? What is the path followed by the producer? I shall try to answer these questions.

D. COMPLETE AND INCOMPLETE COMPETITION: TOTAL REVENUE CURVE AND THE COMPLETE TRANSFORMATION CURVE.

When I studied briefly the shape of the Faretian curves I said that Pareto distinguished two types of curves. These, as in Fig. 5, represent the production of one good B of which the cost of production increases when the total production increases. In Fig. 5 the equilibrium is reached by tangency (and competition is called incomplete). That tangency solution occurred because of the shape of the curves, which depends on the increasing cost. So it is natural to relate that concept of "incomplete competition" to the increasing cost of production of good B.

The other curves, as in Fig. 6, page 64, represent a decreasing cost of production of good B in terms of A. On such an indifference may the equilibrium occur at a terminal point. In fact no path can be tangent to any of those curves. The terminal point is therefore the

only solution. Here competition is complete. "Complete competition" is obviously related to decreasing cost of production of good B in terms of A.

It is evident here that the term competition, used in this sense, has nothing to do with a market. Pareto used "competition" in a very different sense from the modern meaning of the word.

Triffin notes that the Robinsonian theory of imperfect competition developed from a discussion of the logical difficulties implied in the association of pure competition with decreasing costs.¹¹ Pareto on the contrary related "complete competition" to decreasing costs. Why did he call competition "complete"? Undoubtedly because when costs are decreasing, in order to maximize profits, the producer endeavors to expand production indefinitely. On the contrary, when costs are increasing, producers have to limit their production (at the tangency point) in order to maximize their profits. Then competition is called incomplete.

We see that the Paretian sense of competition is very different from the modern one. And Pietro Bonelli, one of the followers of Pareto, had the opportunity to talk about a "monopolistic producer operating in complete competition."¹²

If we want to find something similar to the modern meaning of competition we must go to paragraph 89, chapter III. In this paragraph, Pareto, as for the consumers, distinguished model (1) and model (2) of phenomena (plus model 3). Model (1) is that in which producers

¹¹R. Triffin, Monopolistic Competition and General Equilibrium Theory, p. 58.

¹²Quoted by R. Triffin, Ibid., p. 53.

cannot influence the market prices (competition in the modern meaning of the word). Model (2) is the one in which one producer may influence the market-price and acts according to that goal. This, according to Triffin, is nothing "but the behavior of Mrs. Robinson's or Professor Chamberlin's monopolistic competitor."¹ But for Pareto the variable is the price rather than the quantity. This seems to be the reason for which the producer operating according to Pareto's model (2) is rather similar to an oligopolist whose behavior is shown by a kinked curve.

The other question we asked ourselves was to know what was the path followed by the producer. Pareto says that when this path is tangent to an indifference curve there is equilibrium at the tangency point. But we saw that the indifference curve is nothing but a productivity curve of the factor A. We can consider also this curve as representing the total cost of production of good B in terms of A. The slope of this curve represents the marginal cost of production. This slope, when there is equilibrium, is equal to the slope of the path. We also know that, at equilibrium, the marginal cost of production is equal to the marginal revenue. So the slope of the Paretian "path" is very likely to represent the marginal revenue which at equilibrium is equal to marginal cost. Since the slope of the path is the marginal revenue, the path itself is the total revenue curve of the producer. We can verify this if we consider the price of B in terms of A. Each point on the path shows the quantity of A obtained for a certain quantity of B.

¹Ibid., p. 56.

When the cost of production is decreasing, in complete competition we can also verify that the path is the total revenue curve. When the cost is decreasing the producer tries to sell the largest quantity of good B possible, in order to obtain the largest profit in terms of A. He will only be stopped on the hill of profit by obstacles, at "a terminal point". Here, with Henry Schultz, we must remember that "the term obstacles is used by Pareto to designate any impediments--social, natural or economic."¹

The matter of the complete transformation curve needs also some explanations:

The curve of complete transformation has the index zero. When the producer operates on this curve he does not make any profit. The complete transformation curve expresses the production function. When Pareto studies the processing of wine into vinegar he says that one unit of wine is necessary to produce one unit of vinegar. So the complete transformation curve which expresses this production function is a straight line going through the origin. It has a slope of 45° on both axes. No profit is made on this curve.

The purpose of the other curves, of positive or negative index, is only to give a tool to study profits and competition. They do not represent directly the production function. This function appears only in their shape since they are parallel to the first curve of index (0) or of "complete transformation". We may see that they do not represent the production function since on curve (1) two of wine

¹H. Schultz, Theory of Measurement and Demand, p. 24.

would only give one of vinegar. These curves, related to the "complete transformation curve", make a profit appear. This profit is expressed in terms of A or in our example in terms of wine. This idea appears rather difficult to explain. However, we shall attempt an explanation here.

When the total revenue curve is tangent to the complete transformation curve there is equilibrium. We also said that the marginal revenue, or slope of the total revenue curve, was equal to the marginal cost, or slope of the total cost curve. When the total cost curve is the curve of complete transformation the cost of B produced is expressed in terms of A. Besides, all the quantity of A (which is the cost of B) has been necessary to produce B. One unit of vinegar has been obtained at a cost of one unit of wine. Also one unit of wine is entirely necessary to produce one of vinegar. So in the cost of good B, in terms of A, all of A went into the production of good B; this is what the complete transformation curve means. When the equilibrium position of the producer is on this line, his marginal revenue is equal to the marginal cost. The marginal revenue is here equal to the average revenue since the total revenue curve is a straight line.

On the curve of index (1) what is changed? At the equilibrium position will the marginal revenue always be equal to the marginal cost? Yes, but on the total cost curve of index (1) the calculation of cost is based on another assumption.

The cost of one unit of vinegar, on that curve, is not only one unit of wine, as expressed by the curve of index (0), but also an extra unit of wine which is counted in the cost of production in order to represent the profit. This profit will always be one at any point of the curve of index (1). This idea is perhaps difficult to understand because this profit is expressed in terms of wine. But on Figure 7 we may say that on axis Oa we measure money. An amount Oa of money is then necessary to produce a quantity ac of commodity B. On the complete transformation curve one unit of money is necessary to produce one unit of B. On this curve the quantity of money is entirely necessary for the production. The curve of index (1) shows that besides the quantity of money necessary for production there is, in the cost of production, an extra unit of money which is the profit of the entrepreneur. So on the curve of index (1) and on all the curves of higher index we must realize that the cost of production includes the profit of the entrepreneur. And here we must understand extra-profit. The example of the process of wine into vinegar is not a very good one because, in this case, on the complete transformation curve no salary is paid to the entrepreneur. We shall see that according to Pareto, in a competitive market, the firms operate on the complete transformation curve. That is to say that pricing is on a marginal cost basis.

According to the analysis of Pareto, pricing is always on a marginal cost basis. The difference from modern analysis is the basis of calculation of cost of production. In Pareto's writings the cost of production on all the curves of index higher than (0) includes a

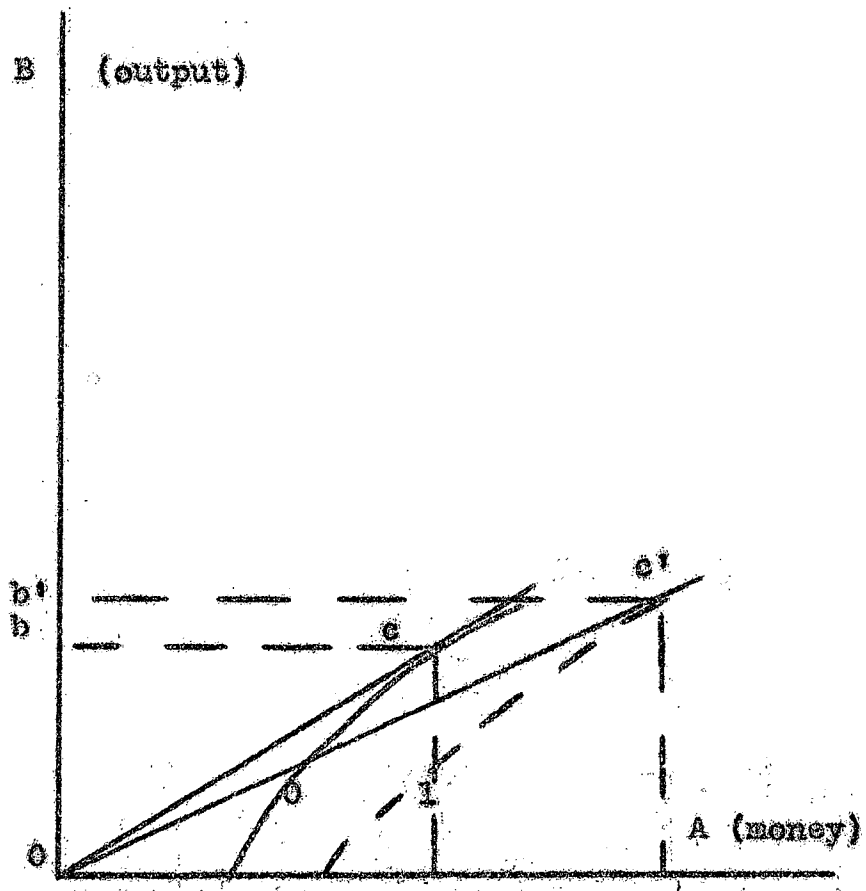


Fig. 7

profit, besides the salary of the producer. The difference between each one of the profit-indifference curves of Pareto is the difference between the parts given to profit in each one of the calculations of costs. Each curve represents a way of calculating cost, the part of the profit being expressed by the index of the curve. We can see more clearly now the nature of the bill of profit on which the producer wants to reach the highest possible point.

Let us consider now the other positions of equilibrium. On the curves of index (1), index (2), etc. there will be equilibrium when the total revenue curve is tangent to the curve of index (1) or (2), or (n). We can see that when the total revenue curve becomes tangent to a higher profit-indifference curve, its slope on OA (Fig. 7), decreases so that for the same quantity of good B the total revenue increases. The total revenue is then the product quantity of B times the quantity of A; this will be clearer if we suppose that A is money (see Fig. 7). So, since the total revenue increases and the quantity of B remains constant, the price of B has to increase. This corresponds to the increase in the cost of production arising from a larger profit, since at the equilibrium position marginal revenue (or price on a straight total revenue curve) is equal to the marginal cost.

All of this will be very useful for us to understand Pareto's theory of economic equilibrium in case of competition or of monopoly. We can already see that in the case of competition, the producers will start operating on a high profit-indifference curve, that is to say, at a high price. But because of price cutting, they will have to operate on lower and lower indifference curves until the time when they

operate on the curve of index 0. We shall study this in detail in the part of the thesis dedicated to equilibrium (Chapter VI).

Before starting the study of general equilibrium let us summarize our study of the producer's equilibrium: the producer will be in equilibrium at the point where his total revenue curve (defined by the price he applies) crosses the curve of highest profit (locus of the tangency points we just studied). This is in the case of increasing costs in the short run.

In the case of decreasing costs in the short run, (which in modern theory is not very often considered): the equilibrium position will be at the highest point which can be reached on the hill of profit when one goes along the total revenue curve.

All that I said about the profit indifference curves with a positive index can be used for the curves with negative indices. When the producer operates on a curve with a negative index, his total revenue curve is tangent to the profit indifference curve. That is to say that the price he applies is not high enough to cover all the expenses which are necessary to produce the quantity of products sold. As a matter of fact, on curve (I) the cost of production is calculated including a loss of 1, in the same way as on curve (I) it was calculated including a profit of 1.

At first sight such a theory of production may seem complicated. I think that as it is developed by Pareto in his book it is hardly understandable at once. This is for several reasons. First, the study of production which is made by Pareto in Chapter III essentially

is destined to furnish the theoretical tools necessary for a further study of general equilibrium. Because of this, it is not as complete as a study of production which is the only topic of a chapter or of a book. In chapter V of the Manual, the study of production is more complete. In this chapter Pareto studies the division of labor, coefficients of production, capital and its revenue and the problem of rent. He already said that some of those studies, like the ones of coefficients of production, reached the modern conclusions.¹ This part of the Manual does not offer many difficulties and is based on the reckonings made in the Appendix (especially for the variations of coefficients of production).

Second, for an economist with a modern Anglo-American theoretical background, the study of Pareto is very different from anything he knows. As a matter of fact, we just saw that after a study with the help of modern theories it becomes clearer. But along with Triffin we must acknowledge that sometimes "Pareto's own account is difficult to follow, owing to its extreme technological and terminological apparatus."²

The third reason is that in this part of the book the author still does not want to use the concepts of total revenue, total costs, price, etc. which are all based on the existence of money. Once more we see that what Pareto wanted to have a precaution only made his reasoning more difficult to follow. Having said this, one must also

¹Pareto, Manuel d'Economie Politique, pp. 329-330.

²Triffin, Monopolistic Competition and General Equilibrium Theory, p. 61.

acknowledge that in many points Pareto reached the same conclusions as the modern theorists.

I shall now study the modern indifference curve approach to the study of production.

CHAPTER V

MODERN INDIFFERENCE CURVE APPROACH

TO THE THEORY OF PRODUCTION

Although indifference curve analysis of consumer demand has many similarities with the indifference curve approach applied to production problem, there are nevertheless some points of difference. Furthermore in production theory there is both an output and an input aspect of the problem.

First, we may consider the producer (the firm), as a consumer, but a consumer of a special type. He would not be immediately seeking pleasure or personal utility out of the commodities he buys, but he would consider them as factors of production and buy them in order to get a maximum profit by their combination. So one can draw an indifference curve between two factors of production bought by the entrepreneur because with them he is able to produce some commodity or service. This analysis is going to be similar to the indifference curve of the consumer. Here, as in demand analysis, we are dealing with demand problems; more exactly, we are on the demand side of production: the producer is looking for the best way to buy and to combine the elements he needs for his production. However, there is a big difference: the goal of the producer is to get more and more production, at least during a certain time, or to decrease his production; the goal of the consumer is to get more and more pleasure or utility. The producer is concerned with production, the

consumer with utility. We see here the difference: production is directly and easily measurable; utility and its measurability have been the objects of long arguments which ended by the acknowledgement that utility is neither directly nor indirectly measurable. Utility is of ordinal nature; production is of cardinal nature. So at first sight it seems easier to deal with indifference curves between factors of production than with consumer's indifference curves. However, the production-indifference curves present their own difficulties.

This is the buying side of production, but we may also apply the indifference curve analysis may be applied to the selling side. An indifference curve may be drawn between two commodities which are the output of a firm. On that curve, the producer, from the viewpoint of cost, is indifferent to producing any of the combinations of these two commodities. This kind of approach will be helpful to solve the problem as to which combination of two commodities is the producer going to establish if prices of the market are given?

A. INPUT ANALYSIS:

I am going first to study input analysis. The basic assumption is that one way or another input determines output. If there is a function, or an equation, or in other words a series of conditions which shows how this assumption works in the case of a particular production this is a production function of that case. The production function shows in which way input determines output. As a mathematical function, the production function may be illustrated graphically. It

is here that the indifference curve approach is useful. This function is the theoretical representation of the technical conditions in that particular sector of the industry.

From a production function one may consider that we get two kinds of information: first, the way in which the different factors must be combined, and second, the way in which the output varies when input varies. For instance, will the ratio of the two factors be left constant when we want to increase output? The production function will show the way the ratio of input to output changes when production increases or decreases.

In the first kind of problem two cases may be distinguished (from now on we consider only two commodities because of the use of the indifference curve approach). There may be a certain freedom of choice allowed by the conditions of production in which several combinations of two factors may give the same output. In this case the factors are called substitutional. Some other factors may be called limitational; for those factors the proportions are fixed rigorously. So when the quantity of output is once decided the quantity of those factors is automatically decided. I am going to examine the indifference curves for that type of factor which we may call complementary for production, in the same way that we called some commodities complementary for consumption. If, as in the case of a pharmaceutical product, one needs one unit of factor A and one unit of factor B to get one unit of output, and because there is no substitution possible, the indifference curve will be represented by two straight lines crossing at the

point representing one of A and one of B. These two straight lines will be parallel, one to the A axis, the other to the B axis. On this curve there will be indifference, since when we are for instance (Fig. 1) at C we have one of B and two of A, but we would just as soon be at C' as at C, since the extra unit of A is of no use. The same thing would happen if we were at C' with an extra unit of B. This is the special indifference curve for limitational factors (Fig. 1).

Consider now the case of two substitutional factors. What will be the general shape of the indifference curve? In the part where the substitution is effective (when the producer requires more A if he wants to keep the same output and give up some B) we can see that the general slope of the curve is from left to right, downward. We can also see that if the two factors are perfect substitutes the curve is going to be a straight line from the B axis to the A axis because the same output may be produced by using only B or only A and because the marginal rate of substitution is constant. In fact, factors are not completely substitutional because in that case they could not be distinguished from each other. That is, they would be two units of the same factor.

What will be the slope of the curve if the two factors are not perfect substitutes? The slope of the indifference curve shows the marginal rate of substitution of one factor for the other; that is to say, the quantity of one factor necessary to replace one unit of the other when we go along the curve (when the quantity of one factor increases the quantity of the other decreases). This corresponds to

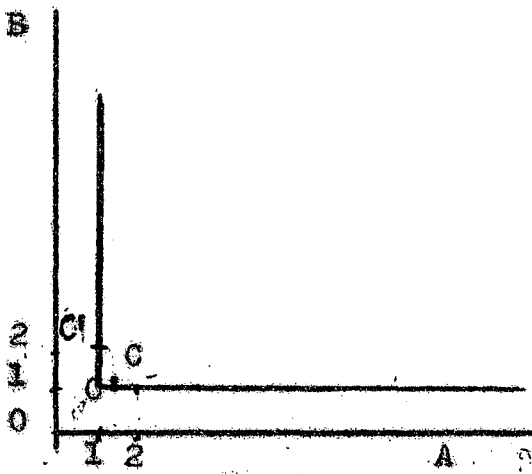


Fig. 1

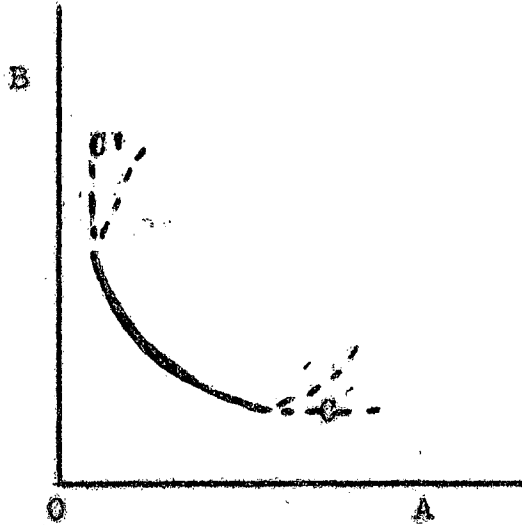


Fig. 2

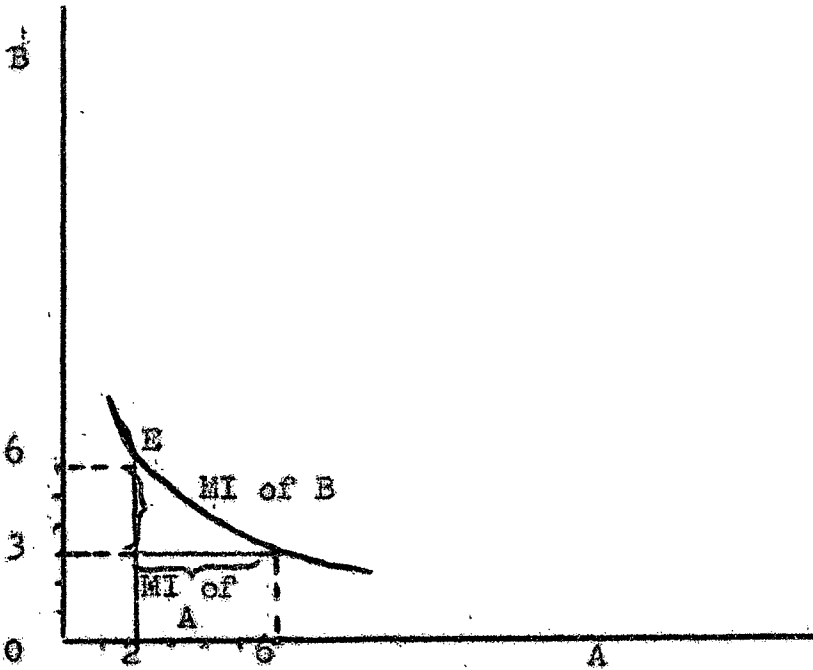


Fig. 3

the consumer's marginal rate of substitution. When A increases and B decreases the presence of factor B is going to become more and more important for the firm and we need a larger and larger quantity of factor A to replace one unit of factor B to keep the same output. If B increases the reasoning is reversed. The presence of factor A is going to become more and more important for the firm and it needs more and more of B to replace one unit of A. All this happens because the two factors are not perfect substitutes, so the marginal rate of substitution (represented by the slope of the curve) changes; i.e., the slope of the curve is not constant. All these conditions are fulfilled on a curve sloping downward from left to right and convex toward the origin. That curve will represent various combinations of functions A and B, but at a constant output. A time comes, however, when a decrease in the quantity of factor B cannot further be replaced by an increase in quantity of factor A; so that if we reduce the quantity of factor B further the output is going to decrease. Because it must remain constant on the same indifference curve, from that point on, the quantity of factor B must remain constant (Fig. 2). If we keep increasing the quantity of factor A the curve will be horizontal and parallel to the A axis. So we are in a case similar to the one of limitational factors, and as a matter of fact, after having been a substitutional factor, and because it reached a minimum, factor B has become limitational. So the curve becomes parallel to the axis beyond the limits of substitution. But we may see that the flat portion of the curve is not interesting for the

producer, because on that part he adds quantities of factor A and the quantity of factor B remains constant; there, the total cost of producing a given output increases.

Some theorists like Scitovsky¹ think that when more units of factor A are added (beyond the limits of substitution) more units of B should be added in order to keep the output constant so that the difference curve would then slope upward from the left to the right. This would happen because beyond a certain point factor A would become cumbersome, the authors who assert this should say "in some opportunities". As a matter of fact, for some types of factors this would not work. If factor A represents a kind of machine which produces a minimum output as soon as it is worked, and if we add more machines and more men to work them, the output is going to increase. On the other hand, if the extra machines are so cumbersome that they decrease the output (on the flat part of the curve), we may question the fact that more men, added in order to work them and to maintain the output constant, would be efficient. It seems that more men would increase the encumbrance and cause a further decrease of output. Anyway, for our further reasoning it is enough to assume that the curve becomes flat beyond the limits of substitution. What happens further depends on the factors, is not very well known, and is not of interest because we know that the entrepreneur will avoid that part of the curve. (The same would happen on the curve of two limitational factors; the producer would not remain at O or O'.)

¹In the chapter of Welfare and Competition which is dedicated to the theory of the firm.

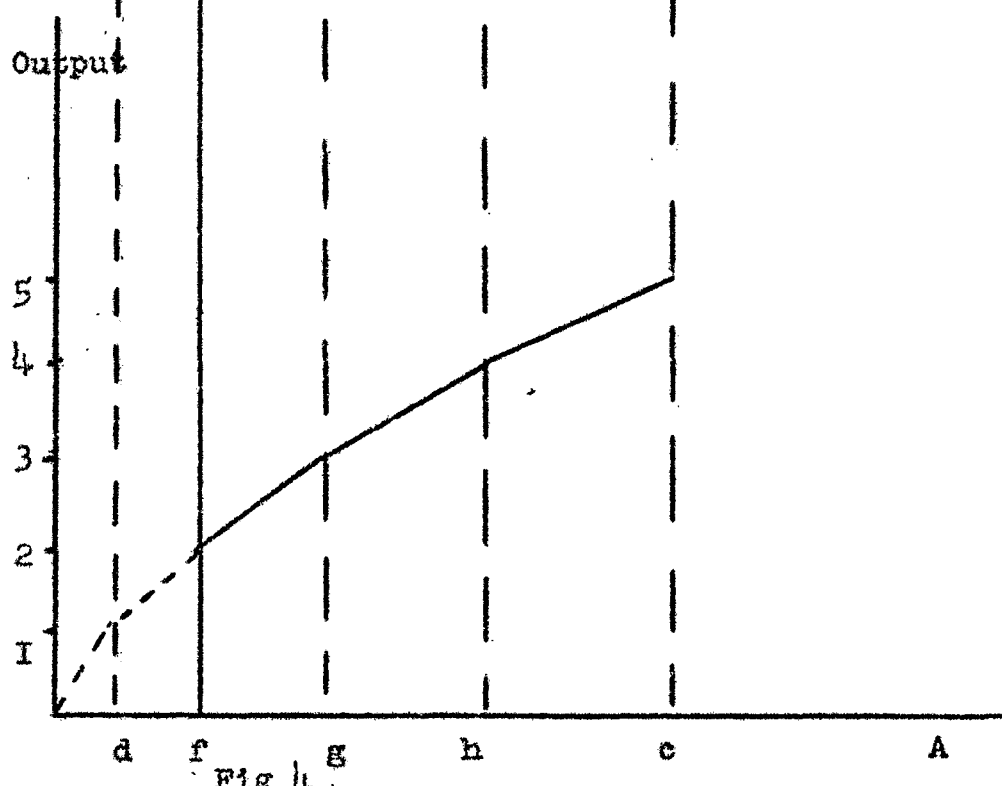
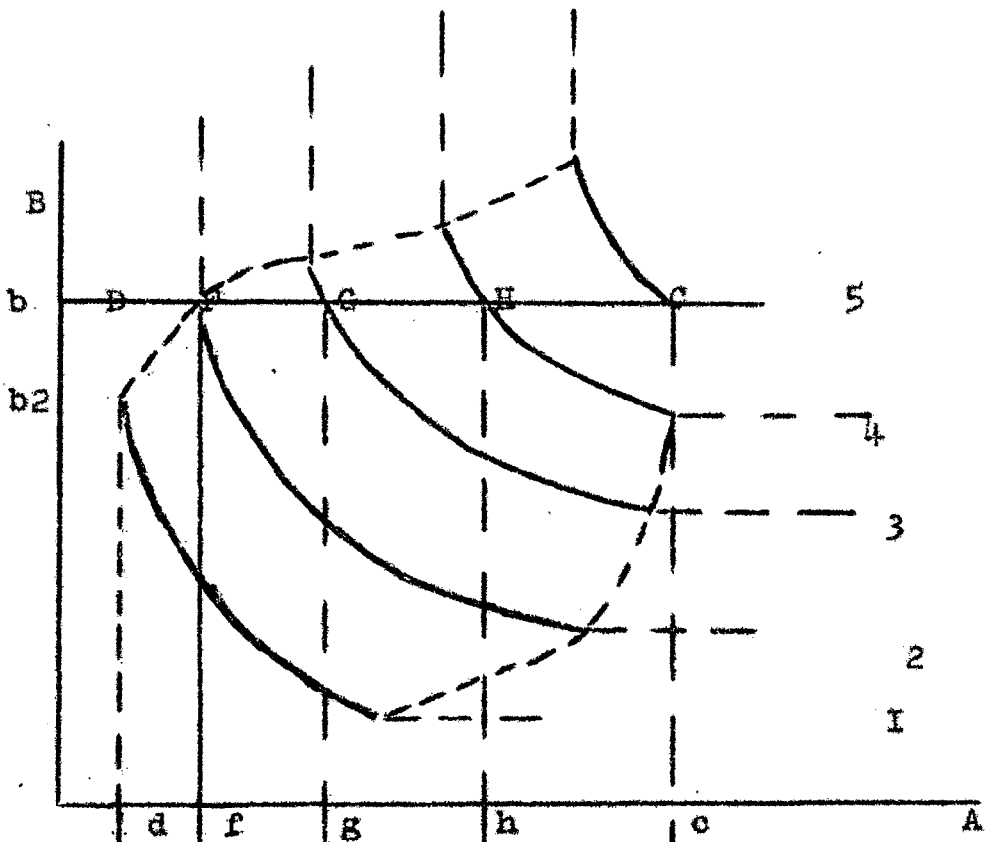
Let us study now the marginal rate of substitution in relation to the indifference curve. The marginal rate of substitution shows us the quantity of factor A necessary to replace a decrease of the quantity of factor B. There, it is related to the marginal productivity of factors A and B. In fact, if we want to keep the same output the marginal productivity of A multiplied by the increment of A must be equal to the marginal productivity of B multiplied by the decrease in the quantity of B. So we see that the marginal rate of substitution is related to the marginal inputs of factors A and B. A glance at the diagram will show that (Fig. 3, page 82).

If at point E, 3 units of B and 4 of A are the marginal inputs of B and A, we know that if we add 4 of A and subtract 3 of B the output will remain constant. So the marginal rate of substitution between B and A is $\frac{MI_a}{MI_b} = \frac{\text{marginal input of A}}{\text{marginal input of B}} = \frac{4}{3}$. "It expresses the rate at which A can be substituted for B...without changing the output."¹

But we know also that (MP = marginal productivity)

$MI_a \times MP_a = MI_b \times MP_b$, so $\frac{MI_a}{MI_b} = \frac{MP_b}{MP_a} =$ Marginal rate of substitution between B and A. In the last equation $\frac{MI_a}{MI_b} = \frac{MP_b}{MP_a}$ we may also see that the marginal input of a factor is the reciprocal of its marginal productivity. This also corresponds to the shape of our curve: when the quantity of a factor decreases this factor becomes more important for the firm. When the total quantity of B increases the quantity of B necessary to compensate for a decrease in A increases, the marginal input of B increases and the marginal productivity of B decreases.

¹T. Scitovsky, Welfare and Competition, p. 119.



Until now we considered only one indifference curve. But as for the indifference curves of consumers, we may have a whole map of indifference between two factors. Each indifference curve represents a level of output which may be obtained with any of the combinations on that curve. Let us consider one of these maps (Fig. 4). Factor A represents labor; factor B is equipment. Let us suppose that in the short run the entrepreneur can only vary the quantity of labor applied to a plant represented by a quantity of equipment b . The possibilities of new combinations are now represented by a straight line parallel to the A axis. When the entrepreneur moves toward the right he reaches higher indifference curves. So more and more labor applied to the same plant increases output as far as point C, because then, when the producer reaches the flat part of a curve, new workmen do not increase output any more, as we say before.

At point D the producer is in such a position that if he used less equipment he would have the same output at a lower cost (at E with b_2 equipment). At point D, he is not using enough men in his plant. If he is able to employ more men he will go farther to the right on line DC. We can therefore draw two lines connecting the limits of substitution. So we have an area in which the producer will try to operate; in this area he will try to reach the higher indifference curve.

On the line DC we may observe the variations of the marginal productivity of factor A when its quantity increases. When we move from D to F by increasing the quantity of factor A by df , the production

increases by one unit (total production rises from 1 to 2). Then when we move from F to G by using 1g more of A, the production increases by one more unit (from 2 to 3), but the increase in A has been much larger than from D to E. In the same way it will be larger from H to C (hc) than from G to H (gh) for the same increase of one unit in output. When the quantity of A increases its marginal productivity decreases.

To represent this we can draw a productivity curve of factor A when factor B is constant. The two axes represent factor A and quantity of output Y. The productivity curve shows the increase in output for an increase in factor A. Its maximum will be at quantity oc of factor A. Beyond point C it is likely to stay horizontal for a certain time and then to fall because of the encumbrance. If we consider that the indifference curves may slope upward beyond the limits of substitution, the productivity curve will fall immediately because on line DO we shall reach lower indifference curves beyond point C. We shall not study further this aspect since it tells us nothing really new.

Let us recall what we said about the production function. The indifference curve analysis permits a graphic representation of the way two factors must be combined. It also permits us to see the way in which output varies when one factor of input varies. We have also some idea about the way the ratio of input to output changes when production increases or decreases. The two latter questions are very

close to each other and may be considered as two aspects of the same problem.

The other question to be examined is: in what way does output vary when both factors vary, for instance, when the monetary resources of the firm increase or when the price of one or both factors varies? This is what Scitovsky calls the "market behavior of the firm"¹ and what we are now going to study.

Like the consumer who is concerned with the price of the commodities he consumes, the producer is concerned with the prices of the factors he uses. As in consumer's analysis, we can draw on our diagram a price line depending on the budget of the firm and on the price ratio of the two factors. This price line will represent the market rate of substitution of the two factors. Since the prices of these factors are considered as constant on the same line, the market rate of substitution stays constant and the line will be straight. When it meets the axes it shows how much of factor A or of factor B may be bought with the total budget of the firm.

When the price line is tangent to an indifference curve, the point of tangency shows the quantities of factors A and B which will be used. As a matter of fact, at any place else on the price line the firm would be on a lower indifference curve, and produce less with the same cost. The tangency point, therefore, is the optimum (least cost) combination (Fig. 5.)

¹Scitovsky, Welfare and Competition, p. 125.

At that point the slope of the price line and that of the indifference curve are the same. In other words, the marginal rate of technical substitution is equal to the market rate of substitution.

In fact, one may also think that on the price line the firm will stop when the quantity of factor A which is added will have the same cost as the quantity of B which is subtracted; that is to say, when $MI_a \times P_a = MI_b \times P_b$. From that equation we get

$$\frac{MI_a}{MI_b} = \frac{P_b}{P_a} = \text{market rate of substitution, but } \frac{MI_a}{MI_b} = \frac{MP_b}{MP_a}$$

Marginal rate of technical substitution. So the price ratio or market rate of substitution equals the marginal rate of technical substitution. We may also say that, since $\frac{MP_b}{MP_a} = \frac{P_b}{P_a}$, the marginal productivity of the two factors are in the same ratio as their prices.

At the point of tangency, costs are at a minimum since with the same price ratio any other position on the indifference curve would need a larger budget than at B or C. On the same price line one would operate at the same total cost, but with a lower product; therefore the average cost would be higher. Professor Scitovsky proves this in an easy mathematical way.¹ If $P_a \times MI_a > P_b \times MI_b$ we are not in a minimum cost situation since a part of the cost could be saved by using more B and less A and leaving the output unchanged at the same time.

Here we overcome one of the drawbacks of the indifference curve analysis which was to consider only two factors. We can apply our results to any number of factors.

¹Scitovsky, Welfare and Competition, p. 124.

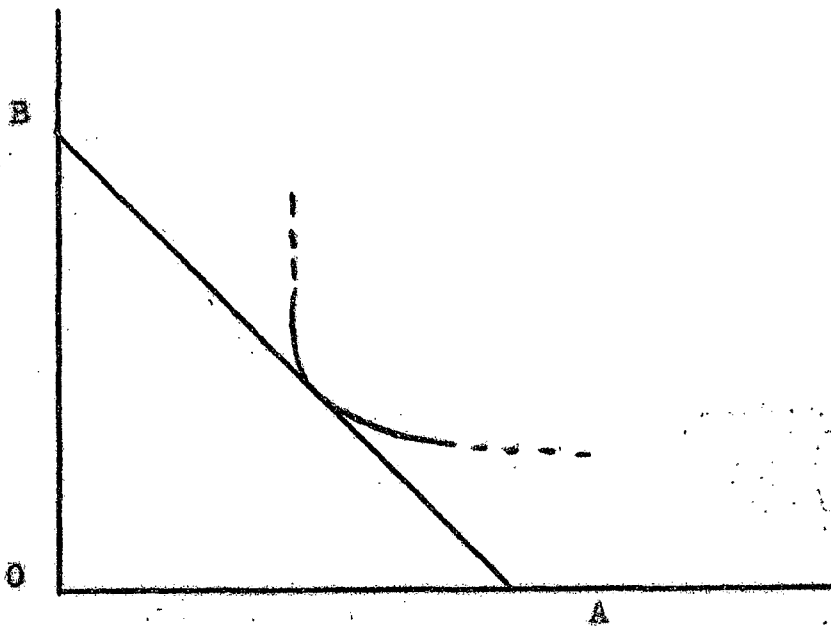


Fig. 5

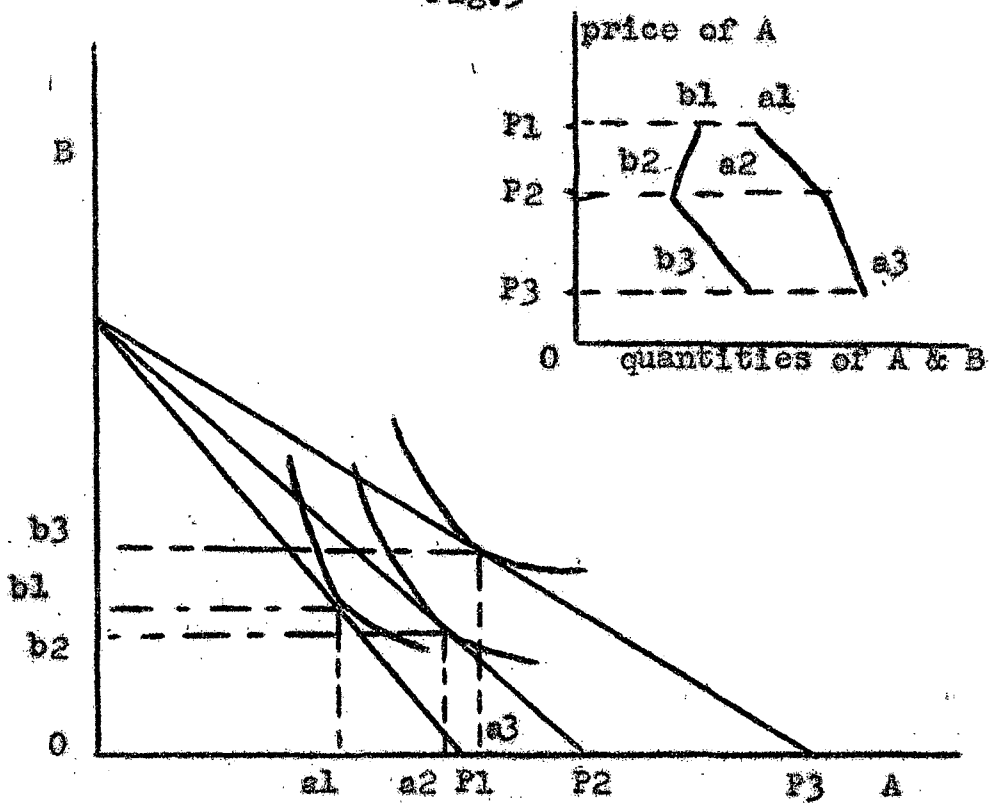


Fig. 6

The condition of equilibrium, or of lowest possible cost will be when the market cost of the marginal input of every factor is the same; that is, when $MI_a \times P_a = MI_b \times P_b = MI_c \times P_c \dots = MI_z \times P_z$ (with small quantities, if we make one more unit of product by adding only b or only a: marginal cost = $MI_a \times P_a = MI_b \times P_b$). Practically, the firm is not able to fulfill those conditions at any time because it cannot change the proportions of factors it uses at any time.

Until now we have considered stable prices for each factor. We know that the condition of equilibrium is that the market costs of marginal inputs are equal for all the factors (let us come back to only two of them, to simplify). So if a price-change occurs for one of the factors the firm is going to try to change its inputs until the equilibrium condition is fulfilled. Graphically, we may represent this change on our indifference map (Fig. 6). If the price of factor A decreases, more A may be bought with the total resources of the firm so the new price line is tangent to a new and higher indifference curve and we may see that more of factor A is bought. With only a small price decrease of A the quantity of B bought is going to decrease but if the decrease in the price of factor A is greater, perhaps more of B will be bought because of the income effect. In fact, in this case we can distinguish between a substitution effect and an income effect as we did in consumer analysis.

From this formulation we may get a demand curve for factor A in relation to its price; we can add also a demand curve for factor

B in relation to the price of A. Let us consider now a change in the budget of the firm. If the budget of the firm increases, more A or B will be bought, assuming that the total budget is spent on A or B. So without any change in price ratio more A and more B will be bought at the tangency point of the new price line since a higher indifference curve is reached. Concretely, what is the increase in the budget of the firm? It is, in fact, an increase in the scale of the firm. So if we have several price lines, representing several increases of the firm's budget, we obtain, by joining the tangency points to the indifference curves, a scale line (Fig. 7).

If the scale line is a straight line the optimum ratio of the two factors remains constant as determined by the tangency points. Some theorists (Joan Robinson) think that it must be this way: if one finds the optimum ratio and if one doubles the input, then one doubles the output, according to Euler's Theorem.¹ Other writers, like Chamberlin, whose line of thought I prefer, think that the optimum ratio is a function of scale; so the ratio is not constant and the scale line would be a curve. The firm will move along the scale line which is the locus of the tangency points which are the lowest cost points for each plant. Obviously the shape of that scale line will depend on the general indifference map which itself depends on the production function. Here, the problem we had noticed when we talked about the

¹Joan Robinson, "The Problem of Distribution," Economic Journal, Sept. 1934, or Economic Essays (London: Macmillan & Co., 1954), pp. 1-13.

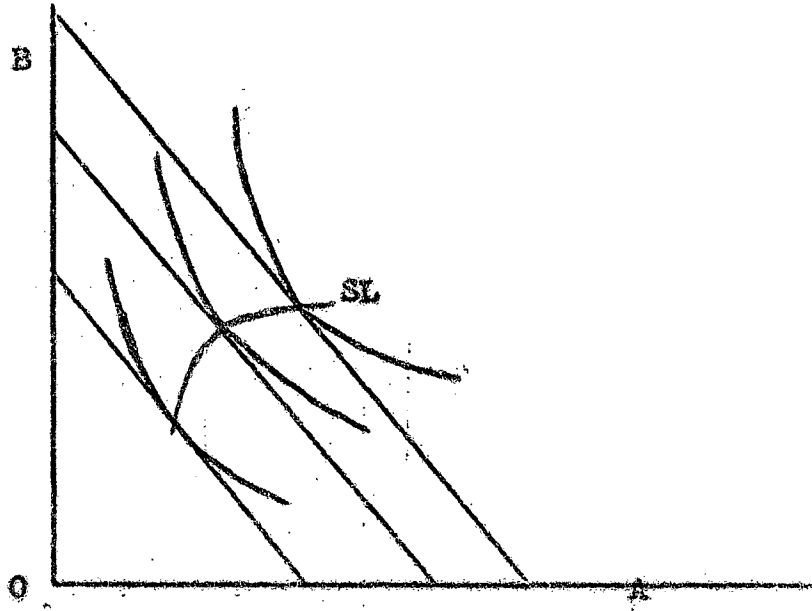


Fig.7

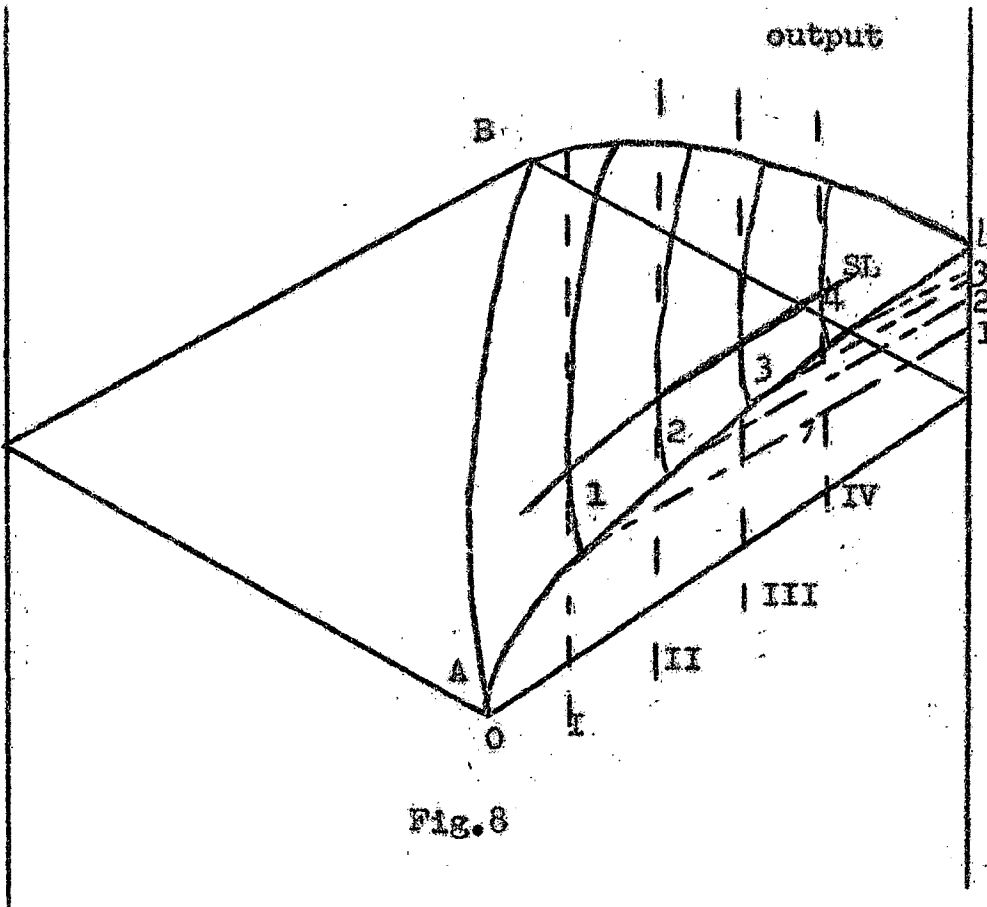


Fig.8

Production function appears again: "shall we leave the ratio of factors constant when we want to increase output?" We may have a space representation of the scale line on a "hill of production" on which the output is represented by the vertical dimension. (Fig. 8)

So we see that the indifference-curve analysis is useful when we want to study the production function; it permits us to show graphically a set of rather abstract problems. Thus far, we have been concerned primarily with the "input" side of the production process. We will now consider essentially the same problem from the side of "output". We move, as it were, from the "factor" market to the "product" market.

B. OUTPUT ANALYSIS:

If a firm produces two different products, one question it must answer is what quantity of each product it is going to produce. This question is similar to the one we studied in the first part of this chapter: what combination of factors should the firm use? We may consider a series of combinations of the two products which will be produced at the same total cost. Other series of combinations will correspond to higher or lower cost levels. In the same way that we represented a series of combinations of two factors by an indifference curve, we may now represent a series of combinations of two products by an indifference curve.

If on one axis (Fig. 9) the quantity of product A is measured and on the other the quantity of product B what is the shape of this indifference curve? The curve will meet the A axis at the point which

shows the quantity of A which alone would be produced with the cost represented by the curve. (Let us call this OX in Fig. 9). In the same way the curve will meet the B axis at the point which shows the amount of B which alone could be produced if the firm spent its entire production budget on the making of product B. On the other hand, since this curve represents a constant cost, when more A is produced, less B must be produced and vice-versa so that the general slope of the curve will be downward from left to right.

Let us examine more precisely the shape of this curve. On this line we have a constant total cost; the cost of the new quantity of A that we produce must be equal to the cost of the quantity of B which is replaced by A. So we have the equation: marginal cost of A x marginal output of A = marginal cost of B x marginal output of B, or $MCA \times MOA = MCB \times MOB$, or $\frac{MOA}{MOB} = \frac{MCB}{MCA}$ or $\frac{MOB}{MCA} = \frac{MCB}{MOA}$. On this curve, when we move along it, we are transforming product B into product A (downward; upward it would be A into B) keeping the total cost unchanged. This is why this kind of curve is commonly called a product-transformation curve. But the rate of transformation of product B into product A is conditioned by the equation $MCA \times MOA = MCB \times MOB$, or also $\frac{MOB}{MCA} = \frac{MCB}{MOA}$. As a matter of fact, the ratio of the marginal output of A to the marginal output of B is nothing but the rate of transformation of B into A. But on a curve sloping downward from left to right the ratio of the marginal output of A to the marginal output of B is nothing but the slope of the curve on the B axis, so one can say: the slope of the product-transformation curve is equal

to the marginal rate of transformation and is equal to $\frac{MO_a}{MO_b}$, or also to $\frac{MC_b}{MC_a} = MRT_{ba} = \frac{MC_b}{MC_a} = \frac{MO_a}{MO_b}$. Consider the marginal costs of B and A. Let us assume that as we increase the production of a product its marginal cost of production increases. When B is transformed into A then the marginal cost of A is going to increase as we increase its production. On the other hand, the marginal cost of B will decrease as we produce less and less of B. So if we want to keep the two ratios equal, i.e. $\frac{MC_b}{MC_a} = \frac{MO_a}{MO_b}$ the first ratio decreases since MC_a increases and MC_b decreases; therefore, the second ratio will have to decrease also and MO_a will decrease and MO_b will increase. The rate of transformation is decreasing: $\frac{MC_b}{MC_a} = \frac{MO_a}{MO_b}$. If the curve is concave to the origin this condition is fulfilled: for the same output of B the corresponding output of A decreases when we transform B into A.

Suppose now that when the production of a product increases the marginal cost of that product decreases. When B is transformed into A, the production of A increases so the marginal cost of A decreases. So if the two ratios must be kept equal, when MC_a decreases and MC_b increases (since less and less of B is produced), MO_b will have to decrease and marginal output of A to increase. This occurs on a curve which is convex to the origin. But in the short run the marginal cost is increasing on the relevant part of the marginal cost curve; i.e. on that part above the average variable cost curve. However, Scitovsky says:

"Only if the production of one commodity is hampered rather than facilitated by the same firm producing also the other commodity can it happen that the transformation curves become convex to the origin."¹

¹ Scitovsky, Welfare and Competition, p. 136.

So from now on we shall only consider concave indifference curves, that is, with increasing costs. On the same diagram where we have an indifference map for two products produced by one firm, we may also draw some price lines (Fig. 10, page 101).

Let us consider the problem this way. For each state of the market there will be a series of price lines which will be determined by the ratios of the prices of the two products and by the amount of money which can be received by selling any combination of the two products represented by that line. Each line will be an equi-revenue curve. The slope of each line is the ratio of the product prices. Each line meets axis A at the point which shows the quantity of A alone which can be sold. The same applies to B at the point where the price line meets the B axis. This type of line is called an iso-revenue or equi-revenue curve.

Suppose now that we have an indifference map with product-transformation curves representing a cost of 80, 100, and 120. Let us suppose that the firm produces at a total cost of 100. Which one of the combinations represented by the curve 100 will be produced? To one curve a price line is tangent. It represents all the combinations which permit reception of a certain revenue. The point of tangency of the price line to the indifference curve will determine the quantity of A and B which will be produced (at C in Fig. 10, page 101).

If A and B were produced in D or E the iso-revenue line would be lower. If they were produced at F or G for the same revenue the

cost would be higher. At the point O the slopes of the two curves are equal, and the price ratio is equal to the marginal rate of substitution. In other words, the price ratio is equal to the ratio of marginal costs.

One sees that this is related to the general theory of production which says that the producer tends to equate the marginal cost of his product to its price. When the producer equates the marginal costs to the prices, the price ratio is always equal to the ratio of marginal costs.

Consider a change in the price ratio (Fig. 11, page 101). Suppose that the price of A increases. The equality of the price and marginal cost of A will no longer exist and the producer will produce more A without decreasing the production of B . But a higher output of A will change the marginal cost of B and another modification will be necessary for the output of B until we reach equilibrium. At this point the ratio of marginal costs will be equal to the prices of A and B . An idea of this is given on the diagram.

One may consider Oa as the price of A in terms of B . If the price of A increases (let us say doubles) Oa_2 (equals $2Oa$) will be the new price of A in terms of B and ab_2 the new price line. Here we represent the increase in price in a way different from the one we use for input analysis. In input analysis one must consider the budget constant, at least for the part of the problem where we study a change in the price ratio. But here one is allowed to consider an increase in the revenue, which one may consider not to be directly

limited for the producer. So in the graphic representation when the price of A doubles in terms of B, the revenue is also doubled since the meeting of the price line and the A axis does not change. (Six A were sold at ten cents per unit; now 6 are sold at 20 cents per unit).

It appears that the second price line is tangent to a higher indifference curve and that more A is produced and less B. So when the price of one of the products increases the firm tends to increase the production of this product if it wants to keep the costs of production of the products equal to their prices. This is represented by the fact that the price line remains tangent to an indifference curve.

The indifference curve analysis also seems to give an idea of what is called technical complementarity. When a rise in the price of A raises the output of B it is said that A and B are technically complementary. Thus the indifference curve analysis is not only practical for the study of substitutability (when the price of A raises proportionally less B is produced).

These results have been obtained for a firm which produces two products. The analysis may be extended to include the case of more than two products as we have shown in respect to multi-factors of production. This, then, is the use of the indifference-curve analysis in the study of production. It permits to get a good graphic representation of the input and output analysis and that it is as useful as it was in the study of consumption. But these are not its only uses. Indifference-curve analysis is also very helpful in studying market equilibrium, efficiency of the firm and international trade, to which I now turn.

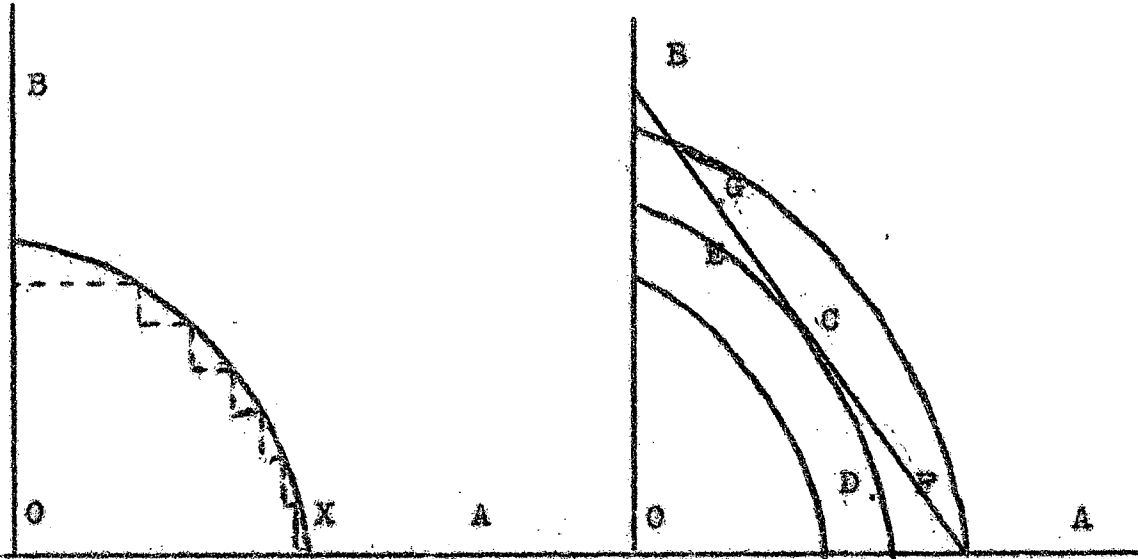


Fig.9

80 100 120
Fig.10

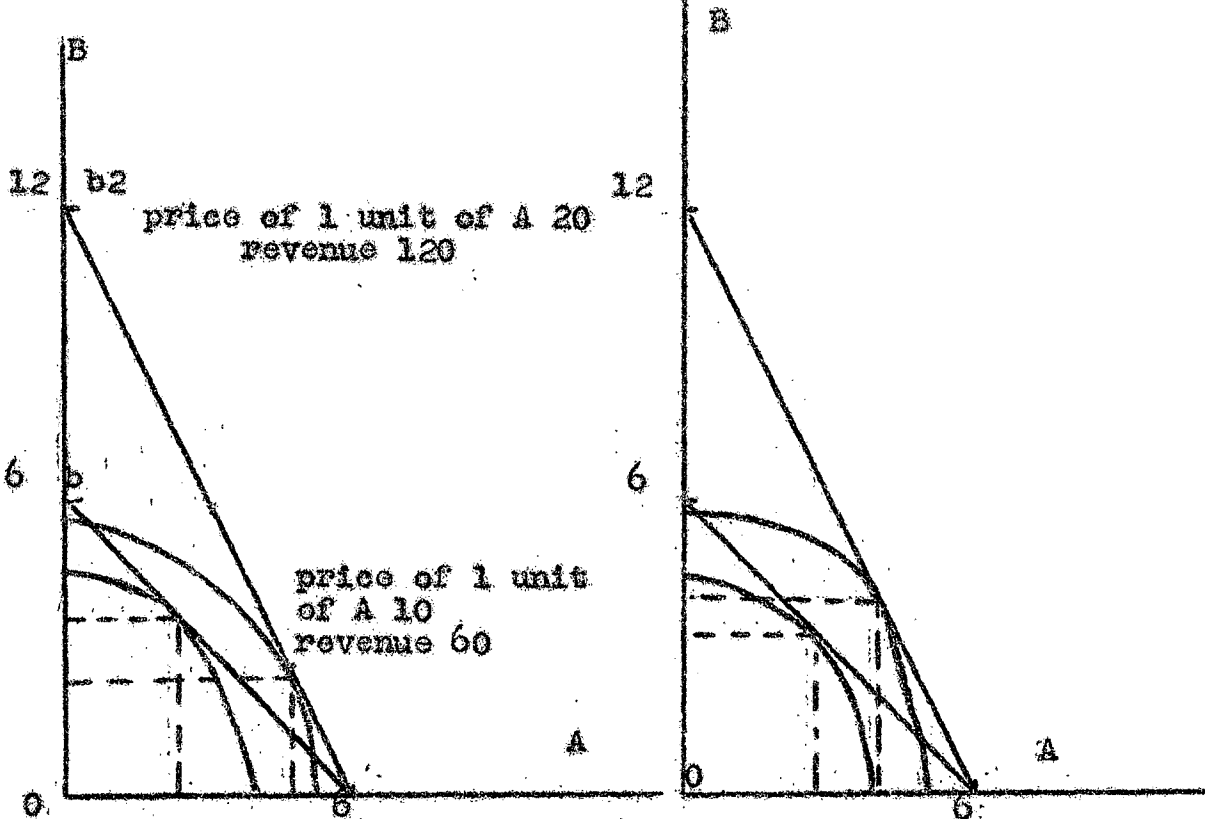


Fig.11 A & B technical substitutes

Fig.12 A&B technical complements

PART IV

EQUILIBRIUM AND LATER DEVELOPMENTS

CHAPTER VI

PARETO'S THEORY OF ECONOMIC EQUILIBRIUM

Pareto used the indifference curve approach to the study of the equilibrium of several producers, several exchangers, or several producers and exchangers. I shall examine Pareto's works on these matters in the order which is presented in the Manuel with but one exception. As usual, Pareto makes his demonstrations without using the concept of prices. When he is through with his reasoning he explains the concept of prices and applies it to his previous demonstrations. When I examine Pareto's theories I shall use the concept of price immediately. This will make their understanding easier.

A. PRICES AND SECOND TYPE OF OBSTACLES:

Before getting into the study of equilibrium, we are going to examine a paragraph which may improve our understanding of the shape of Pareto's "path". In modern theory one is accustomed to seeing price lines which are straight. Pareto feels that some price lines may be curved if the price varies as exchange occurs between the two commodities.¹

If the individual who wants 300 gr. of bread buys 100 gr. at 60¢, 100 gr. at 50¢, and 100 gr. at 40¢ the price line will be a curve since there is a different slope for each separate price. In

¹Pareto, Manuel d'Economie Politique, p. 212.

this case for each operation which tends to bring an equilibrium there is a different price. Let us take an example easy to describe graphically (Fig. 1).

On OB we measure money, on OA, the quantities of commodity A. If we want to buy 3 of A we buy the first unit of A at a price such that 4 of A equals 8 of B, so the slope of the price line will be the slope of a line joining 4 of A to 8 of B. This slope will go on as far as one unit of A. For the second unit of A the price will be 3 of A equals 9 of B. The slope of the price line will be the same as the one of the line joining 3 of A to 9 of B on this for one unit of A. The third unit of A will be bought at 2 of A equals 4 of B so the slope of the price line will be equal to the slope of a line joining 2 of A to 4 of B. The slope of the general price line changes in C and in D.

Pareto also considers prices changing from one complete operation to another. In that case there are two different price-lines. If in each operation the prices are constant, both price lines will be straight; if in each operation the prices change for the successive quantities both price lines will be curved. This must be added to what has been said about the "paths" in the section of this paper dealing with consumption. Therefore, theoretically, the path 'mn' in Fig 3 of chapter II may be considered as a price line, although I think that actually such changes in prices would seldom occur. If mn is considered as a price line point c, as a tangency point, would be

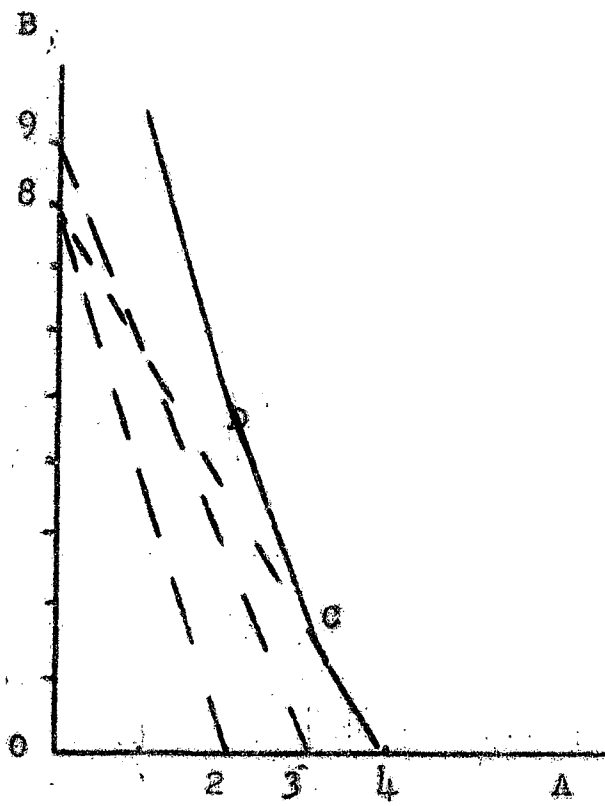


Fig.1

a point of equilibrium. Besides, if the income of the individual did not allow him to face one of the successive changes in price which are represented by the changes of the slope of the curve, we may consider that there would be a terminal point on such a price line. This is an example of the surprises which await the reader of Pareto's economic writings. Sometimes a reasoning or a theory which was not clear at all at first sight is explained fifty or a hundred pages later.

How does Pareto use the consumer's indifference curves to study the equilibrium of exchange? If we consider Fig 1 of chapter II we may suppose that the individual whose tastes are represented by the indifference curves has om of A and no B . If he wants to get some B in order to be on a higher indifference curve, he will exchange some A for more and more B along the path mn of which the slope represents the price of B in terms of A or vice versa. When he is at the equilibrium point c we shall say that the supply of A is $a'm$ and the demand of B $a'c$. So "the exchange curve EE' can also be called a curve of supply and demand."¹ These considerations will directly help us when we consider the equilibrium of exchange.

B. EQUILIBRIUM IN TASTES AND OBSTACLES:

Pareto begins by examining the equilibrium of tastes and obstacles. For this purpose he presents a diagram which is not very easy to understand. This diagram (Fig. 2) is represented as in the Manuel. It is composed of the axes OM and OB which are the axes of tastes; md is the price line of which the movement produces the

¹Ibid., p. 221.

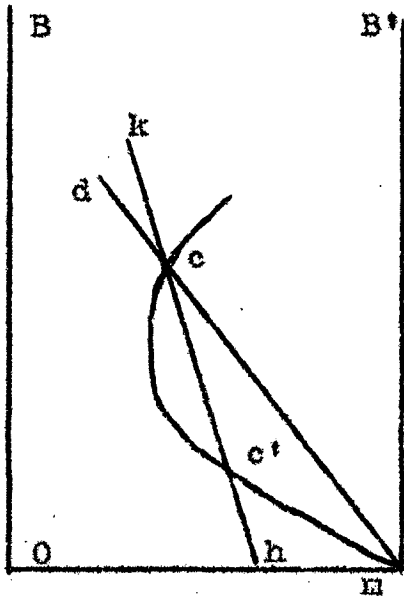


Fig. 2

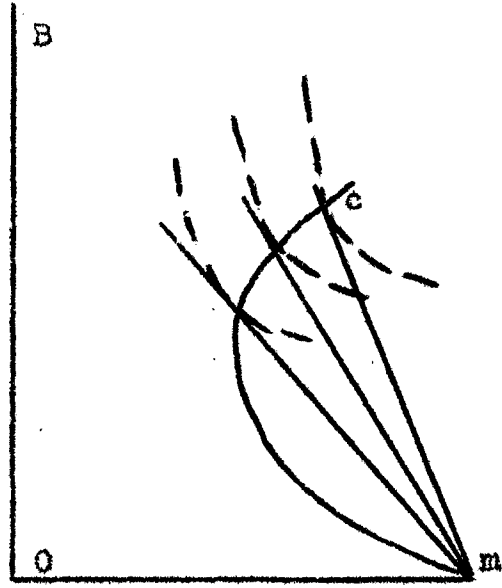


Fig. 2a

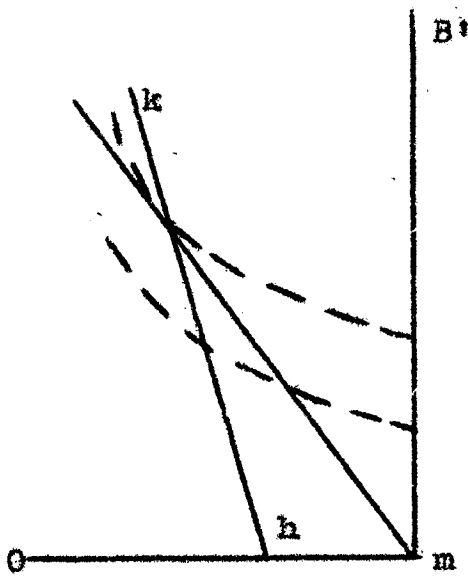


Fig. 2b

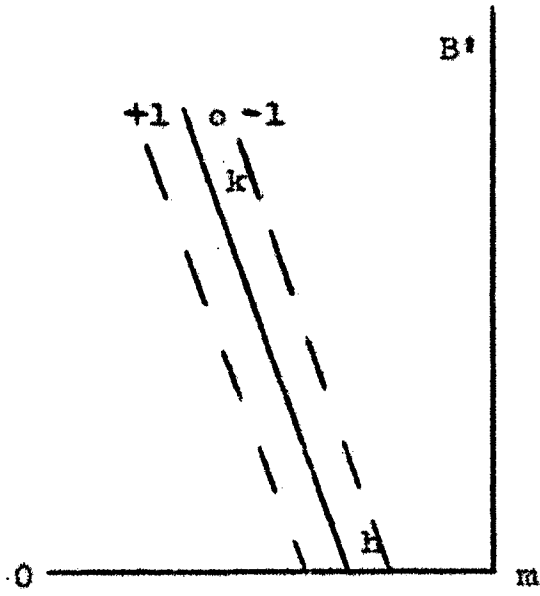


Fig. 2c

exchange curve mn which here is a price consumption curve. We describe that operation in diagram 2a. Let us say that commodity A is measured on Om , commodity B is measured on OB ; the slope of the price line nd on Om represents the price of A in terms of B. In Fig. 2b the axes of obstacles are nB' and nO . Commodity B is measured on nB' . Commodity A is here the factor which is transformed into A. (Commodity A is measured along Om). The "path" nd here corresponds to a total-revenue curve as we noted earlier. Its slope on Om represents, as in the case of tastes, the price of A in terms of B. On this part of the diagram hk is the curve of highest profit.

Pareto adds that the same curve hk for the commodities with a decreasing cost of production is the complete transformation curve. For this we have to consider diagram 2c, turned towards the left.

If things go on according to model (1) (competition) producers will stop on the highest profit curve hk and precisely at the point where the highest profit curve crosses the exchange curve of consumers. So the path, or price line, followed by consumers and producers will be nd (Fig. 2). But in competition the selling price is equal to the marginal cost of production, so the producers will operate on the complete transformation curve. They will operate at the point where the exchange curve crosses the complete transformation curve (because on the exchange curve the consumers are in equilibrium). At that point the path is tangent to the complete transformation curve and also to an indifference curve of tastes (this is the condition for the existence of an exchange curve). At that point the complete transformation curve will be tangent to an indifference curve of tastes.

then Pareto explains that he does not want to consider an isolated single individual. He considers this single in a community.

If the equilibrium of exchange, if the indifference curve of the individuals are placed in such a way that the price (or price lines) coincide, the equilibrium point will be at the crossing of the exchange curves of the two individuals. The path going through these points will be tangent to an indifference curve of the first individual and to an indifference curve of the second individual, since it crosses both exchange curves (since a path of price lines crosses an exchange curve it is tangent to one of the indifference curves which determine that exchange curve).

In this section Pareto is put down a general theorem:

"For phenomena of social life a point exists at which a path can cross by the individuals in regard to the indifference curves of these individuals (products of commodities) this point is an equilibrium point."

This has been shown for the equilibrium of production and for the equilibrium of exchange.

Pareto sets another theorem for the case where there are no tangency points:

"The equilibrium curve of the crossing of the capital stock curve of houses and of the equilibrium curve of production (exchange curve and highest profit curve). These curves are the loci of the tangency points of paths to indifference curves or the loci of tangency points of these paths."

Then one individual operates according to social life the equilibrium occurs at the point which is the best for that individual: one of the

Figure 2-100.

Figure 2-100.

crossings of the paths and of the locus of possible equilibrium points. We have already seen that we must be careful with the use of these "terminal points" which cannot occur on ordinary price lines of straight paths.

This gives us a brief idea of the way Pareto uses indifference maps of consumption and of production in order to give a graphic representation of the equilibrium. But I must acknowledge that the reading of this part of the Manuel is rather difficult because many explanations are omitted. After having given a general idea of his assertions Pareto then examines the details of the equilibrium.

9. TYPES AND FORMS OF EQUILIBRIUM IN EXCHANGE:

The diagram which is used here is the "box diagram" formed by two indifference maps placed in such a way that the paths coincide (here the paths are the price lines). Let us consider Fig. 5. On this figure individual (I) has the axes OA and OB and the indifference curves $t, t', t'',$ etc. Individual (II) has the axes wa and wB; his indifference curves are $s, s', s'',$ etc. Both of them follow the path mn . Individual (I) owns an amount of A and transforms it into B by exchanging it for the B of individual (II) who does not have any A. If the obstacles of second type we talked about previously allow changes in the direction of the price line, both individuals will try several prices until the time when they find the price represented by the slope of mn . At this position mn is tangent to an indifference curve of individual (I) and of individual (II). The tangency point is also the crossing of both the exchange curves of the individuals. This point is an equilibrium point for both individuals.

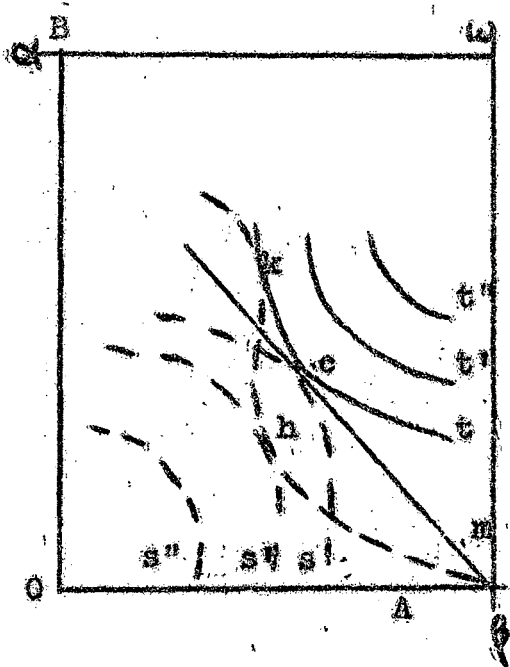


Fig. 3

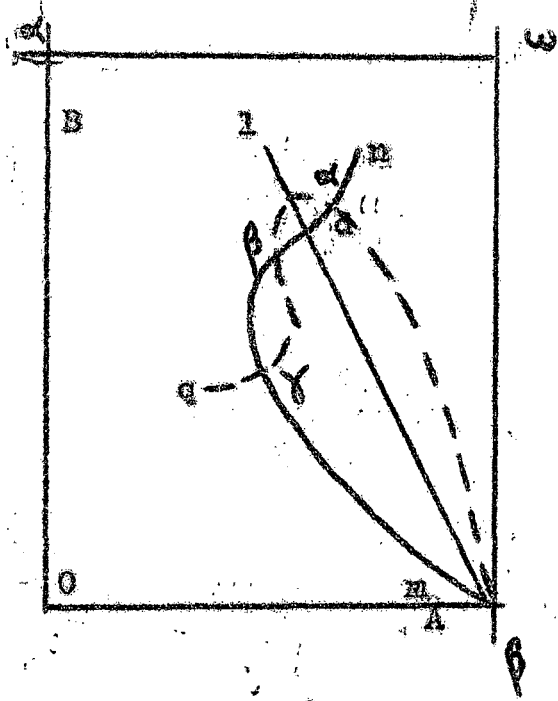


Fig. 4

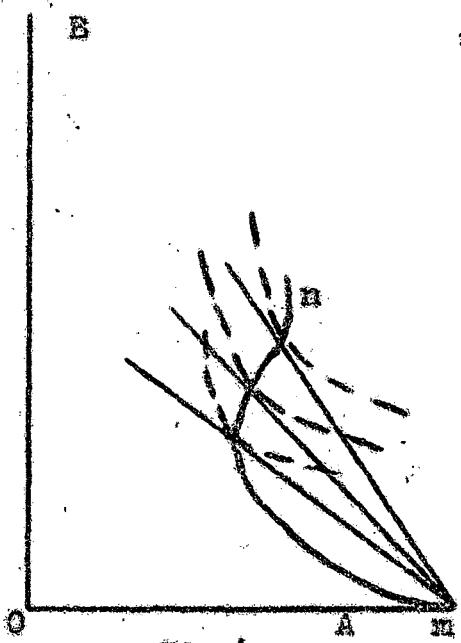


Fig. 4a

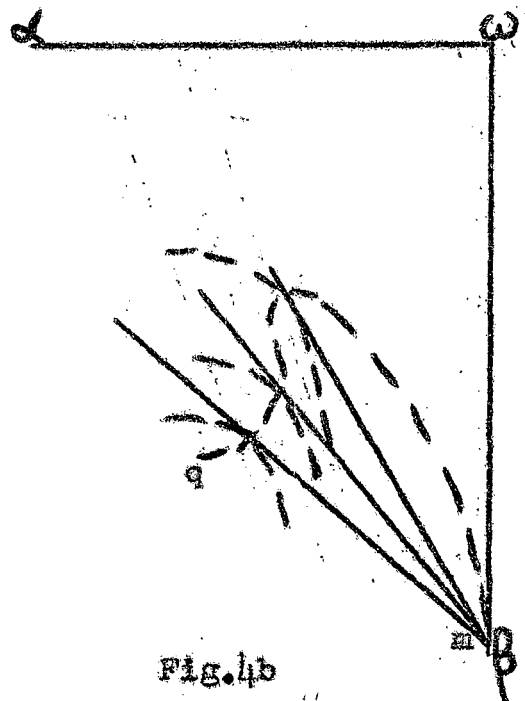


Fig. 4b

If obstacles allow only a path similar to mhk (theoretically such a price line may be imagined), the tangency points are different for each individual. If neither one of the individuals acts according to model (II) the problem cannot be solved. If one individual acts according to model II he will force the other to operate at the tangency point which is the highest on his own hill of pleasure. For the other individual that point will be a terminal point on the path. I do not think that such a demonstration, based on such a curved-price line is very useful to describe what may happen. The variations of price on such a price line would be more complicated than anything we know.

Pareto now examines the case of stable and unstable equilibrium. He presents a box diagram which may be decomposed in two parts. I represent this in Figs. 4, 4a, and 4b. On these diagrams only the exchange curves of the individuals are drawn. The axes OA and OB and the exchange curve mn are the first individual's. The axes va and wb and the curve mq are for the second person. On such a diagram the exchange curves are price consumption curves.

There are three equilibrium points α , β and γ that Pareto represents on different diagrams (Figs. 5a and 5b). On these diagrams cd is always the exchange curve of the first individual and hk will be the exchange curve for the second individual. The path is ml .

At first sight one sees that points α and γ are similar; in these cases curve lh is before curves ld when one comes from m on path ml . Because of the tastes of the second individual, the first one will not be able to operate, as he would like to, on curve ld .

He will have to operate on the exchange curve of the second individual. The point h will be a terminal point for him on path lm . That same point h will be a tangency point for the second individual. The movements which lead to equilibrium are going to occur because the first individual is not satisfied, since he is not on his own exchange curve.

From now on the demonstration of Pareto is very complex. Pareto seems to confound the case where both individuals trade A for B with all the other individuals of the collectivity, and the case where one individual exchanges B for A with another one who exchanges A for B. Briefly the explanation of Pareto is that both individuals are going to decrease the price of A in terms of B in order to gain more customers. So the slope of the path on nm will decrease as far as the equilibrium point i .

Here I am trying an explanation which is much simpler and applies to the case of the first individual who exchanges A for B trading with the second one who exchanges B for A.

Consider Fig. 5a. With a price line such as nl the two individuals operate at l . If the first individual wanted to operate on his own equilibrium curve the other one would not trade. At l , a of A is exchanged for a' of B. The first individual, with such a price, would like to exchange a'' of A. So he is going to decrease the price of A in terms of B and the slope of nl on ms will decrease. The difference between a and a'' will decrease and become equal to zero at the equilibrium point d . The same reasoning but reversed is valid for the part of the diagram where the second individual is not satisfied. Point d (or δ) is a stable equilibrium point.

Consider now the point G of diagram K and let us represent it on a diagram $5b$ similar $5a$. It may be seen that the exchange curves are in a position different from the position we just examined. On path ml the first individual has to operate at point l on the exchange curve of the second individual. At such a price he would like to exchange ma' of A (ms) so he is going to decrease the price of A in terms of B and the slope of the price line is going to decrease. So points l and l' go farther and farther from point G (at l_2 and l_2'). Again it would be possible to make a similar reasoning for the part of the diagram where the second individual is not satisfied. So G is an unstable equilibrium point.

This was valid for model (1) (competition). Pareto now examines model (2), (Fig. $5c$). Suppose that the first individual acts according to model (1) and that the second individual acts according to model (2). mn is the exchange curve of the first individual, mq the exchange curve of the second individual. Since the second individual acts according to model (2) he may impose a price, that is, he may determine the price line. Let us, then, consider price line ml . The quantities of A and B which are exchanged are determined by the crossing of the price line with the first individual's exchange curve. The equilibrium point cannot be beyond d on the price line because then the first individual would no longer trade. The second individual even operating according to model (2) cannot make him trade on the part of the price line which is beyond d . But, on the contrary, the second individual could trade on the part of the price line which is before d .

The first individual would trade also because he would be going higher and higher on his hill of pleasure, toward his equilibrium line. So an equilibrium is possible on mn . If the second individual is operating at d he is going to compare what he gets at that point and what he would get at any other point. For the second individual the best point will be the highest point on his pleasure hill, at which mn is tangent to an indifference curve. Pareto notices that this is practically very difficult and the second individual often tries to get the maximum of d ; this would happen at e . If A is a superior good x and t will be very close together.

In chapter VI Pareto adds a few words about the equilibrium of exchange. At the tangency point of two indifference curves both individuals have the maximum "ophelimity". If they move from this point, the ophelimity index of one will increase and the ophelimity index of the other will decrease. One will be going up on his pleasure hill, the other one will be going down. From an equilibrium point no movement can increase both ophelimities.¹

B. AN APPLICATION OF PARETO'S THEORY OF GENERAL EQUILIBRIUM OF EXCHANGE TO WELFARE ECONOMICS.

The last statement of Pareto which we noted--that no movement away from an equilibrium point can increase both ophelimities of the individuals--has been used by Pareto in welfare economics and later by Hicks. Let us see how it has been used.

¹Ibid., p. 356.

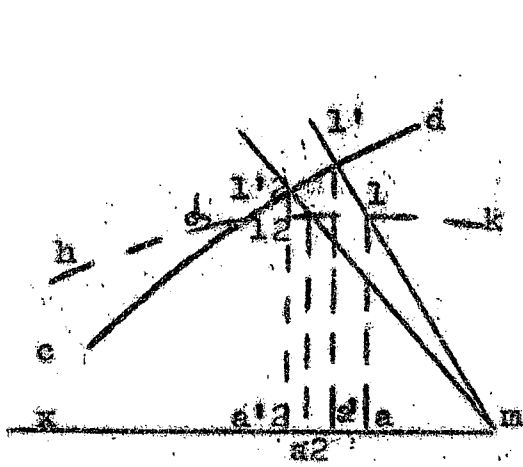


Fig. 5a
(cases α & γ)

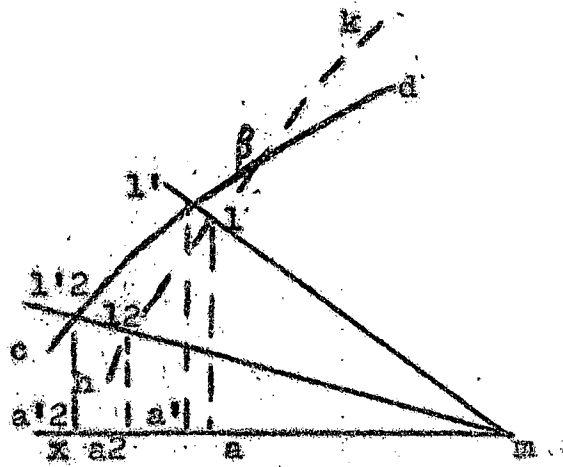


Fig. 5b
(cases β)

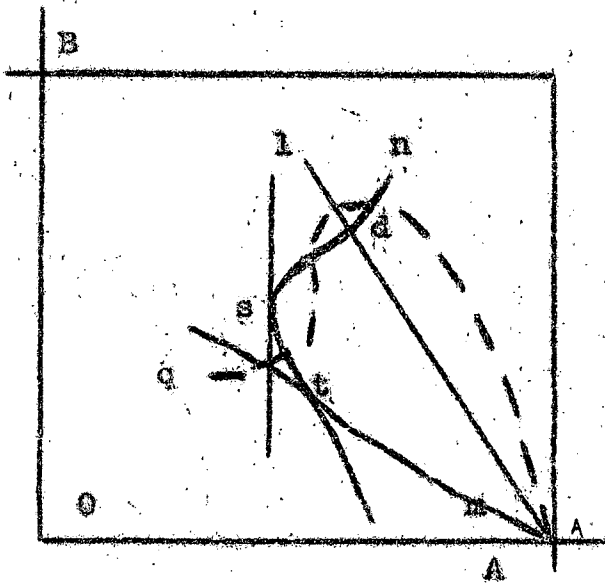


Fig. 5c

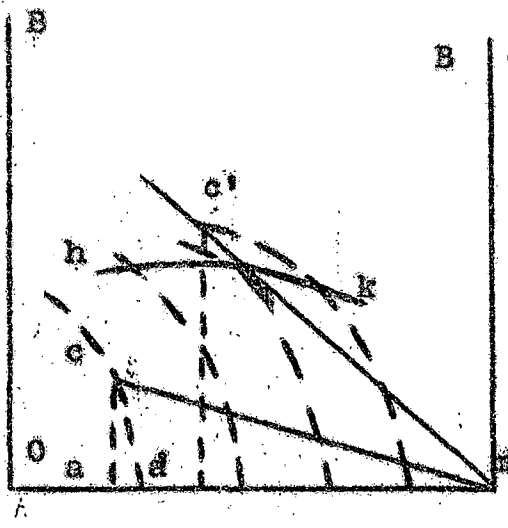


Fig. 6

Pareto's writings which are related to this problem are in section 28 of the Economie Mathématique included in the Encyclopedie des Sciences Mathematiques.¹ This long article is famous among Pareto's writings and is a kind of a summary of the mathematical appendix of the Manual. It is available, translated into English, in International Economic Papers.² Here, briefly, is what Pareto says:

If we want to maximize the welfare of a community there are two problems to be solved. The problem of the rules of distribution belongs to sociology. The problem to find the best position for the community according to those rules is solved by economics.

Pareto considers the welfare of a community in terms of good A. He then considers a very small movement from an earlier position. If the welfare of all the individuals is increased the new position is better for the collectivity, and vice versa if the welfare of all is decreased. If the welfare of some remains constant and the welfare of others increases we shall also consider the new position as better than the previous one. "But if on the contrary, this slight movement results in increasing the welfare of some individuals and diminishing that of others we can no longer affirm that to make this change benefits to the whole of the community."³ The fact that this change may benefit to the whole of the community depends on each individual change.

¹ Tome I, Vol. 4, Fasc. 4, Paris 1916.

² No. 5, (London Macmillan, 1953), p. 87.

³ Ibid., p. 87.

Here we are helped by the equation presented by Pareto. If for individuals 1, 2, and 3 ϕ_1 , ϕ_2 , and ϕ_3 are the total index functions and ϕ_{1a} , ϕ_{2a} , and ϕ_{3a} the elementary index functions of good A; (here the welfare is calculated in terms of good A and d represents "change in..."), we have the equation: $dS = \left(\frac{1}{\phi_{1a}}\right) d\phi_1 + \left(\frac{1}{\phi_{2a}}\right) d\phi_2 + \dots + \left(\frac{n}{\phi_{na}}\right) d\phi_n$ dS is the change in the total supply of A. The expressions similar to $\left(\frac{1}{\phi_{1a}}\right) d\phi_1$ represent the changes in the quantities of A owned by each individual. If dS is positive we can say that the change has been favorable. If it is negative the collectivity as a whole has lost.

The sign of dS is determined by the sign of each expression $\left(\frac{1}{\phi_{1a}}\right) d\phi_1$, etc., which represents the changes in the quantities possessed by individuals. I shall examine first the case in which the welfare of some individuals increases and that of others decreases. That is to say when some expressions of the sums increase while others decrease.

If the sum of the positive changes is higher than the sum of negative ones, dS will be positive and we can consider that the total ophefimity of the collectivity has increased. If the contrary occurs there will be a loss for the collectivity. If these two sums are equal, or if each individual change is equal to zero (that is to say when the individuals are satiated), dS will be equal to zero. When dS equals 0, that is if everybody is satiated, or if, when we want to increase the share of some individuals, it can be done only by reducing the share of others, the total ophefimity of the collectivity is maximum. This condition is necessary to have an economic equilibrium.

This is all that Pareto says explicitly. But the equation is full of meaning that we can develop here. Let us give some value to the expressions $\left(\frac{1}{QIa}\right) d\phi I$, etc. We have $dS = +4 + (-2) + (0I) + 6$
 $dS = +4 + 6 - (2+1) = 7$, the value 7 representing the increase for the whole collectivity. If we observe what has been done we see that before making a gain appear for the whole collectivity the increase of $4 + 6 = 10$ had to balance the decrease of $-(2 + 1) = -3$. This was necessary to maintain the welfare of the people who previously suffered from the loss. The remainder represents the gain of the whole collectivity.

The case of equilibrium would be something similar to
 $dS = +4 + (-7) + (-3) + 6$ or $dS = +4 + 6 + (7 + 3) = 0$. When one moves from an equilibrium position all that is gained by some individuals is necessary to balance what is lost by others: the gain of the community is zero.

Hicks, in a lecture¹ presented at the "Institut de Science Economique Appliquée" on the 26th of June, 1946, develops this idea of the Paretian equation:

It will seldom occur that a simple change in the economic organization... will produce by itself a non-equivocal benefit, because most of the people will perhaps get an advantage, but some will lose. The change should nevertheless be considered as an improvement in Pareto's sense, if the persons benefited gained enough in order to compensate those who lose and still have a gain left... the Paretian optimum is an optimum in which no gain of that kind is possible.²

¹Hicks, "L'Economie de Bien Etre," Economie Appliquée, No. 4, Oct-Dec 1948, (Presses Universitaires de France).

²Ibid., pp. 435-436.

Hicks shows that it is not necessary for a compensation to be paid.
 "It is clear that a series of improvements would finally give an economic optimum position in which no new improvement is possible."¹

Pareto said that pure economics does not deal with redistributions of income. Hicks should have remembered this when he said that there are several optimum positions, one for each system of distribution, and that movements are possible from one optimum position to another one. Such movements will inevitably increase the share of some people and decrease the share of others. The pure economics of Pareto just does not consider such movements since the phenomenon is studied on a given sociological basis. In Pareto's theory the system of distribution is fixed. If we want to consider another system, we have to solve a separate problem; this is all that pure economics can do in Pareto's system.

Therefore, if we confine ourselves to pure economics I do not quite agree with Hicks when he says that the Paretian theory of the optimum is therefore a theory of the optimum of production and not of the optimum of distribution." In any event, Hicks admits that, from this point on, the problem is a part of welfare economics and not a part "of the logical development of the Paretian logic."²

B. EQUILIBRIUM OF PRODUCTION:

We have just examined equilibrium in exchange. Let us now examine the Paretian "production equilibrium".

¹Ibid.

²Ibid., p. 433.

The producer tends to stay on the highest profit curve in the same way as the consumer tends to stay on his exchange curve. Let us consider Fig. 6. The price line mc does not cross the highest profit curve. The producer wants to move to the highest possible point on his profit hill, that is, to transform a maximum of A and to sell a maximum of B . But the consumers may not want to buy more than ac of B at the price represented by the price line mc . So the producer is going to offer a lower price for B and this will decrease the slope of mc on OB , or increase it on mo . But the other producers suffer from such a decrease in price. So they are going to decrease their prices and the price line will finally bring them on the highest profit curve. This occurs in a market working according to model (I).

If the producer operates at e' on another price line mc' he will have an interest in decreasing the quantity ma' of A that he transforms into B . If he does so, following the price line he will go higher and higher on his profit hill as far as his highest profit curve. This may occur without any price change. In this demonstration I considered a production with increasing costs. There was a highest profit curve.

I am now going to consider a decreasing cost situation, a case in which there is no highest profit curve, since as Triffin says, "every competitive firm under complete competition (decreasing costs) tends to expand output indefinitely without reaching any true equilibrium position."¹

¹R. Triffin, Monopolistic Competition and General Equilibrium Theory, p. 60, Cambridge H. U. P. 1947.

As a matter of fact there is an equilibrium position: at the crossing of the complete-transformation curve and of the exchange curve of the consumers. Pareto distinguishes two cases. The first is a stable-equilibrium case. Let us consider Fig. 7a, page . . . At a on price line ma the producer is in a favorable position: he is in the region of positive indexes. The consumers are in equilibrium since they are on the exchange curve cd .¹ But the producer will try to move higher on his profit hill, beyond oa . At such a price, however, the consumers do not want to trade, since they would be going lower on their pleasure hill. So the producer is going to decrease the price of B in terms of A, and the slope of ma on mx will increase. But the other producers will follow this decrease in price and so all of them will reach the complete-transformation curve at l . They will not go beyond l along cd because they would enter the non-profit part of the hill. They cannot leave the exchange curve because then consumers would not trade any more. So l is the equilibrium point.

Let us consider Fig. 7b. We see that the exchange curve cd is in a different position. The part cl is in the non-profit region of the diagram. So if the producers are on cl they are going to increase their production and decrease the price of B in terms of A. But they will not stop on the complete-transformation curve at l , because they can proceed along the consumer's-exchange curve toward high profit indexes by decreasing prices (increasing the slope of mc on mx) and increasing production. This appears to be profitable

¹Pareto considers that cd is the sum of all the exchange curves of the community. See the section of the paper on social indifference curves, chapter VII, D.

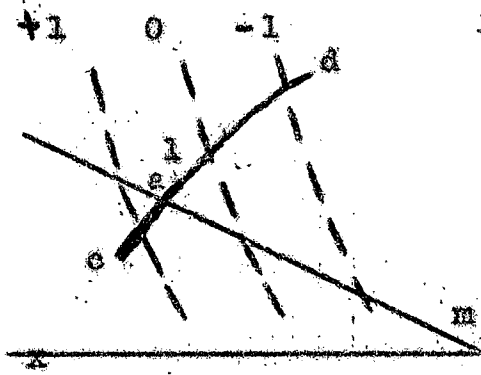


Fig. 7a

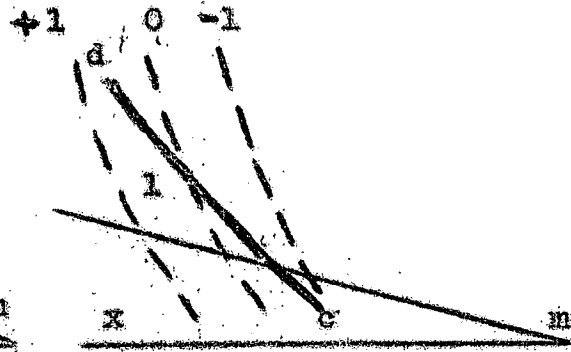


Fig. 7b

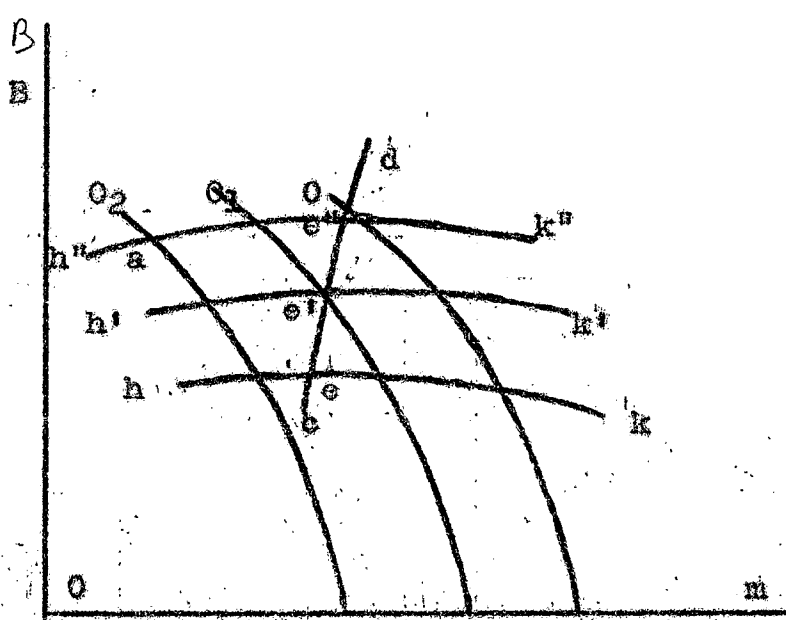


Fig. 8

since costs are decreasing. So at L there is an unstable equilibrium; Pareto says: stable on one side (in the non-profit region), unstable on the other (the profit region).¹ Concerning this Pareto says: "we described the phenomenon as it occurs in the long run. The producers may be losing in the short time."²

It is interesting here to report a note of Triffin which is related to this point: "It is not quite clear from Pareto's exposition whether this limitation of total output of the competing sellers is reached through a spontaneous limitation of output by every individual firm or through a continuous expansion of output by some firms, compensated by the bankruptcy and disappearance of others."³ This is related to case (a). Then Pareto examines what happens when the number of producers has an influence on obstacles. This is a case of "new-comers" competition.

Once more the exposition of Pareto is not clear. In figure 6 we consider the highest profit curve for one producer. If a second one arrives it is necessary, in order to keep the highest profit, to double the quantities (or maybe, to increase them in some other proportion). The highest-profit curve is then displaced to a new position Hc' . The complete transformation curve is also displaced to Cl . In modern theory we would say that "free entry of firms eliminates profits."⁴ Pareto does not say in which way the movement

¹ Manuel d'Economie Politique, p. 206.

² Ibid., p. 51

³ Monopolistic Competition and General Equilibrium, Theory

of the complete transformation curve occurs. But we can see that the base of the new complete-transformations curve O_2 will be further from m (the origin) than before.

Pareto says "as long as it (the exchange curve or equilibrium curve of consumers) crosses the highest profit curve at a point different from the one at which it is crossed by the complete transformations curve, the equilibrium may occur at the crossing of the exchange curve and of the highest profit curve."¹ Then he adds: "until the time when the highest-profit curve does not cross the exchange curve any more." This explanation is misleading because as we can see on Fig. 8 these two curves will obviously always cross. But if they cross in the non-profit part of the diagram, as with a new complete-transformation curve O_2 , the equilibrium will not occur there because the producer would be incurring a loss. It will occur at a point a where there are no losses, but no profits, on the complete transformation curve. When he was alone the first producer was able to sell with a profit but if newcomers arrive he will finally have to operate on a marginal cost basis as any competitive producer.

Consider model (2), or monopoly. In this respect I shall be very much helped by the work which has been done on the matter by Triffin. This economist shows a remarkable understanding of the writings of Pareto.

For the study of the equilibrium in model (2) we are going to

¹Manuel d'Economie Politique, p. 206.

use Fig. 10 which is a diagram similar to the one used for the equilibrium of obstacles and tastes in Fig. 1. mn will be the exchange curve. The producer who acts according to model (2) is similar to the exchanger who acts in the same way (Fig. 5). So along the curve mn which is the equilibrium curve of the part of the contract who acts according to model 1, the producer will try to reach the highest point possible on his profit hill. In that case costs are decreasing or in other words competition is complete. So equilibrium will occur at the tangency point of the exchange curve to an indifference curve of producer (this will show the highest profit attainable in such a market). Triffin says: "The intersection of marginal cost and marginal revenue might express a position of minimum rather than maximum profit."¹ (Here Triffin is talking about the intersection of a complete-transformation curve and an exchange curve.)

The great merit of Triffin's analysis is to consider the exchange curve as the total-revenue curve. As a matter of fact, when I studied the equilibrium of the producer I said that he follows a path which is a total-revenue curve. Here the path followed by the monopolist is an exchange curve (equilibrium can only occur on the equilibrium curve of the non-monopolist) so it seems normal to consider such a curve as a total-revenue curve. This corresponds to Fig. 10, in which the equilibrium occurs at a point where total

¹Triffin, Monopolistic Competition and General Equilibrium Theory, p. 59.

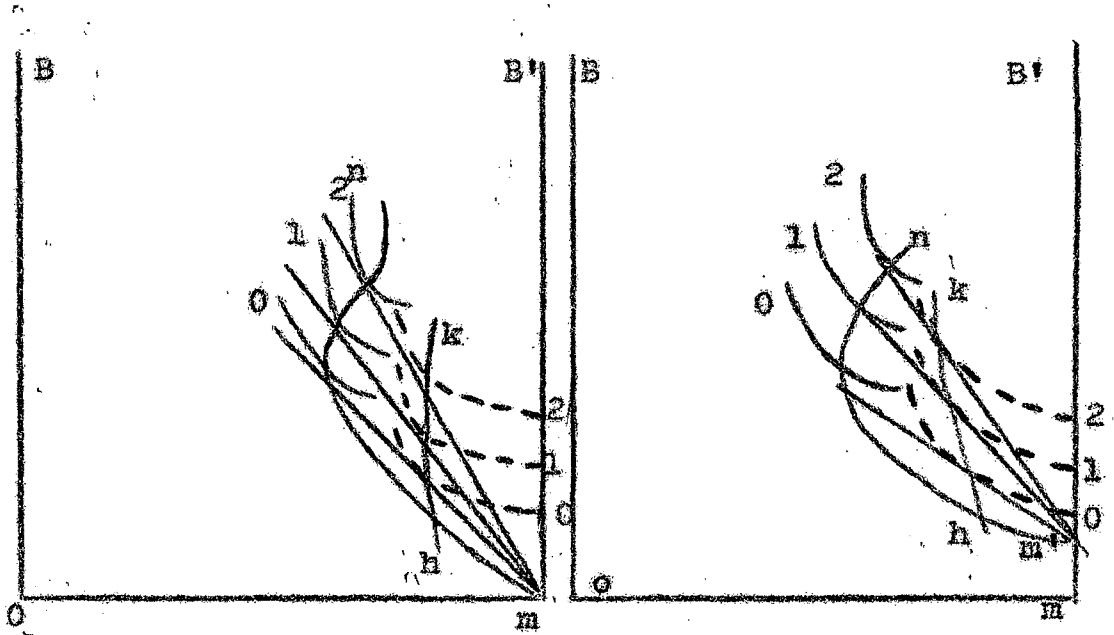


Fig.9

Fig.9a

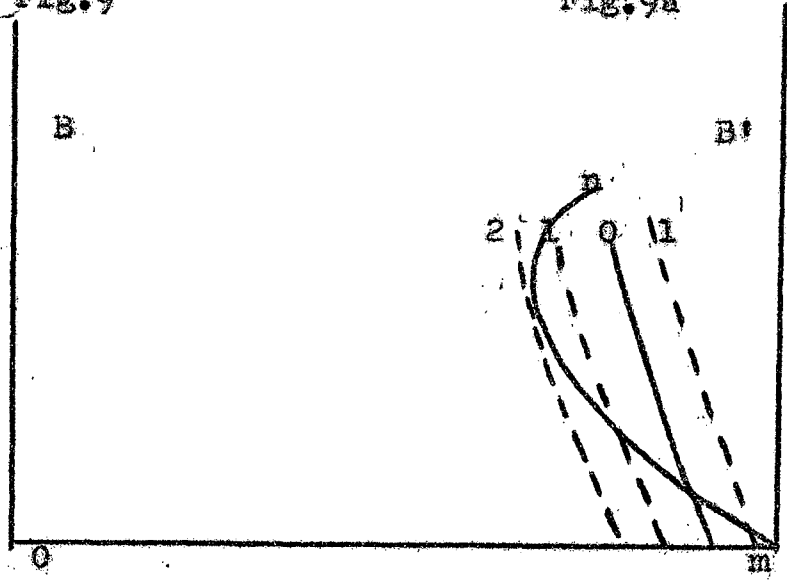


Fig.10

revenue is still increasing (along the exchange curve mn). In this case the total-revenue curve is different from each price line.

Triffin notes that when Pareto says that for an increasing cost industry, the equilibrium point will be at the tangency point of the exchange curve and of the highest-profit curve, this does not make any sense. In fact, since in that case the exchange curve is considered as a total-revenue curve, and since the highest-profit curve by definition crosses all the total-revenue curves, they cannot be tangency to each other.

Even without any explanation of this kind we see in Fig. 9 and ga that Pareto's condition is impossible. The curve hk cannot by any means be tangent to mn if only because both of them are determined by the same kind of price line (built around g). If mn moves, hk will move also. If the consumer acts according to model (2) the producer will have to operate on the complete-transformations curve because the consumer will be imposing the lowest prices possible.

P. PARETO'S CONCEPTION OF ECONOMIC EQUILIBRIUM:

When he studied economic equilibrium Pareto seems to have been very cautious in his graphic constructions. It is true that it was the only means which could portray his ideas without resorting to mathematical economics. Some economists do not have the background necessary to understand so advanced a mathematical approach. The principle part of Pareto's system can also be found in Walras' writings-- for instance, the system of equations which gives a representation

of the economic equilibrium.¹ Pareto also gives a system of equations. When he does this he is following the Walrasian tradition. Walras in his representation of general interdependence went even further than the modern theorists. As Triffin noted: "Consideration of the general interdependence of the economic system is, however, still limited by the new theorists to the group of industry, rather than extended, as in Walrasian economics, to the whole economic collectivity."² This, according to Triffin, arises from the fact that: "In opposition to Anglo Saxon economics, the French tradition in its presentation of the workings of economic equilibrium, had always started the firm rather than the industry."²

Walras considered the case of free competition.³ Pareto wanted to go further in the same way; he says as much at the beginning of the third chapter (paragraph 31). In the appendix of the Journal (pp. 613-617) he takes the monopolist into account in his system of equations, side by side with all the other sellers, competitors or other monopolists. Marshall has studied the profit maximization of the isolated monopolist with the "sales curve". Pareto, like Chamberlin, considers a group of monopolists. But while the theory of monopolistic competition³

¹ Walras, Elements d'Economie Politique Pure ou Theorie de la Richesse Sociale, edition definitive, (Lausanne: F. Rouge, 1926)

² Monopolistic Competition and General Equilibrium Theory, p. 9.

³ See p. . . first quotation of Pareto, and footnote 1. A very interesting discussion of this part of Pareto is given by Triffin in Monopolistic Competition and General Equilibrium Theory pp 70-75

¹ Walras, Richesse

² Monopol

³ See p. interest in Monop

is the study of a partial equilibrium within the limits of a group. Pareto's theory includes all the interrelations in the whole economy. If one wants to consider a complete general equilibrium, and to use a system of equations to represent its working, it appears that one is attacking a problem which cannot be solved. The obstacle is the behavior, inside of the general equilibrium, of a system of oligopolists. If the influence of one oligopolist on the others was perfectly definite, if the other sellers took his influence directly as a factor in their decisions, there would be no trouble. But the other sellers may have an influence on the oligopolist. And this influence, as the first one, is anything but definite. In the case of an oligopolistic system the reactions of the competitors are perfectly unpredictable in theory. These reactions may yield an indeterminate solution. We have the example, before World War II, of the American Tobacco Company which raised its prices and was not followed. This is an example of immobility. Another example is that the Reynolds Tobacco Corporation raised its prices in the 1920's and was followed by the whole tobacco industry. These examples are given by George Stigler to show the traditional problem of oligopoly with an obtuse kink in the demand curve did not exist. The same examples were taken over by H. Hotelling to show that, according to the general conditions of the economy, the kink could be "obtuse" or "reflex"; that there could be immobility or mobility. And here we leave static economics in which we were working

¹"A Note on Kinked Demand Curves," Amer. Econ. Rev., Vol. XXXIII, No. 1, Pt. 1 (March, 1943).

and we enter the domain of dynamics. And we are even out of the range of dynamics; tacit or explicit agreements are possible, bringing restriction to competition. Besides price competition we must consider all the other types of differentiation, from advertising or service to financial backing and political influence. And pure economics cannot deal with these. This problem is dealt with by Pareto in his study of duopoly (which we may extend to oligopoly without any change in the reasonings).

"It is useless to ask of pure economics what will happen if two individuals dealing in the same commodity want to act according to model (2) on the same market. Pure economics, after having taught us that they cannot act monopolistically, cannot tell us anything more. The observation of the facts will show us the rest... We must not think that it will show us one solution. There are an infinite number of solutions."¹

Mathematically, Pareto shows that this problem cannot be solved. He considers two individuals acting as monopolists. He finds three equations which are not compatible, so he says:

"mathematically it is not exact to say... that in the case of two monopolists and one commodity the equilibrium problem is not determined. On the contrary, it is too determined, because the conditions exposed are inconsistent."² In this problem the supply of one individual should have been a maximum when the price and the supply of one individual should have been a maximum when

¹ Manuel d'Economie Politique, pp. 601-602.

² ibid., p. 597.

the price and the supply of the other vary and vice versa. This, Pareto says, is impossible; such individuals cannot act according to model (2).

Triffin must think that this is implicit in Pareto's theory, for I have not found any statement of that kind in the book. Pareto simply left the problem of oligopoly undetermined and passed to the problem of "monopoly of two individuals and of two commodities." Here the big difference, mathematically, is that we have two independent prices, thus enabling us to solve the problem. Economically this is to say that each monopolist seeks profit in an autonomous way. If Pareto actually thought that a little differentiation was enough to make the sellers independent he was obviously wrong; the cases of Lucky Strike and Reynolds Tobacco prove it. There is always some differentiation between the brands of cigarettes and tobacco. As Triffin says, "There is between the two extreme cases of perfect interdependence (the "one commodity" case) and perfect isolation a long series of intermediate possibilities."¹ But I repeat that I did not find obviously that Pareto considers only a little differentiation. He does not seem to want to avoid a difficulty, but to deal with another problem. At any rate, it is right that Pareto acknowledges that pure economics may be applied to equilibrium only when oligopolistic relationships are excluded. At any rate, it is right that Pareto acknowledges that pure economics may be applied to equilibrium.

¹ Monopolistic Competition and General Equilibrium, Theory,
p. 73.

only when oligopolistic relationships are excluded.

G. THE SYSTEM OF EQUATIONS:

Pareto has made the most advanced attempts toward the formation of a complete concept of economic equilibrium. He realized that pure economics is not enough to study such a concept. We shall now take a brief look at his system of equations. This will be very rapid and superficial because it is somewhat beyond the scope of this paper which is to study the graphic approach of Pareto. It is nevertheless necessary.

The latter part of the third chapter of the Manuel, from paragraph 196 on, is dedicated to the study of equilibrium in the general case. It includes a part about the system of equations. The equations Pareto presents in this chapter deal only with the case of phenomena of model (1) with complete competition, that is, with decreasing costs. In other words, this will be a very brief example of what the general system is as it is developed in the Appendix.

The first type of equations, called type A, represents the fact that each individual satisfies his tastes directly. The individual is in equilibrium when he exchanges ten kilograms of bread for 5 kilograms of wine. If he keeps exchanging he will receive, for instance, 5 gr. of wine for 10 gr. of bread. If the ophelimity index of the 10 gr. of bread was smaller than the ophelimity index of 5 gr. of wine he would actually make this

exchange. The equilibrium position would be 10,10 of bread for 5,5 of wine. So since the equilibrium position is at 10 for 5 the ophelimity index of 10 gr. of bread is not smaller than the ophelimity index of 5 gr. of wine.

It cannot be larger because the equilibrium position would be at 9.990 kg. for 4.995 kg. (The individual would not exchange as much as 10 of bread for 5 of wine.) So the two ophelimity indexes are equal, if we reason on small quantities. The first type of equation expresses that the "poised elementary ophelimities" of bread and wine are equal. "Poised elementary ophelimity" in modern theory would be marginal utility divided by marginal quantity divided by price. So the individual will stop in his transformations when the "poised elementary ophelimity" (equals p.e.o.) of the goods he consumes are equal. This is valid for more than two commodities:

$$p.e.o. A = p.e.o. B$$

$$p.e.o. A = p.e.o. C$$

$$p.e.o. A = p.e.o. D$$

So there are three equations for four commodities. For n commodities we have $n-1$ equations because in general one commodity is used as money.

The second type of equation (or type B) is formed by the equations which express the equality between the expenses and receipts of each individual. There will be as many equations as individuals.

If there are 100 individuals and 700 goods, there will be 699 equations of type A for each individual, a total of 69,900 equations for the collectivity. We also have 100 equations of type B giving a total of $69,900 + 100 = 70,000$ equations. There are 699 prices, since one commodity is money and 70,000 quantities of commodities exchanged by the individuals (700 goods x 100 individuals), a total of 70,699 unknown elements. Therefore 699 equations are still needed.

In this case of exchange the obstacles arise from the fact that the total quantities of commodities are constant (what is given by one is received by another). So the sales of the collectivity balance its purchases.

But the B conditions permit us to consider only 699 commodities, because, if one knows the quantities sold and bought of each commodity but one, it is possible to find these quantities for the last commodity, since total receipts equal total expenses. There will be 699 equations of type C which express that the remainder of purchases or sales of the collectivity for each commodity but one is equal to 0. Thus there are enough equations to solve the problem of exchange.

In paragraph 205 Pareto studies production. 699 equations must be found to replace the 699 (of type C) found for exchange. Suppose that 200 commodities are transformed into 500 others. At the equilibrium position the producer operates when the price equals the cost of production. We have 500 equations of type D expressing

that equality for each commodity produced. We must also show that 200 commodities have been transformed into 500 others: the quantity of the first ones has disappeared and has been replaced by the quantity of the 500 others. For the same reason as before that condition may be indicated only for 199 commodities. We obtain 199 equations of type E.

Thus finally there are 699 (500 - 199) equations as in the case of exchange which will take the place of type C in the study of production. The equations may be related to the diagrams we already studied for two goods. Type A equations express only that equilibrium occurs at the point where the price line is tangent to an indifference curve of tastes. Type B equations show the budget of the individual, or the path run over, or the price line. Type C equations indicate that the quantity of commodity given up by one is received by the other, or in other words, show that the paths run over coincide, as in the "box diagram". Type D equations represent the fact that the producer operates at the tangency point of the cost curve and of the total revenue curve: the cost of production equals price. The E conditions stand for the fact that in model (1) the producers operate on the complete-transformations curve: there is no remainder of the commodity which is processed.

This is a summary of the system of equations presented by Pareto. Following the summary, Pareto deals with stable and unstable equilibrium, relates his theory to the ideas of "literary economists" and concludes his theory of value.

I have one remark to make about this. I do not see why Pareto said at the beginning of this part of the work that he studied a case of complete competition, that is to say of decreasing costs industry. It seems to me that all the equations he presents would as well be valid for an increasing cost industry.

Before finishing this brief study of Pareto's concept of general equilibrium it would be good to look at the modern developments in this field. We shall see that basically nothing has been changed. Hicks has dedicated an important part of his book Value and Capital to the study of equilibrium of exchange and of equilibrium of production. The basis of Hicks' work is nothing but the system of equations of Walras, the same as Pareto. The main changes are found in the consideration of the equilibrium of the firm and in a more advanced study of the relations of factors to products. Another difference is that the study of complementarity is given greater emphasis.

Let us quote Hicks: "To some people...the system of simultaneous equations, determining a whole price system, seems to have vast significance....I have myself very considerable sympathy with this point of view; I believe that we can get considerable insight just by extending Walrasian system of equations."¹

But this is what Pareto did.

Later Hicks says that a certain sterility arises from the fact that the work which had been done was mainly in the domain of statics and so no opportunity for a comparison with real world is provided. This is the reason for which Hicks dedicated a large part of his book to the study of dynamics.

¹Hicks, Value and Capital, p. 60.

As for the deficiencies of the general equilibrium theory,
Hicks agrees with Pareto:¹

Its main deficiencies may perhaps be classified as three in number. First, it pays no attention to monopoly and imperfect competition; as I have explained, I do not think the importance of this defect should be exaggerated. Secondly, it abstracts from the economic activity of the State... (this is, of course, a deficiency of economic theory as a whole). Lastly, it abstracts from capital and interest, savings and investment... this is a vital defect, which we must try to remedy in the later part of this book.

And this, of course, is in the domain of dynamics.

¹ Ibid., pp. 99-100.

CHAPTER VII

OTHER USES OF THE INDIFFERENCE CURVE

APPROACH TO EQUILIBRIUM THEORY

First I shall examine Scitovsky's theory of the efficiency of the firm in a market, and I shall study the body of theories based on Edgeworth's and Marshall's diagrams applied to international trade.

A. SCITOVSKY'S THEORY OF THE EFFICIENCY OF THE FIRM:

In order to study the efficiency of a combination of products put out by a firm, Scitovsky, in Welfare and Competition¹ uses a very simple diagram. This diagram may be imagined even before studying Scitovsky's book:

The transformation curve shows all the combinations of two products of output which are possible with a same total cost. This curve is a production-indifference curve for the producer. We know that in perfect competition the producer will operate at the point where that transformation curve is tangent to the price line, that is, when its slope is equal to the slope of the price line, when the marginal rate of substitution between the two products is equal to the market price ratio.

It is also known that the consumers equal the rate of substitution of the products to the market price ratio (i.e., operate

¹P., 161-163

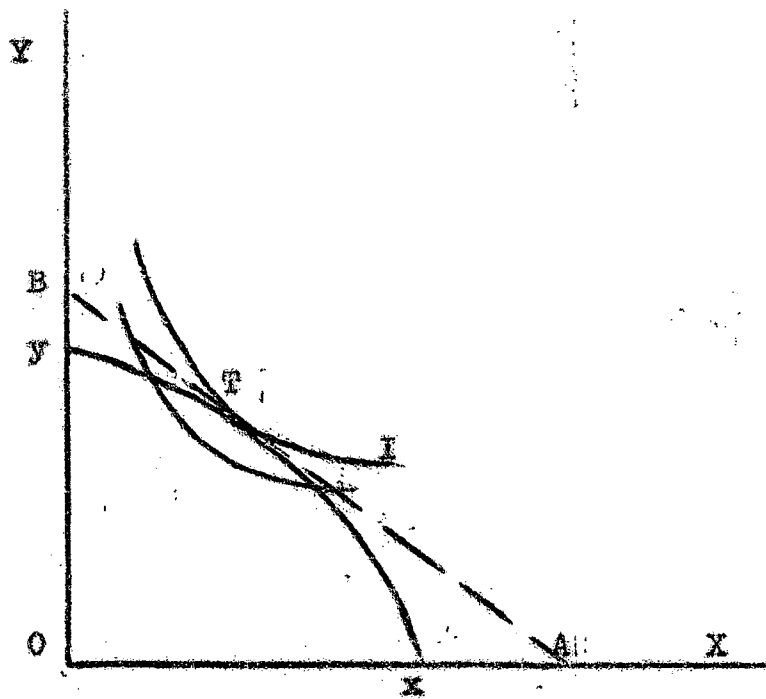


Fig.1

at the point where a consumer-indifference curve is tangent to the price line). In a competitive market the marginal rate of transformation of x and y is equal to the price ratio of x and y : $MRT_{xy} = \frac{P_x}{P_y}$ and the marginal rate of substitution of x and y = price ratio of x and y : $MRS_{xy} = \frac{P_x}{P_y}$. Therefore, in a competitive market we have $MRT_{xy} = MRS_{xy}$. When this equation is satisfied, the total cost of production of x and y is shared in the best way possible between the production of x and of y , since at that cost, x and y are produced in the proportions wanted by the consumer.

Diagrammatically, this is represented by the tangency point of the production curve to an indifference curve of the consumer (Fig. 1). At that point the slopes of the two curves are equal and $MRT_{xy} = MRS_{xy}$. As a matter of fact, if the producer operated elsewhere on the production curve, the indifference curve of the consumer would be lower than the one which is tangent to the production curve. But in competition, both producer and consumer operate at the tangency point; therefore competition brings the maximum of welfare to both elements. These results may be extended to many producers, consumers, and commodities produced, as we extended our diagrammatical results for two factors of production to many factors.

Scitovsky draws two conclusions from this approach. First, he shows that competition among consumers is necessary for the economic efficiency of the firm as far as welfare is concerned. Such competition secures "optimum distribution" (in static conditions) of the products. The relation $MRT_{xy} = \frac{P_x}{P_y} = MRS_{xy}$ shows that in a competitive

market equilibrium is established in accordance with the structure of factor and product prices.

The second conclusion reached by Scitovsky is that in this demonstration we considered a given cost expenditure. If one wants to derive something interesting for welfare economics we have to consider a given productive effort and to show that a given cost expenditure represents a given productive effort. This is easily shown. One can consider the same diagrammatical approach as before. The transformation curve will represent an indifference between the production of two factors or productive services; it will then be an opportunity-cost curve. The indifference curve will represent the indifference of the producer in the use of two factors, with a constant output. The point of tangency will be the equilibrium point in a competitive market. In fact, the producer will try to make the rate of technical substitution equal to the price ratio, $MRS_{xy} = \frac{P_x}{P_y}$ (if x and y are the factors). The producer will equate the price ratio of these factors to the marginal rate of transformation represented by the opportunity-cost curve $\frac{P_x}{P_y} = MRT_{xy}$. So one has: $MRS_{xy} = MRT_{xy}$. But the combination represented by the tangency point corresponds to a given cost expenditure. It corresponds also to a given productive effort (since it is situated on the opportunity cost curve). Thus the two notions correspond in a competitive market. In the same way as before this reasoning may be extended to more factors and more individuals.

I think that Scitovsky's indifference-curve approach to this problem is very good. It represents the model of what should be a valid diagrammatic approach.

B. DEVELOPMENT OF EDGEWORTH'S AND MARSHALL'S APPROACH IN THE MODERN THEORY OF INTERNATIONAL TRADE:

Edgeworth, both in an article published in the Economic Journal in 1894 (reproduced in Volume II of Papers Relating to Political Economy)¹ and in the Presidential Address delivered to the British Association in 1889², demonstrated a new approach to the study of economic equilibrium. This graphic approach, applied to the study of the equilibrium of international trade, was taken over by Alfred Marshall in 1925.³

In diagram 2, the quantities measured on the axes are expressed in terms of bales. Each bale "represents uniform aggregate investments of her labour (of the country) and her capital". In diagram 2, on OA I measure A-bales and on OB I measure B-bales. In such a diagram one may draw a curve OP for country I which shows the quantity of A bales and B bales which would be exchanged at each price by country I. In other words, the curve shows the quantity of A bales (or vice versa). In the same way one may draw a similar curve for country II. In such a diagram the lines similar to Op, Op', Op'' are price lines of which the slope represents the price. The two

¹ Pp. 3-60.

² Ibid., pp. 273-319

³ See his Money, Credit and Commerce (London: Macmillan & Co., 1923), p. 157.

agree on the price of A bales in terms of B bales when they have the same price line. The price lines coincide when their position becomes OE . In fact at E the two offer curves cross each other. At the equilibrium point E both countries agree on the price and on the quantities to be exchanged.

Such curves may also be derived on an indifference map, from the points of tangency of the price line Op , to the indifference curves (O, O', O'') of country I for A bales and B bales as in figure 3, and from the tangency points of the price line to the indifference curves (t, t', t'') of country II for A bales and B bales (Fig. 4).

So, if figures 3 and 4 are put together one obtains the same diagram of the famous "box diagram" (Fig. 5) representing the exchange between two economic units. (This apparatus was used by Edgeworth before Pareto and is a representation of Figure 2. In the box diagram the locus of the possible points of equilibrium is the locus of tangencies of the two families of indifference-curves (Fig. 5). In diagram 1, the equilibrium point is at the crossing of the two offer curves. So it can be inferred that point E will be on the locus of tangency points. One can see that since at E the price line is tangent to an indifference curve of country I and to an indifference curve of country II, E will be on MM which is the locus of tangencies; in fact, these two indifference-curves are tangent to each other at E . This is the graphic apparatus used by Edgeworth, Marshall and their followers. We shall now show what the uses of such a diagram are.

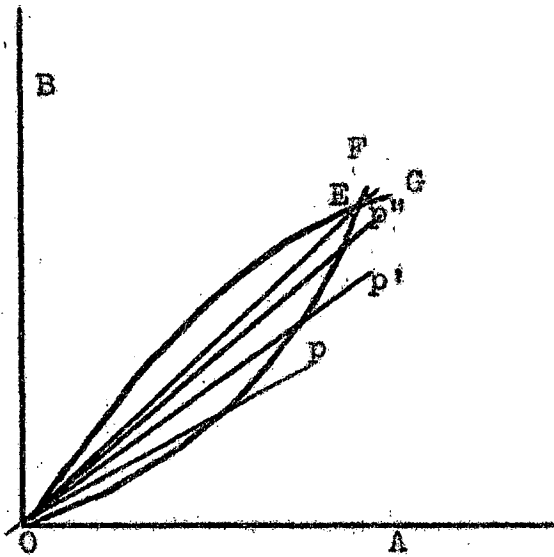


Fig. 2

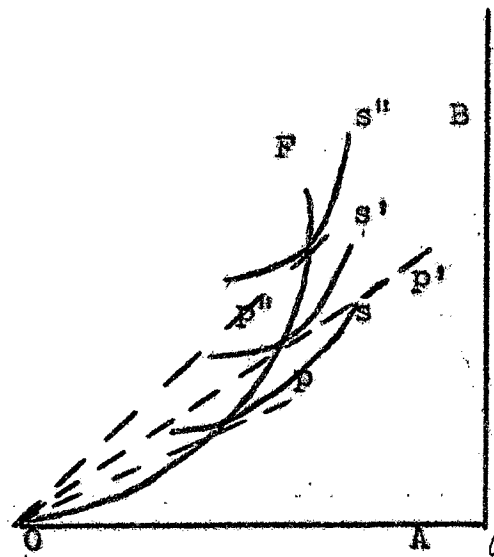


Fig. 3

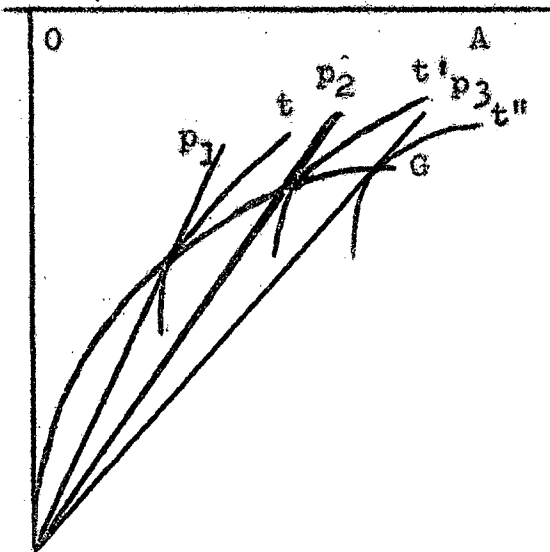


Fig. 4

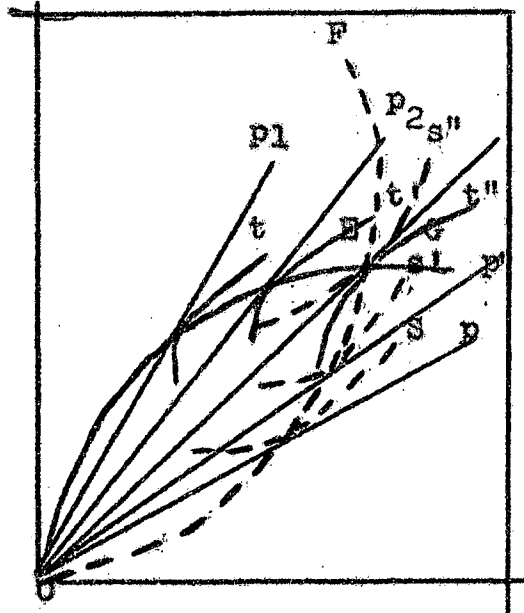


Fig. 5

Paul Samuelson, in his "Welfare Economics and International Trade,"¹ defines exactly what is represented by such a diagram: two countries in bilateral monopoly trading in free trade. We have seen that the two offer curves determine on the contract curve (locus of tangencies) of the two countries, the point at which the two countries will trade. On such a contract curve no movement is possible without injuring one of the countries. In fact, if one country reaches higher indifference curves, the other reaches lower indifference curves. So Samuelson says: "There is absolutely no presumption whatsoever that (the free trade) equilibrium point is superior in any sense to any other point on the contract curve."² In fact, in economics welfare judgments are not possible. I think that, at least, we can say in favor of the free-trade equilibrium point that it is reached in a natural way. But it is right that such a point would not be the best for any of the countries because it is not on the highest possible indifference curve.

The second point shown by Kaldor on such a diagram is that if one country behaves like a competitor the other has an interest to behave like a monopolist and to move the first country on its offer curve to a point where its offer curve is tangent to an indifference curve of the country acting monopolistically. This is shown in Figure 6. Kaldor says when country G imposes a tariff its offer curve is distorted as in GG^1 . The best tariff will be the one which will bring

¹Amer. Econ. Rev., Vol. XXVIII, No. 2 (June, 1938), pp. 261-266.

²Quoted by Stevens in "New Ideas in International Trade Theory," American Economic Review, Vol. XII, No. 3, (June, 1951), p. 372.

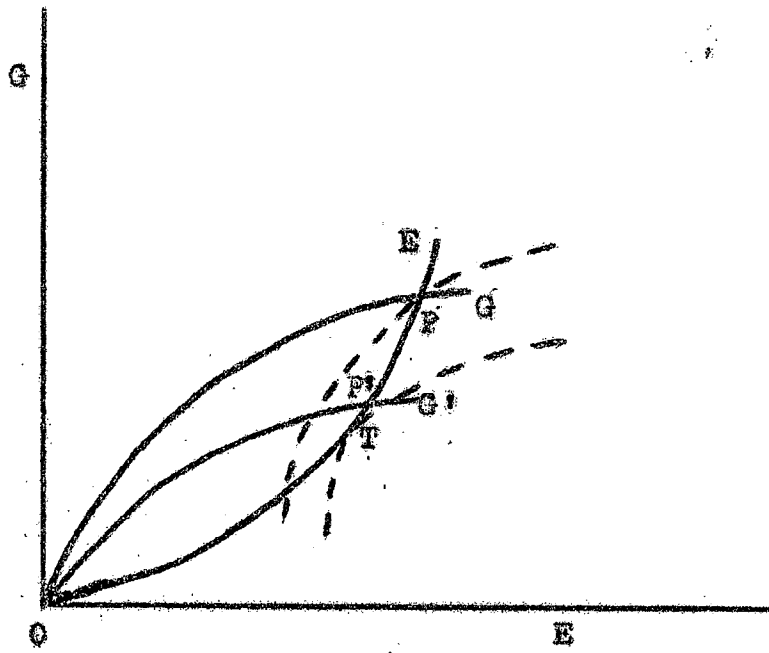


Fig. 6

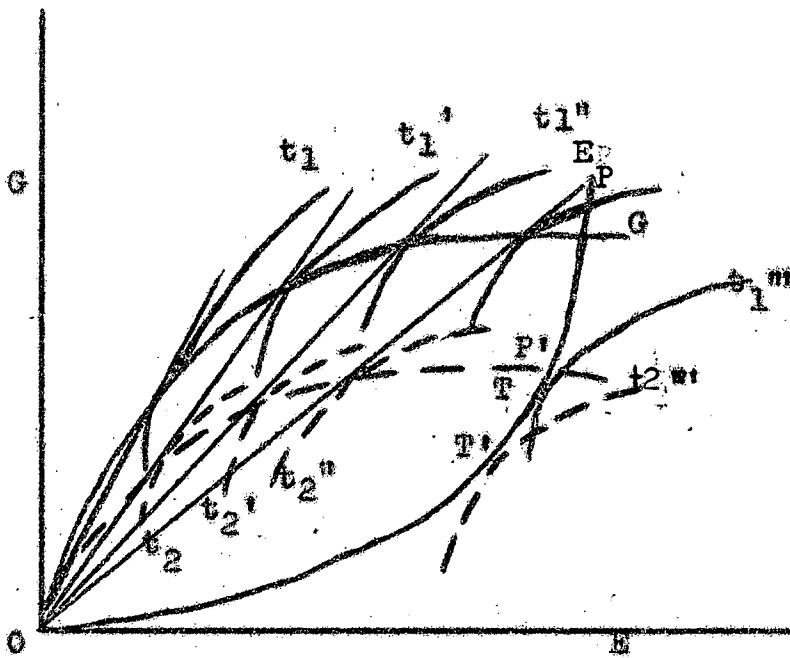


Fig. 7

the equilibrium point P at the tangency point t of OE to an indifference curve of G . This indifference curve will be the highest which may be reached by country G .

I think that if we take into account the construction of the offer curve we gave previously (Figs. 3 and 4) it is evident that such a movement is impossible. OG , offer curve of country G , is the locus of tangency points of price lines based on O , to the indifference curves of country G . If it is assumed with Kaldor and with several other theorists, that the imposition of a tariff distorts OG into OG' we see that it must be assumed that the complete indifference map of country G is changed. So the indifference curve which was tangent to OE when the offer curve was OG will not be at the same position when the offer curve is OG' . Kaldor's diagram showed the contrary. Besides, as OG' is distorted the tangency point T gets closer to O along OE , preceding point P which also moves toward O (Fig. 7). Point P and point T will never coincide. We can intuitively see that they will coincide when the quantities are equal to zero, when the whole diagram has become a point (co-terminus) with the origin.

I think that the fact that a tariff is imposed by G has for its effect a decrease in the slope of the price line which represents the terms of trade. This is represented in Figure 8. I do not see why the imposition of a tariff would change the tastes of the consumers and henceforth the shape of the indifference map of the country which applies the tariff. A change in the offer curve is only possible if one assumes such a change of the indifference map. In the consumer's

analysis we saw that a change in price never changes the indifference maps. Therefore I think that the two offer curves are constant as long as the indifference maps are the same. The equilibrium point P on Figure B is a point which results only from the confrontation of the systems of tastes of two different countries. Point P is the only one of this type. There are no new points P' , P'' , etc., at least when a country imposes a tariff. A new Point P' , P'' , etc., may be conceived when the indifference map of one or both countries changes. The offer curves OB and OC and the equilibrium point P belong exclusively to the domain of free trade in which only tastes have any influence.

On figure (let us try to reconsider the problem of tariff imposition and of retaliation. Country A is a monopolistic seller of commodity G expressed in units g . It buys commodity E from country B which buys commodity G. Commodity E is expressed in units e . When the two countries operate at the crossing of their offer curves P , 12 g 's are exchanged for 12 e 's. Then country A is on its indifference curve t_0 . Country A will try to reach P' where it will be on a higher indifference curve t_1 . t_1 is the highest indifference curve which may be reached by A since it is tangent to OB (since no equilibrium can occur but on OB). This has been shown, of course, when we were studying Pareto. So country A, which is a monopolist, in order to reach P' will decrease the slope of OP on Oe , that is to say will change the terms of trade by imposing a tax upon the exports of G. When the intersection of OP' and of OB comes to P' country A has reached its goal. At that point 8 g 's are exchanged for 10 e 's. At point P , in

free trade the price was 12 g's for 10 e's or 4 g's to 5 e's. At point P we had the ratio $\frac{4}{5} = 1$; at point P' we have the ratio $\frac{5}{4} = 1.25$. The price of g in terms of e has increased and it gets more e's in proportion to the g's it gives up.

If country B acts in a non-monopolistic way, that is, remains in free trade as far as it is concerned, the position P' will stay stable. But, as a matter of fact, country B is now on an indifference curve lower than in P and maybe it wants also to reach the highest indifference curve which may be reached. It may realize that it can also act monopolistically. So for country B the point to be reached is P''. Country B will increase the slope of OP on OX as far as OP''. A tax will be imposed on the export of B by country B's traders. The price of B will increase. As a matter of fact, at P'', 10 g's are exchanged for 8 e's; the ratio of point P' is reversed.

What does such a diagram show? It only shows that if each country retaliates to a tax imposed by the other, there is no stable equilibrium. If we consider two monopolistic countries, P is a point of unstable equilibrium. This corresponds to what we already saw for two economic units, each one operating as a monopolist. It is impossible to determine the equilibrium position. Pareto had similar troubles in his system of equations for general equilibrium.

In the approach used by Kaldor, the successive distortions of both offer curves showed that the volume of trade decreased for each retaliation. Here we do not have anything like that. As a matter of fact, the route followed by both countries will be P' P P'' on the offer curves so that the lowest limits of trade would be P' and P''.

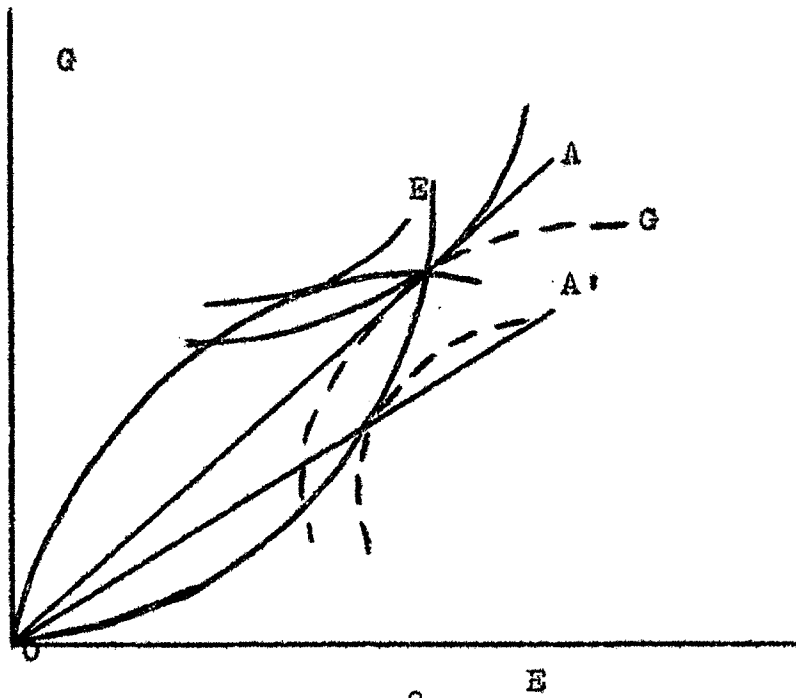


Fig. 8

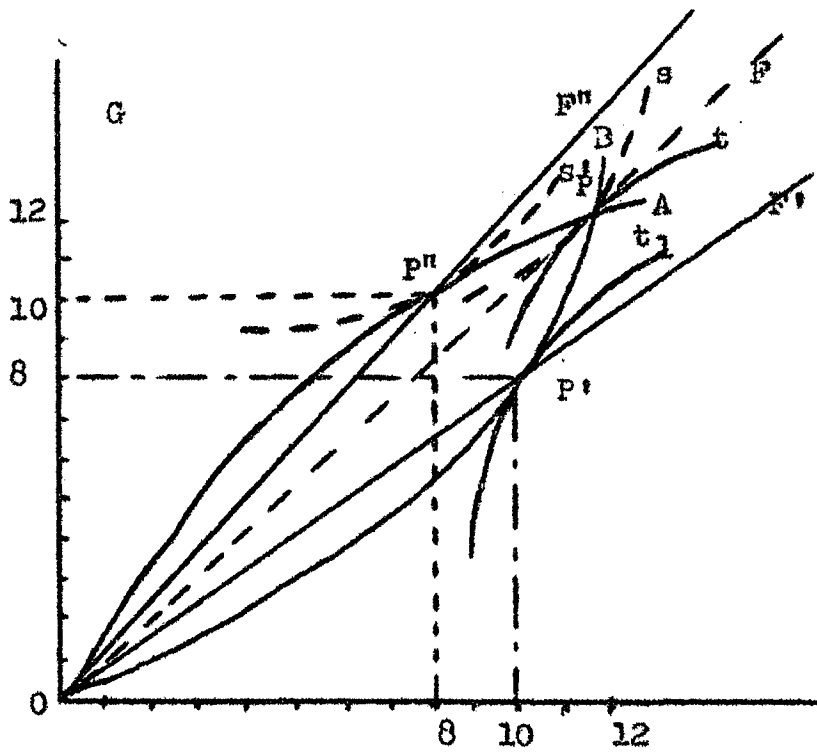


Fig. 9

We can see that when a country tries to reach its optimum point, the volume of trade decreases because of an increase of the price of one of the commodities exchanged. But as long as point P moves from the point of equilibrium in free trade to the tangency point of an indifference curve to the offer curve of the other country, the reduction in the volume of trade would be compensated by the fact that the country imposing the tariff would go higher and higher on its "pleasure hill".

I think that this is about all we can get out of such a diagram. But before leaving the question, I want to demonstrate an idea of Professor W. O. Thweatt.

Let us consider Fig. 10. The aim of country A is to get to point P' on the highest possible curve, tangent to the offer-curve of country B: OB. We saw that if it tries to do so by imposing a tariff it will be the beginning of a series of disequilibria because of successive retaliations. Besides the fact that the volume of trade is much reduced by the imposition of a tariff must be taken into account. This can be avoided and the retaliations can also be avoided.

All that the country which acts first wants is to get to P' . The first country which acts is the one which is first conscious of acting monopolistically. If no country believed itself to have that power, free trade would prevail. P' is on the indifference curve t' . But for both countries the best points are on the contract curve DD' (locus of tangencies of the two indifference map). So let us consider point c. At C country A would still be on indifference curve t' and

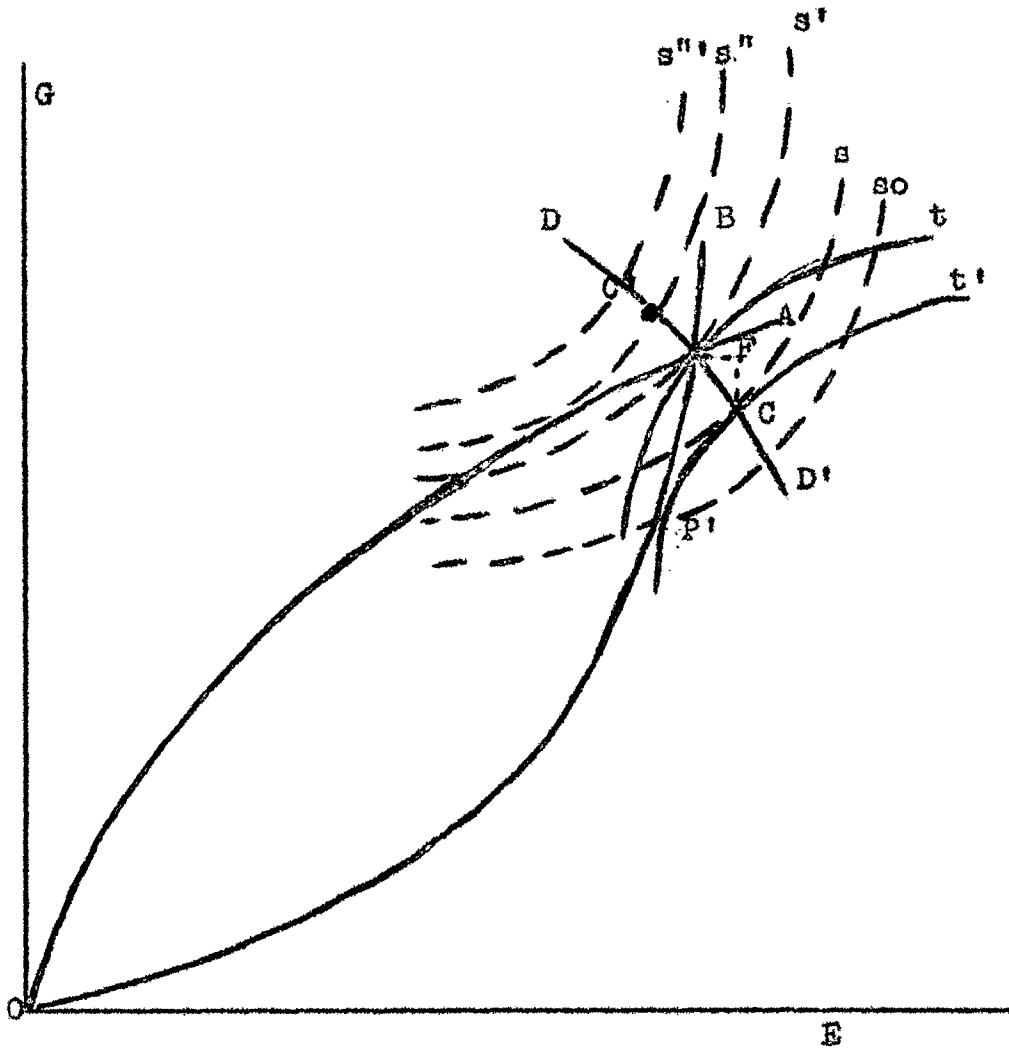


Fig.10

at G country B would be on an indifference curve S which is higher than indifference curve D_0 on which it would have been at P^1 . Besides, at G the volume of trade would be larger than at P^1 . So all these considerations together make us think that at G there would be less risk of retaliation than at P^1 . If there was a little difference between the two countries, if country A was a little stronger than country B, G could be a point of stable equilibrium. This would occur because country B is a little less strong than country A and because B does not lose much. Even with the same proportions of strength, at P^1 , country B is more likely to retaliate than at G .

How does country A get to G ? Consider a period of trade of the unit of time in which the quantities represented by point F (equilibrium in free trade) are exchanged. During this time the exchange would be carried on on a free trade basis; then at the end of the period country A would make a special transaction: the exchange of PQ of G for PF of F . The difference from the P^1 solution is that the loss of country B is smaller (both in volume of trade and in position on the pleasure hill) so that there are some chances for country B not to realize that it would be better to act monopolistically.

But still we suppose an inferiority of country B, so it is in a weaker position and it has to accept the extra transaction, or it is in such conditions that it cannot see that it would be possible and profitable to act monopolistically. But if we assume that countries A and B are in the same position, have the same knowledge and

power, we are in the same case as previously. There is no reason for country A to endeavor to reach C before country B endeavors to reach C¹ by the same process.

The whole of this approach to the problems of foreign trade is very theoretical and many authors have felt that it has some drawbacks.

Apparently it was Edgeworth's personal opinion that this graphical exercise should not be taken too seriously, for he promptly sought to minimize the implications of his conclusion for policy. In commenting upon his analysis he quoted with approval a remark by J. S. Nicholson that "some analytical demonstrations are 'part of the casuistry of economics, like the discussions of moral philosophers concerning the occasional justification of mendacity. Free trade, like honesty, still remains the best policy'.¹

The statement of Nicholson corresponds to the results of our analysis when we see that if both countries act monopolistically, only free trade will provide an equilibrium position. Before ending this section, let us report the opinion of Haberler:

"It is out of the question to deduce an argument for tariffs (on this basis)...whoever does so proves only that he has not realized the full complexity of the problem."²

C. LEONTIEFF'S APPROACH:

In the Quarterly Journal of Economics³ W. Leontieff, of Harvard University, published an article entitled: "The Use of Indifference Curves in the Analysis of Foreign Trade". In this article the author

¹Loc. cit., p. 375n.

²Haberler, Theory of International Trade (Allen and Unwin, 1937), pp. 294-295.

³Vol. XLVII, May 1933, pp. 493-503.

tries to apply the indifference-curve approach to the study of "the linkage connection between the 'national' and the international elements of economic equilibrium."¹

The first part of the article states nothing new. It is only the application to one country of a consumer's indifference map. The price line in that particular case represents "the terms of trade". If the terms of trade are changed, that is to say, if the slope of the price line changes, one will obtain a curve equivalent to what has been called a price-consumption curve. This curve, when one deals with international trade, leads directly to the Marshallian demand-supply curve. It shows the quantities of the two commodities which are offered when the terms of trade change.

A price line may be looked at from another standpoint. (See Fig. 11). If the nation in question produces commodities A and B we have on the same diagram a curve similar to the one studied in output analysis. This curve represents the ability of the national economy to produce commodity A and commodity B at a constant total cost. Such a curve may be a straight line if the opportunity costs are constant, that is, if the returns of both industries are constant. It will be a curve concave to the origin if both industries are subject to diminishing returns.

If the entire resources of the nation are used in the A industry, the amount of A produced will be Oa ; if some resources are diverted into the B industry the quantities of A and B produced will be showed by the curve ab (production curve). From now on I shall call the

¹p. 229 of reprint in Ellis and Metzler, Readings in the Theory of International Trade, (Philadelphia: Blakiston Co., 1949).

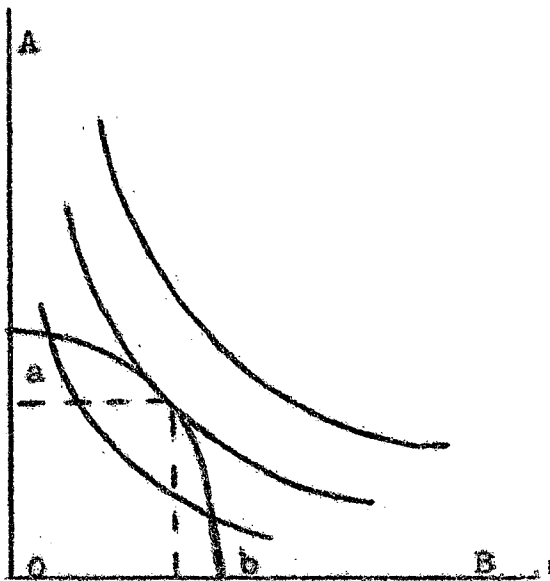


Fig. 11

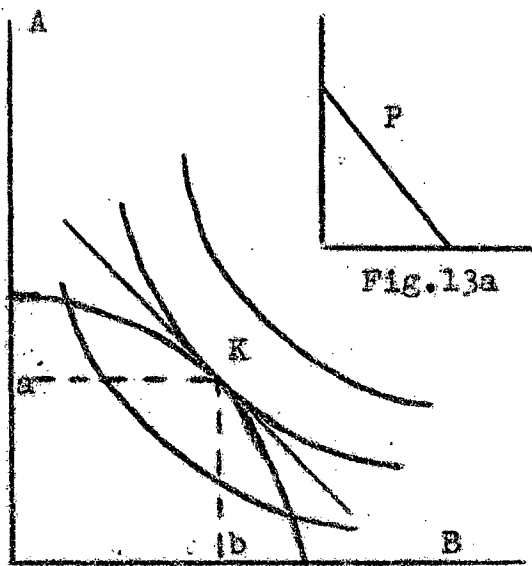


Fig. 13

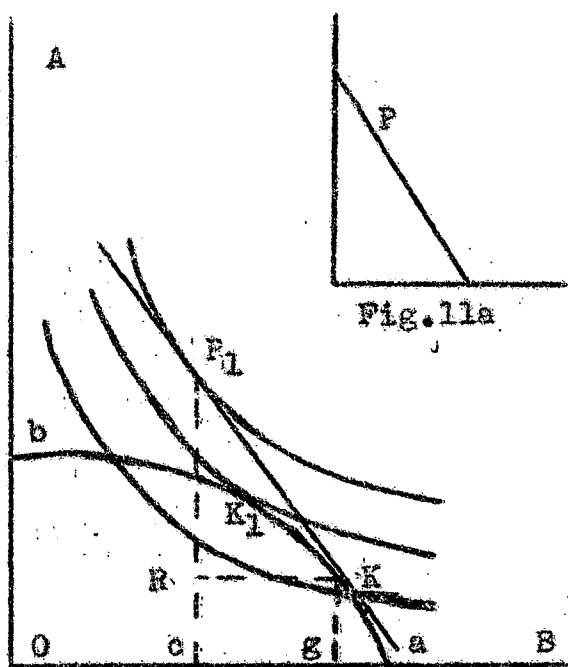


Fig. 12

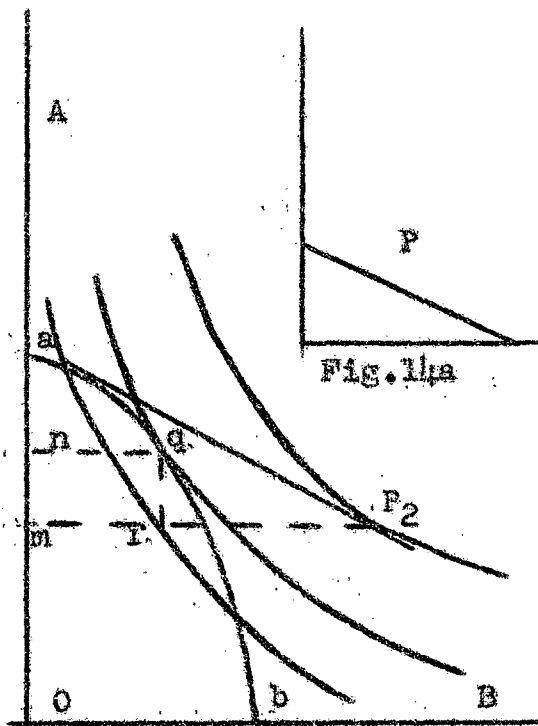


Fig. 14

curved line a production curve and the straight line an exchange curve which represents the terms of trade with another country.

Leontieff in his construction then brings together the exchange curve and the production curve. So, both international and national elements are represented in the production and in the exchange of A and B.

If the production curve is ab , if the terms of trade are represented by a slope equal to the slope of curve F on Fig. 11, the best solution of the problem is shown in Fig. 12. The nation will produce A until it arrives at the tangency point (of the production curve) to a curve having the same slope as the one which represents the terms of trade. From that point on it is more profitable to acquire A by exchanging B along the exchange curve. The indifference curve reached is higher than that which would be reached if the nation followed its own production curve.

The equilibrium position will then be at P_1 . K_3 of A is produced at home and P_1B is imported by exchanging Og (equals KK) of B. OC of B will be kept at home. The ratio $\frac{P_1B}{Og}$ represents the terms of trade, and the slope of the exchange curve; it is also the ratio of the quantity of A imported to the quantity of B exported, of the price of A in terms of B.

Suppose a change in the terms of trade. If the price of A in terms of B increases the slope of the exchange line will decrease. So the tangency point of the exchange line to the production curve will move to the left. When the tangency point coincides with an indifference curve it will not be profitable to exchange further, since

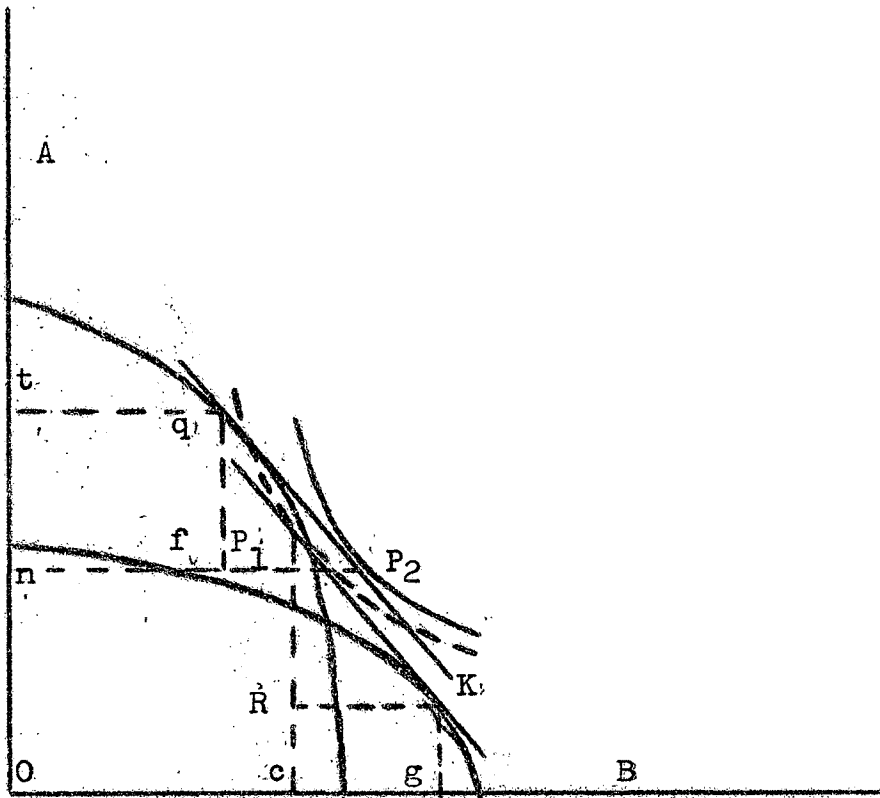


Fig.15

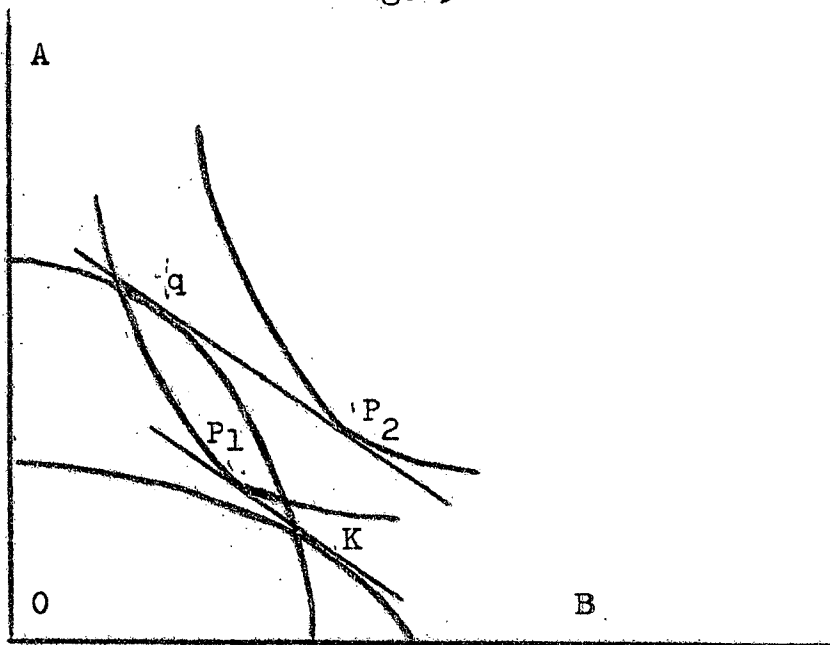


Fig.16

the exchange rates will only reach indifference curves lower than that which is tangent both to the production and exchange curves. Both commodities 1 and 2 will be produced at home. The nation will be an autarky (Fig. 13).

If the price of 1 in terms of 2 increases further, the slope of the exchange line will increase and then it will be more profitable to get the other way round. It would be profitable to export 1 and to import 2 (Fig. 14). If (equals q_2) of 2 is produced at home and q_1 is exchanged for q_2 of 1 (equals q_1). An of 1 will be kept within the nation's borders.

Identical analysis now the relations of two nations on the same diagram. First, assume that both nations have the same indifference curves but different production curves. Above the indifference curve and production curves, the terms of trade may be found.

Consider Figure 15. The quantity of 2 given by nation 1 must be equal to the quantity of 2 received by nation 2 and vice versa for commodity 1. The terms of trade are equal, that is to say, that the exchange curves are parallel. These conditions in Figure 15 could be represented by the equations $q_1 = p_1 R$, $RK = p_2 R$, and of course $\frac{q_2}{q_1} = \frac{RK}{R}$. Identical wants to find that q_1 is equality states a determination when for both the nation and the terms of trade. Now the diagram as he stated, "The terms of trade and the quantities produced and exchanged can be determined." This may appear strange at first.

¹ Ibid. p. 254.

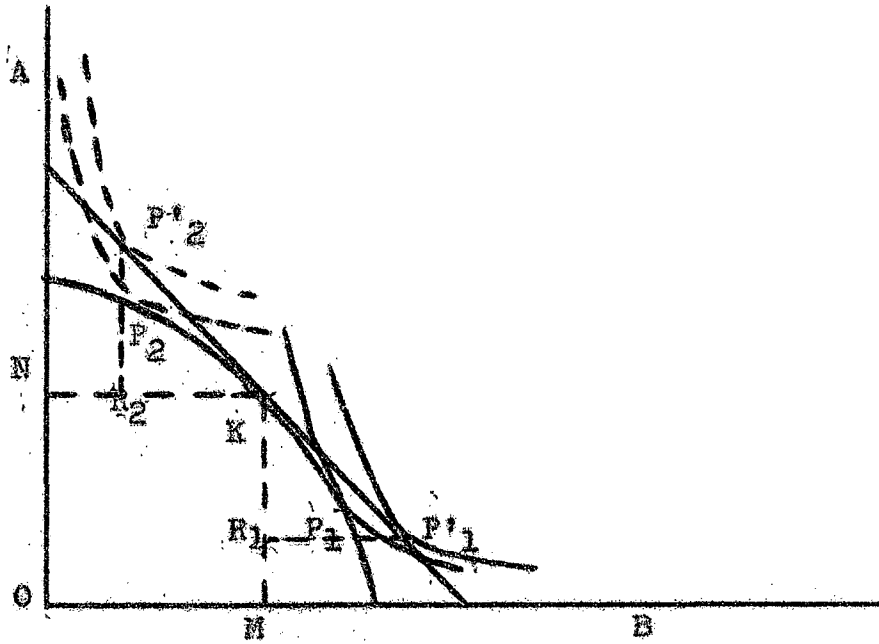


Fig. 17

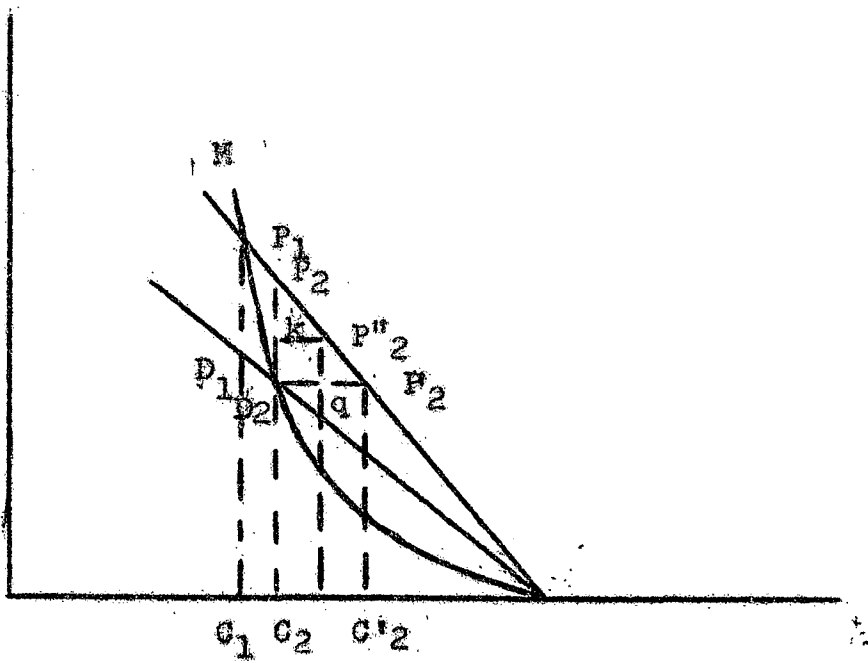


Fig. 18

because one may wonder whether there are several positions in which the two exchange lines are parallel and yield a balanced trade for both countries. As a matter of fact, it can be shown that there is only one such position (Fig. 15).

The interesting conclusion of all this is that trade may arise between two countries having the same cost structures but differing only in respect to tastes. (Fig. 17). The solid curves are of country I, the dotted ones are of country II. In self-sufficiency, country I would be at P_1 and country II at P_2 . But by entering into trade they both reach higher indifference curves. Since they have the same production curve, their exchange lines, parallel and tangent to the production curve, will coincide. The position at which the exchange curve is tangent to the production curves and the indifference curves of each nation will be the equilibrium. Country I will be in equilibrium at P^* and country II at P'^2 (Fig. 17). Both countries will produce OP^* of B and OP'^2 of A, but $P^*P_1P_2$ of A will be exchanged for $P'^2P_1P_2$ of B.

Finally, Leontieff studies the way the tax on imports may be paid. (Fig. 18.) Without taxes the price line (or exchange line) is CP_1 , and the equilibrium point is P_1 where it crosses the demand curve (or price consumption line). With a tax, the new exchange line is CP_2 . P_1P_2 is the amount paid to customs when C_1D_1 is imported. Actually, only D_2C_2 will be imported. Then the amount paid to customs will be P_2D_2 in terms of A. In terms of B it would be D_2P_2 . In fact, this is equivalent. The quantity of A imported is always $P_2C_2 = P_1C_2$. At P'^2 the tax will be paid both in terms of A and B.

D. A NOTE ON THE VALIDITY OF SOCIAL INDIFFERENCE CURVES:

In this section we shall be concerned with curves which are said to represent the behavior of a group of individuals or of a nation. The use of such curves is based on the assertion that we can consider social indifference curves as having the same properties as individual indifference curves: nonintersection, and convexity to the origin. During a certain time such curves were used and most of the authors using them did not question whether such use was justified or not. We shall attempt to determine whether social indifference curves are a valid analytical tool. Let us state the position taken by several economists in this matter. Pareto used the curves as representative of a group of individuals. In paragraph 87 of chapter III (p. 181) of the Manuel, he developed his ideas about the question. This paragraph concerns a "path" or price line which is run over by two individuals; to define the path he starts from two parallel paths, each of them being run over by one individual (two price lines which represent the same price ratios but different levels of income). The collective path is formed by the addition of the quantities represented by the two individual paths. The indifference curve which is related to that collective price line appears to be the summation of the two individual-indifference curves. But we must note that the two individual indifference curves are parallel on the diagram presented by Pareto. That is to say that Pareto supposes that the tastes are the same. So when he says,

"The same reasoning may be applied to any number of consumers who may therefore be replaced by one ideal consumer who represents them in their totality,"¹ he assumes that each new individual who would be added to the collectivity would have the same tastes as the group. Based on such an assertion, the problem does not present any difficulty. But such an assertion is a way of avoiding the problem rather than a way of solving it.

Before or during the life of Pareto, Jevons, Walras, Edgeworth and others were concerned with the problem. Edgeworth used such collective curves for his study of international trade but did not think that there was an absolute need to compare or compound different persons' marginal utilities which would lead to the construction of collective indifference curves. For many theorists of that time who study of group demand was based on the addition of observable market demand curves.

In the late 1930's, the manner in which international equilibrium was shown was to use community indifference curves which would "consolidate" all the tastes of the individuals. This way of building community indifference curves has been shown by Lerner.²

Since 1940, one of the main contributions to this matter has been made by Scitovsky in his article, "A Reconsideration of the Theory of Tariffs."³ In this paper he uses community indifference curves which can intersect each other in many circumstances. The explanation

¹V. Pareto, Manuel d'Economie Politique, p. 162.

²Economica, Vol. XII, (1952), pp. 346-356.

³Review of Economic Studies, IX, 1942 (reprinted in Readings in International Trade Theory)

of this phenomenon is simple enough. Scitovsky considers two cases in which bread and wine are measured. If the price of bread increases, the consumers will buy a smaller total quantity of bread. It is possible to compensate such a loss by a distribution of wine. So the point where the community is, is a point of indifference compared to the first point because "it represents for each member of the community the same welfare that he enjoyed at p_0 (the first point.)"¹

For this demonstration the system of distribution of bread and wine is given and fixed. But Scitovsky notices that there are an infinite number of indifference curves going through the first point. All these indifference curves correspond to different distributions of welfare.

As a matter of fact, the total quantity of wine needed to compensate the loss of bread is conditioned by the distribution of the total loss of bread and by the average rate of substitution between wine and bread. Scitovsky says that at equilibrium all the marginal rate of substitutions are equal since each one of them is equal to the price ratio.² These two elements themselves depend on the distribution of welfare.

One can see that if we start from the same quantity of bread and of wine as in the first case the second point will be higher or lower with a different distribution of welfare. If a great number of people in the community only need a little quantity of wine to compensate the loss of bread the curve will be relatively flatter

¹ Scitovsky, ibid.

² Readings in International Trade Theory, p. 366.

than if a great number of people need a large quantity of wine to compensate for the loss of bread. Obviously, the fact that somebody needs more or less wine to replace a given loss of bread depends on the quantity of bread possessed at the beginning. So the fact that a more or less large number of people possess a relatively large quantity of bread, and therefore need a more or less large quantity of wine to replace a loss of bread will influence the slope of the curve even though we start from the same quantity of bread and wine to be exchanged.

The distribution of income is also important. For a class of people some commodities will be inferior, for another class the same commodity will be superior, so the size of those classes which have different incomes will influence the slope of the curve. In fact, when the price decreases one class may buy more of the commodity, others may buy less.

All this affects the general slope of the curves. And we can say with Scitovsky, "since each community indifference curve represents a given welfare distribution, they have the same geometric properties as individual's indifference curves."¹ That is all that was necessary to know. Until now in the theories I exposed, I did not worry about knowing whether the distribution of income was stable or not. But we see that, in order to use the indifference curves the way we did, I had to assume a given and fixed distribution of income. As

¹Ibid., p. 366.

a matter of fact, this was implicit in my assumptions because we dealt with static economy. I dealt with individual indifference curves I assumed constant tastes; when I deal with community indifference curves I assume constant tastes and a constant distribution of welfare.

If we had not assumed a constant distribution of welfare within the group, I should have introduced an important difficulty which we cannot, however, ignore at least diagrammatically. I say the influence of distribution on the slope of community indifference curves in general. But the general considerations that I have made do not automatically enable anybody to use indifference curves to deal with dynamic problems. Such an approach may be conceived but there is a big difference between conception and realization.

The last contribution I shall mention has been made by Paul A. Samuelson in his article, "Social Indifference Curves."¹ In this article Samuelson's goal is to build another theory about social indifference curves. He begins by showing mathematically that "community indifference contours" cannot exist. Then he explains, still mathematically, what Scitovsky's curves are. Here, I think, it is much better to read Scitovsky's article than to try to understand his theory through Samuelson's explanation. Anyway, I do not know why Samuelson wrote that part because the article of Scitovsky is very clear and does not need the additional diagrams presented by Samuelson. Samuelson concludes this part of his paper by saying

¹Quarterly Journal of Economics, Vol. L, No. 1, (Feb., 1956).

that since Scitovsky's indifference curves cross each other it is impossible to have any theory of group demand without. In all the problems we dealt with earlier in this chapter we were in the realm of statics so Scitovsky's indifference curves were enough for what we wanted to do. As far as Samuelson is concerned, he seems to be willing to work in dynamics and to build a system of indifference curves, taking into account the influence of external elements on distribution of income.

He does this by working on the family unit and then on a group unit. "However, if within the family there can be assumed to take place an optimal reallocation of income so as to keep each member's dollar expenditure of equal ethical worth, then there can be derived for the whole family a set of well-behaved indifference curves relating the totals of what it consumes."¹ The basic conditions for the similarity of such an indifference map to an individual indifference map is that "within the group optimal lump-sum transfers are always made."² The question to be solved is "What is optimal? What is not optimal?" As far as I can see economics does not deal with such a matter; ethics, however, says

Perhaps we would be wise, in any case, to follow Pareto's advice in this respect when he said: "How must the goods possessed or produced by the society be shared among its members...we shall assume that this problem is already solved."³ I think that the assumption

¹ loc. cit. p. 21.

² loc. cit. p. 16.

³ V. Pareto, Manuel d'Economie Politique, p. 362.

that the system of distribution is constant, at least during a very short period. One may say, "But the application of a tariff changes the distribution of welfare in a country!" Undoubtedly. But we cannot do much to that because in each special case a given tariff will influence the distribution of income in a certain way. Here we leave pure economic theory, as when we want to solve the problem of two firms acting monopolistically.

So, as far as we can go in the graphic study of international problems, I think that Scitovsky's explanations of community-indifference curves are sufficient. Samuelson has given us a very interesting intellectual exercise, but I do not think that it offers a better approach to international trade problems than Scitovsky's article. (Samuelson says that he will publish some more articles showing the use of his system for the study of international trade policy.)¹ For the rest, I think we must agree that our graphic approach to these complicated matters may be insufficient for policy consideration. At any rate, it will be very interesting to read the subsequent studies of Samuelson on this topic.

CONCLUSION

Pareto's theory of value and consumption is really the source of all modern theories and I think that this is his most important contribution. At any rate, it is because of it that he is still referred to in most of the modern works. Pareto's theory of production is less important. He reached most of the conclusions of the modern theorists

¹ Samuelson, loc. cit., p. 16.

but by a less clear and less easy approach. Moreover, the graphic representation of the production phenomena is carried on only as far as it is necessary to study general equilibrium. Pareto's graphic approach to production is now usually abandoned for better ways of explaining the firm's equilibrium factors of production, output and costs of production. Modern Anglo-American theorists use much simpler indifference curves to study these problems.

In the study of general equilibrium Pareto may be considered the most advanced economist in the study of general interdependence. Here, however, the importance of Walras' thought as a source for Paretian theory is greater than in production and consumption theories. Walras discovered the system of equations and the importance of interdependence. Pareto tried to carry these ideas further in an always more general way, but was prevented from completing this study by some problems which just cannot be solved by pure economics. Modern theorists are also stopped by the same problems.

Pareto went very far in the graphic representation of equilibria and most of his diagrams are the ancestors of the modern ones. The box diagrams he used for the study of equilibrium of exchange are nothing but the box diagrams used by Edgeworth and Marshall for the study of international trade. The exchange lines of Pareto are the offer curves of Marshall. Moreover, I think that Pareto diagrammatically studied equilibria more exactly and more fruitfully than Edgeworth, Marshall, and their followers.

In his approach no distortion of the offer curve (or exchange curve) appears. I am fairly certain that my approach to the tariff problems in reaction to Kaldor's analysis is similar to what Pareto would have done if confronted with such a matter. Pareto's work is not without mistakes, as he admitted and hoped that his mistakes would be useful to establish more complete theories in the future.

So I reached what could be called the "frontier" of the indifference curve approach. Contemporary economists are always studying new ways of representing graphically the equilibrium phenomena. Modern theorists are more interested in international equilibrium and the representation of international trade. They went also to justify the use of indifference curves for some problems like the study of group behavior. The care for exactitude is always greater and leads to interesting constructions.

More work will be done on these matters, such as the study announced by Spence. The main consideration which the present analysis suggests is that the further the theorists get into the study of the real world, the more difficult becomes the solution of problems by pure economy and their graphic representation. And I think, with most of the modern theorists, that such diagrams must not be given too much importance when one comes to propose policies.

Since Pareto, much has been accomplished but the modern student must be more and more cautious when he advances to the more complicated fields of economic analysis.

APPENDIX
TRANSLATION¹

120. We said that several paths are tried before finding the one which leads to the equilibrium point; let us examine more closely. If we draw the exchange curves of two individuals, we shall see, in very numerous cases, that they have shapes similar to the curves of Fig. 13, and that they meet almost like in that diagram; one gives 3 points of intersection, the other gives one point of intersection. They are of three types that we shall call α, β, γ ; we indicate them with more details in Fig. 14.

The exchange curve for the first individual, for whom axes are on Fig. 13, oa, ob will always be indicated by cd on Fig. 14. This curve will always be indicated by hk for the second individual (axes ua, up). The point of intersection of these two contract curves, that is the equilibrium point, is marked by l .

121. Let us consider the equilibrium for the first individual. In the case of points (x) and (y) the points of the curve lh are before the points of the curve cd and consequently are terminal points (p. 62) for the first individual. Therefore the curve on which equilibrium can occur is oh . For a similar reason, the curve on which the second

¹ From Pareto's Manuel, Ch. III. This is a translation made from French into English in order to give the reader an idea of what Pareto's demonstration of the equilibrium in exchange, dealt with in chapter VI of the thesis, consists of. The translation from Italian into French being obscure, it has been difficult to make this clear.

individual can be in equilibrium, always in the case of the points (d) and (y) is also ok. In the case of the point (b) this equilibrium curve is bid for the first as for the second individual. We only have to consider what occurs on these curves.

122. Let us deal with the points (d) and (y). The first individual is at h in an equilibrium position. Since we deal with the model (1) he compares only the conditions in which he would be at the different points of the path and, and he sees that he would be in better conditions at d than at h; he cannot go to do because this is forbidden to him by the tastes of the second individual. If a large number of individuals are in competition with a large number of other individuals, if our couple is not isolated, the first individual has a wish to go, if not to d, at least to a very close point. He takes a path in d' less inclined than ad on the axis ox , that is to say he gives a larger quantity of A for the same quantity of B. So he takes customers from the second individual, he receives some B from other individuals, and he can go to d' , which is the highest point of the path and at which he is in equilibrium.

Let us examine what happens to the second individual. He was in h , which is for him the highest point of the path. The loss of customers draws him back. They give him less A, because the first individual receives more of it. So the second individual is drawn back to a lower position, in which he would be at the different points of the path and, he realizes that his condition has become worse, that it is profitable for him to try to come back to h , or at least to a fairly

close point. For this purpose he will not like the first individual. He will take a path much closer, but less inclined than mk^1 , and so he will arrive for instance to the point k^2 of the line kh .

Now the first individual must be careful, and he will run over a path less inclined. In that way both of the individuals will get closer to the point l going in the sense of the arrow. Those phenomena are similar if one goes from c on. The second individual which is at c is for him a terminal point--wants to get closer to k , which is the highest point on the path mk . Consequently he accepts a smaller quantity of A for the same quantity of B ; thus he follows a path mk^1 steeper on xy axis than mk . The first individual has to imitate him. Thus progressively both of the individuals get closer to l in the sense of the arrow.

103. The equilibrium point, then, is at l and we shall call it a stable equilibrium point, because if both of the individuals get farther and farther from l , they try to come back to that point.

104. Let us deal with the point β . As we saw it, the equilibrium curve is hd . Suppose that both of the individuals were at d ; the second one would like to get closer to h from that point, which is for him a terminal point. To do it he has to accept a smaller quantity of A for the same quantity of B . That is he runs over a path mk^1 , steeper than mk on xy axis and he will get farther from l . The first individual has to do the same; therefore they will go in the sense of the arrow. The same thing happens on the other side of l .

If both of the individuals are in h , the first one will want to get closer to c . He will run over a path less steep than ac and he will get farther from l . The second individual has to do the same, and so on. Both of the individuals go farther from l . The point l is an unstable equilibrium point.

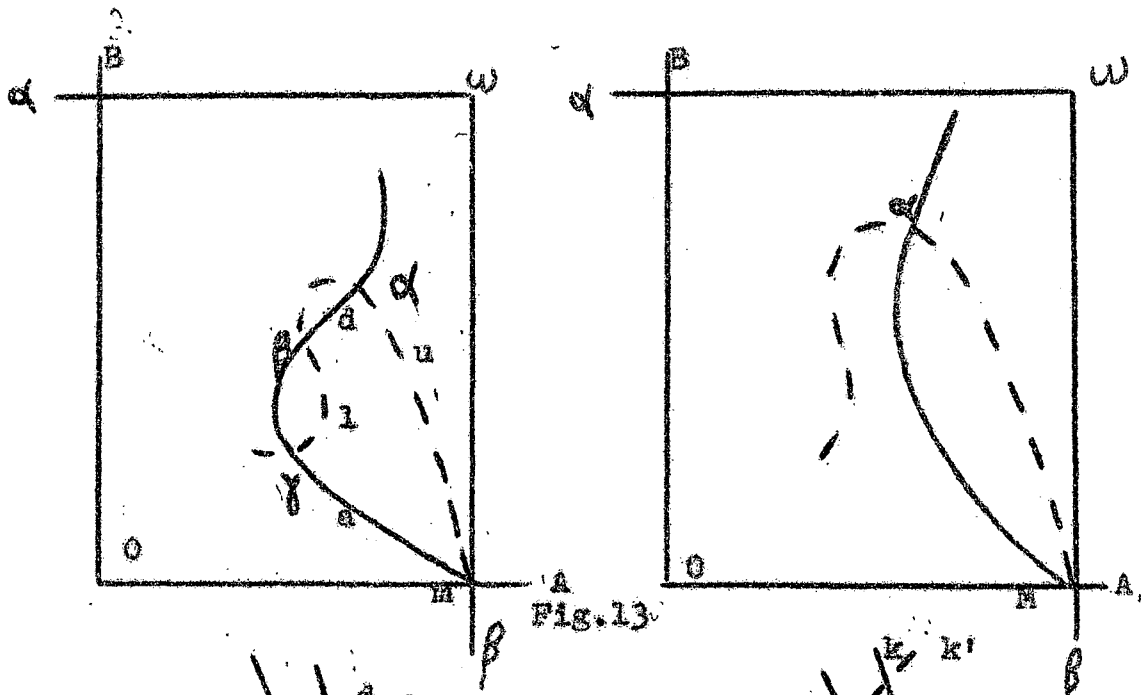


Fig. 13

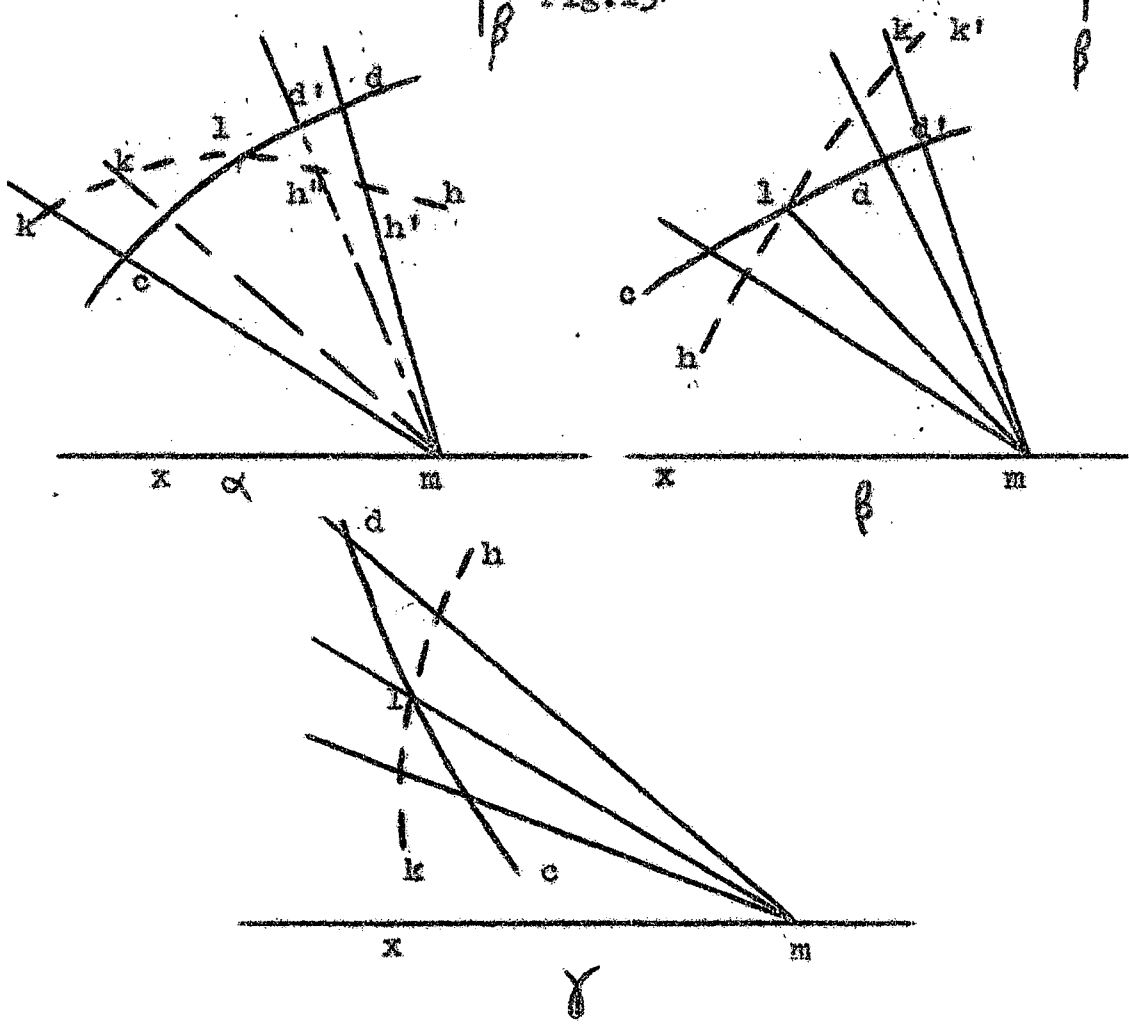


Fig. 14

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