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Teachers and their Educators – Views on Contents and their Development Needs in Mathematics Teacher Education

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Abstract: Finland has scored well in international assessments (e.g. PISA, TIMSS), and the pressure to attain excellent scores has activated a drive toward even more effective mathematics teacher education. This article presents the results of a qualitative assessment of the mathematics teacher education provided by the University of Eastern Finland. In this study, the views held by practicing teachers (N=101) and teacher educators (N=19) are compared so that the outstanding development needs of mathematics teacher education in terms of their contents can be revealed. The data was gathered via an electronic survey and was mainly analyzed using data-driven methods. In addition, framework provided by Mathematical Knowledge for Teaching (MKT) was used to categorize the respondents' views regarding the contents of mathematics teacher education and to develop general guidelines for the reform of mathematics teacher education. The results indicate that mathematics teacher education should include pure mathematical content (Common Content Knowledge, CCK) and mathematical content that will have been designed only for future teachers (Specialized Content Knowledge, SCK). Teacher educators and practicing teachers both held the view that the relevance of CCK studies depend on the connections between university and school mathematics. Pedagogical studies should also be reformed because practicing teachers have realized that effective teaching (Knowledge of Content and Teaching, KCT) requires knowledge about learning mathematics (Knowledge of Content and Students, KCS) that is not offered in the current educational system on a sufficiently broad basis. In this study, suggestions for developing mathematics teacher education were mostly connected to four domains of MKT: (CCK, SCK, KCT and KCS). Interestingly those domains are the same domains which has been empirically tested and better conceptualized.

Keywords: Mathematical Knowledge for Teaching, MKT, mathematics teacher education, evaluating teacher education, contents of mathematics teacher education.

Introduction

Finland has scored well in international assessments (e.g., PISA, TIMSS), and the Finnish school system has been rated as being of top quality. Finnish teacher education has also been evaluated as high in quality from an international perspective (Kivirauma & Ruoho, 2007; Tryggvason, 2009). An important reason for this success is that Finnish teachers are educated both systematically and extensively, and every qualified teacher must have a Master's degree (Tryggvason, 2009). It is claimed that Finnish teacher education is the result of a long-term, research-based development (Tryggvason, 2009). However, the voices of practicing mathematics teachers and teacher educators have not received attention enough in the research field. Are these two groups satisfied with the current contents of mathematics teacher education and what kind of needs of development they see at the moment?

In the present study we focus on practicing mathematics teachers' and teacher educators' views on mathematics teacher education. The practicing teachers participating in this study graduated in the period of 2002–2012 and they nowadays teach at school level, which enables them to evaluate the contents of teacher education from a perspective of the teacher's profession. In addition, when the survey was implemented the teacher educators were actively working as teacher educators. We were interested in discovering how these two subject groups saw the present contents of the Mathematics Teacher Education Program (MTEP) at the University of Eastern Finland and also in how they would develop the teacher education program. We sought answers to the following research questions:

1. How do teacher educators and practicing mathematics teachers regard the course contents of mathematics teacher education?
2. What kind of recommendations would teacher educators and practicing mathematics teachers make for improving mathematics teacher education program?

The views held by practicing teachers and teacher educators play an import role in developing teacher education. There may be a possibility that the contents are not regarded as being as useful as teacher educators assume. It is also possible that practicing teachers and teacher educators hold conflicting views about the contents. Hence, the views of both groups are important in order to be able to form a coherent picture of the current status of teacher education and to construct an extensive basis for the development work.

Our methodical aim has been to test a theoretical framework called *Mathematical Knowledge for Teaching (MKT)* (Ball, Thames & Phelps, 2008) through the process of categorizing practicing teachers' and teacher educators' views. This framework appeared to be promising for categorizing these views, since it has previously worked relatively well in classifying teacher knowledge (see Markworth, Goodwin, & Glisson, 2009; Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012).

Conceptualizing the Teaching of Mathematics

Mathematical knowledge for teaching

There was an increasing interest in the 1980s in teacher qualifications and methods of effective teaching that would influence student learning. Lee Shulman proposed that a teacher also needs to possess other types of knowledge than pure subject matter knowledge in order to teach so that students would understand. In 1986 Lee Shulman introduced a new term, *pedagogical content knowledge (PCK).* According to Shulman (1986), teachers must have an integrated knowledge of subject and pedagogy, some kind of amalgam knowledge. Initially, Shulman considered PCK to be a topic-specific subcategory of content knowledge, which included two further subcategories: knowledge of representations and knowledge of learning difficulties and strategies for overcoming them. Shulman's later model consisted of seven categories, of which PCK was one, with no subcategories (Shulman, 1987). By proposing PCK as one out of seven categories of conceptualization, Shulman neglected the potential for integration among these categories and the hierarchies that might exist between them, and left the task of further development of the concept to other researchers (Hashweh, 2005).

Shulman's conceptualization has been criticized for its restricted and ambiguous definitions of categories (Ball et al., 2008, Hashweh, 2005). Ball et al. (2008) claim that the terms PCK and *content knowledge* are frequently confused with common pedagogical skills. Meredith (1995) argues that PCK as defined by Shulman simply implies one type of pedagogy rooted in particular representations of prior knowledge. Meredith suggests that learners have a built-in competence for constructing their own understanding of subject matter, but Shulman's PCK seems not to encompass alternative views of teaching. Meredith argues that Shulman's definition of PCK leads to teaching methods where the teacher will explain and illustrate procedures while learners practice the procedures by using examples. Thus, the teacher's role can be seen as transmitting mathematical knowledge and helping learners to acquire understanding.

Shulman's conceptualization has also been claimed to ignore the interaction between the different categories, assuming that knowledge is static rather than possessing a dynamic nature (Hashweh, 2005; Fennema & Franke, 1992). Fennema and Franke (1992) argue that teacher knowledge frequently changes in light of classroom interaction experiences, and hence teachers' beliefs should form an important part of the conceptualization. According to Fennema and Franke, teacher knowledge can be divided into four parts: *knowledge of content, knowledge of pedagogy, knowledge of students' cognitions*, and *teachers' beliefs*. At the center of this model is *context specific knowledge*, which can be seen as dynamic knowledge, since it occurs in the context of the classroom. In this model, PCK consists of teachers' knowledge of teaching procedures, such as effective strategies for planning, classroom routines, behavior management techniques, classroom organization procedures, and motivational techniques. Fennema and Franke (1992) see teacher knowledge as interactive and dynamic in nature and they suggest that no single domain of teacher knowledge plays a particular role in the effective teaching of mathematics.

Rowland, Turner, Thwaites & Huckstep (2009) developed *The Knowledge Quartet* conceptualization, which was based on Shulman's conceptualization (1986) with respect to Fenneman and Franke conceptualization (1992). The Knowledge Quartet was generated by categorizing elementary teachers' classroom actions. The main aim of the research work was to investigate the relation between the teacher's subject matter and PCK knowledge. Detailed analysis of the elementary mathematics lessons taught by pre-service teachers resulted in the identification of teacher knowledge framework. Rowland et al. (2009) suggest that the framework can be used to classify teachers' actions in the context of a classroom.

One of the most promising recent efforts in discovering the kind of knowledge and skills that are needed for high-quality mathematics teaching has been the theoretical framework known as *Mathematical knowledge for teaching (MKT)*, as posited by Ball and her associates[1]. In this model, subject matter knowledge is categorized into three domains: *common content knowledge (CCK), horizon content knowledge (HCK)*, *and specialized content knowledge (SCK)* (see Figure 1). In addition, PCK consists of three parts: *knowledge of content and student (KCS), knowledge of content and teaching (KCT)*, and *knowledge of content and curriculum (KCC)*. The domains CCK, HCK, and SCK are subject matter knowledge that requires no knowledge concerning either the students or pedagogy. In addition, the domains of KCS, KCT, and KCC are the kind of knowledge that requires an integrated knowledge made up of subject matter knowledge and pedagogical knowledge (Sleep, 2009), as in Shulman's (1986) conceptualization. According to Sleep (2009), four of the domains (CCK, SCK, KCS and KCT) have been empirically tested and better conceptualized, while two of the domains (HCK and KCC) are still in the earlier stages of conceptualization.

[1]The University of Michigan projects *Mathematics Teaching and Learning to Teach project (MTLT)* and *Learning Mathematics for Teaching project (LMT)* produced plenty of details to form MKT, e.g., Hill & Ball, 2004; Hill, Schilling & Ball, 2004; Hill, Rowan & Ball, 2005; Hill & Lubienski, 2007; Hill, Ball, Sleep et al., 2007; Hill, 2007; Schilling, 2007; Schilling, Blunk & Hill, 2007; Schilling & Hill, 2007; Hill, Ball, Blunk, et al., 2007; Hill, Dean & Goffney, 2007; Hill, Ball & Schilling, 2008; Delaney, Ball, Hill et al., 2008; Stylianides & Ball, 2008; Hill, Blunk, Charalambous et al., 2008; Ball, Thames & Phelps, 2008; Ball & Forzani, 2009; Thames & Ball, 2010.

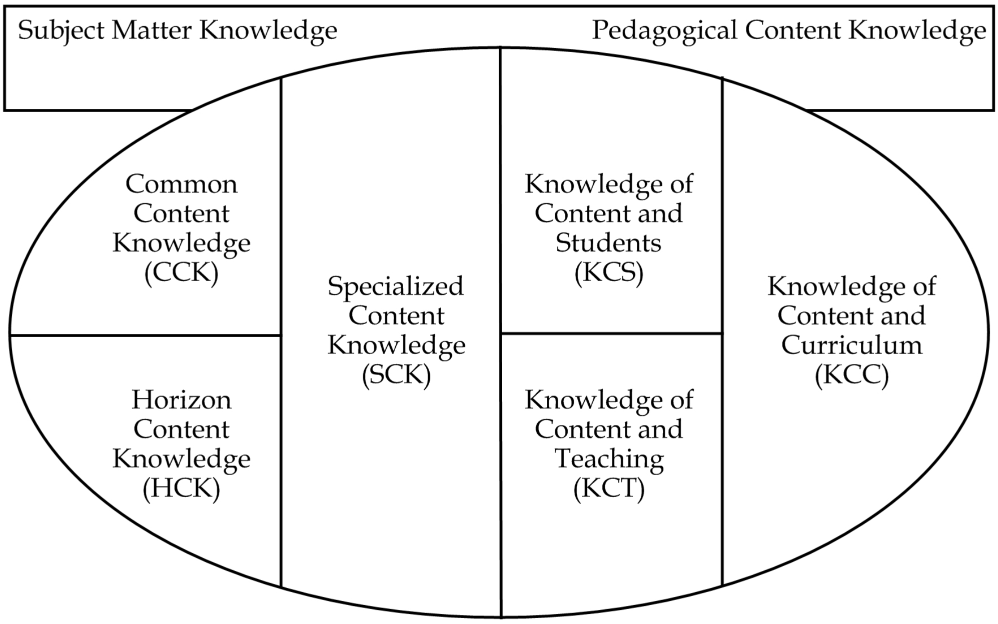

Figure 1. Domains of Mathematical Knowledge for Teaching (MKT) by Ball et al. (2008)

CCK consists of mathematical knowledge and skills used in any settings, including in settings other than teaching, and it includes calculating, solving problems, and other common mathematical knowledge that is not unique to teaching (Ball et al., 2008). SCK is mathematical knowledge and skills that are peculiar to teaching, and is typically not intended for other settings than teaching (Ball et al., 2008). In other words, SCK consists of the mathematical knowledge and skills that a mathematician does not need, while at the same time they are needed by a teacher in order to practice effective teaching. HCK consists of mathematical knowledge of the mathematical structures and also awareness of how mathematical topics are related to each other in a curriculum (Ball et al., 2008). This means that a teacher needs to know how topics are related to each other at different school levels and how mathematics is actually constructed.

KCS consists of amalgam knowledge of students, learning, and mathematics (Ball et al., 2008). A teacher must be able to anticipate students' difficulties, hear and respond to students' thinking, and choose suitable examples and presentations while teaching. A teacher's action in planning and teaching requires awareness of students' conceptions and misconceptions of different mathematical topics. KCT is also amalgam knowledge of teaching and mathematics (Ball et al. 2008). Teachers need KCT knowledge in choosing proper activities, exercises and representations for different topics. Teachers need KCT knowledge for both planning and teaching. One important part of this knowledge for teachers is to recognize situations where teachers should diverge from their original planning, for example, if a student makes a mathematical discovery.

KCC represents amalgam knowledge of mathematics and curriculum. According to Sleep (2009), a teacher needs to know the contents of the curriculum, but Ball et al. (2008) offer only a restricted definition of KCC and hence the kind of knowledge and skills that KCC includes remains unclear. Our preliminary analysis of the data in the present study showed that if MKT is used to organize practicing teachers' and educators' views, the KCC domain has to be modified. The practicing teachers and teacher educators mentioned skills and knowledge related to teaching equipment. Hence, our conceptualization states that KCC also includes knowledge and skills related to teaching materials (including textbooks, other materials, etc.), teaching instruments (blackboard, overhead projector, etc.), and technology (computer, smart board, calculators, software, etc.).

The Evolution of MKT

The development of MKT started with the study of classroom actions with a view to identifying the knowledge needed for teaching mathematics (Ball & Bass, 2003). This work continued with the formation of hypothetical characterizations of MKT (e.g. Ball, Hill & Bass, 2005; Ball et al., 2008). Thereafter, Hill, Schilling, and Ball (2004) developed specific measurements of MKT that could be used to test this hypothetical characterization. In the case of validating measurements, the Michigan group tested measurements against practice (Hill, Blunk, Charalambos, et al., 2008) and also against students' achievements (Hill, Rowan & Ball, 2005). Thereafter, MKT has been used to develop the contents of teacher education in ways that should help teachers to acquire the knowledge required for teaching mathematics (Ball, Sleep, Boerst & Bass, 2009).

Markworth, Goodwin and Glisson (2009) have used MKT to evaluate what student teachers have learned during a teaching practicum course. They coded interview responses and conversational topics on the basis of the domains of MKT. By using MKT in their analysis, Markworth, Goodwin and Glisson (2009) were able to capture more detailed information about the subject matter knowledge and pedagogical content knowledge that student teachers had gained during the teaching practicum course.

In the course of this study, practicing teachers and teacher educators suggested various recommendations for improving the mathematics teacher education program. To identify these suggestions systematically, we used MKT in a similar way to that of Markworth, Goodwin and Glisson (2009). This meant that suggested recommendations for improving mathematics teacher education could be classified in terms of six domains of MKT.

Method

Context

This study was implemented at the University of Eastern Finland, which offers two programs for students of mathematics: one for mathematicians and another for teachers. The programs are almost identical in their respective amounts of mathematics courses, but they differ in minor subjects. In the present study, we concentrate on the program for teachers, Mathematics Teacher Education Program, MTEP.

MTEP includes a Bachelor's degree (180 cp[2]) and a Master's degree (120 cp). Both degrees are required for a student to qualify as a mathematics teacher in Finland. MTEP includes mathematical studies (130 cp), pedagogical studies (60 cp), and studies in one or two minor subjects (60 cp each). Most mathematical studies are traditional mathematics courses, which are compulsory for both future teachers and mathematicians (e.g. *calculus, analysis, algebra, differential equations*, etc.).

The pedagogical studies include theoretical studies focusing on teaching and learning (30 cp), the didactics of mathematics (10 cp), and teaching practice (20 cp). Teaching and learning courses are intended to all subject teachers and courses are concerning teaching and learning in general. However, the following courses which are intended only to forthcoming subject teachers of mathematics enables taking into account the special aspects of mathematics. Teaching practice is undertaken at the university teacher training school. Student teachers plan their own teaching sequences or lessons under the guidance of a subject teacher. Student teachers' lessons are evaluated and feedback is also provided. The amount of student teaching is approximately 50

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[2] One *credit point* (*cp*) is the equivalent of 25 hours of study. The recommendation is to complete 60 cp of studies per year.

lessons. The training school teachers' task is to guide student teachers in addition to performing their own ordinary teaching work.

Student teachers can choose to study any school subject as a minor subject, but the most typical choices are physics or chemistry, or both. In its entirety, MTEP provides students with the competence to teach mathematics and minor subjects at lower or upper secondary schools and vocational schools.

Sample

The data was collected in the course of two separate electronic surveys conducted in 2012–2013. The first survey was aimed at mathematics teachers who had graduated from the UEF during the period 2002–2012. Our sample (N=101) includes 54% of all teachers who have graduated from UEF during period 2002–2012. In the sample, the majors taken by our respondents were 72% (73) mathematics, 20% (20) physics and 8% (8) chemistry, which makes the sample similar to the distribution of graduated teachers according to their major subject. All of the respondents, with the exception of one, had had previous experience of teaching mathematics at school or they were working as teachers when the survey was implemented.

The second survey targeted the teacher educators in mathematics at the UEF, who taught either mathematical studies or pedagogical studies or were guiding teaching practice. Our sample (N=19) includes 79% of all of the teacher educators in mathematics at the UEF. In the sample, 74% (14) of the teacher educators taught mathematics and 26% (5) worked in pedagogical studies or teaching practice. To conceal the respondents' identities, the teacher educators in the fields of both pedagogical studies and teaching practice were placed in the same category.

Instrument and data analysis

The study was implemented with the aid of a survey that included statements about the knowledge and skills learned in MTEP and open questions about the present state and the future of MTEP.

The survey conducted with practicing teachers included three open questions about MTEP.

1. Evaluate the contents of mathematical studies in MTEP, especially with regard to the work of a mathematics teacher.
2. Evaluate the contents of the pedagogical studies and teaching practice in MTEP, especially with regard to the work of a mathematics teacher.
3. Suggestions regarding the development of mathematics teacher education would also be appreciated.

The survey involving the UEF teacher educators included two open questions.

4. Evaluate the contents of the studies you teach, especially with regard to the work of a mathematics teacher.
5. Please make a suggestion regarding the development of mathematics teacher education would also be appreciated.

The data was analyzed using qualitative content analysis (Tesch, 1990; Hickey & Kipping, 1996; Mayring, 2000; Hsieh & Shannon, 2005). Hsieh and Shannon (2005) have identified three different approaches to qualitative content analysis that can be used to interpret meaning from the content of text data: conventional, directed or summative (Hsieh & Shannon, 2005).

Our analysis started with reading the data several times to achieve immersion and obtain a sense of the whole (Tesch, 1990). Then, practicing teachers' and teacher educators' perceptions about the contents of MTEP (Questions 1, 2 and 4) were analyzed with *Conventional Content Analysis* (Hsieh & Shannon, 2005). In the conventional content analysis, coding categories are derived directly from the text data. In our data, respondents' personal experience or more like attitudes towards contents emerged clearly from data. Each respondent was placed in one of these categories (Figure 2).

A majority of the practicing teachers' mentioned only issues that should be developed in the contents of MTEP (Questions 1 and 2), and therefore their responses were placed in the category of *In need of development*. The contents of this category were analyzed with directed content analysis, which is a more structured process than the conventional approach (Hickey & Kipping, 1996; Hsieh & Shannon, 2005). Direct content analysis starts with a theory, which is used for coding text data (Hsieh & Shannon, 2005). Generally, a goal of the directed approach is to validate or conceptually extend a theoretical framework or theory (Hsieh & Shannon, 2005). *Mathematical Knowledge for Teaching (MKT)* framework was a starting point for designing the survey, and each statement was designed to be interconnected to the domain of MKT. In the planning, we noticed a possibility for using MKT for directed content analysis. A pre-analysis of the data indicated that all the issues in question 1 and many of the issues in question 2 can be categorized with MKT. Issues beyond MKT were categorized with the conventional content analysis in case of question 2.

Both the surveys also included blank spaces for other suggestions for the development of the teacher education program (Questions 3 and 5). Many of the respondents did, however, mention the same issues which they already mentioned in the previous question related to the contents. Therefore we used directed content analysis similarly as in the categorization of the suggestions related to the six domains of MKT. Suggestions beyond MKT were categorized with the conventional content analysis. Previous questions in the survey covered the majority of respondents' ideas, and so there were only a few new ideas among these suggestions.

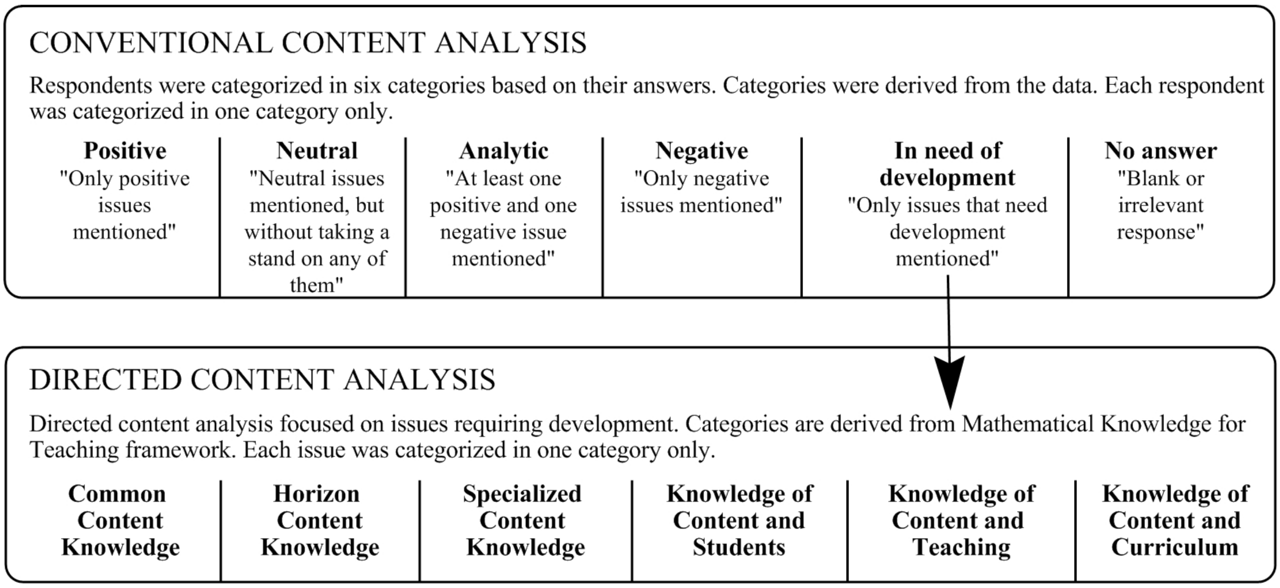

Figure 2. Text data analysis was performed with conventional and direct content analysis (Hsieh & Shannon, 2005).

Results

The results of the study are presented in two parts. First, we discuss how teacher educators and practicing mathematics teachers view the contents of mathematics teacher education. Second, our discussion focuses on teacher educators' and practicing mathematics teachers' ideas for developing teacher education. Suggestions for developing mathematics teacher education will be represented in tables where the categories have been provided mainly by MKT.

Views on the contents of mathematics teacher education

Practicing teachers' views concerning the contents of mathematics studies. The categorization of practicing mathematics teachers' views concerning the contents of mathematical showed that one fifth of the respondents (21%) viewed the contents neutrally. Half of them considered the number of mathematics courses appropriate for teachers, while the other half gave no reasons for their responses. A small minority of the respondents (7%) did not consider the contents of the mathematics courses useful for teachers. In most cases, the reason for this was that the courses were considered to provide too complex a discussion of mathematics in comparison with the mathematics needed in a teacher's work. No fully positive views appeared in the categorized responses.

A majority of the practicing teachers (59%) provided only suggestions related to developing the present contents of mathematics courses. These suggestions were analyzed again by using MKT. Most of these suggestions (79%) were related to improving student teachers' subject matter knowledge, while one fifth of them were concerned with developing student teachers' pedagogical knowledge and skills (see Table 1).

Table 1. *Categorization of practicing teachers' (N=60) suggestions for developing the content of mathematical studies. Each respondent was permitted to mention more than one issue.*

Category	Domain of MKT	f
Subject matter knowledge and skills	*Common content knowledge (CCK)*	
	• Present course contents are not linked with school mathematics	12
	• Present course contents are not the same as in schools	11
	• More geometry	4
	• More financial and statistical mathematics	2
	• Wider knowledge of mathematical concepts	2
	Specialized content knowledge (SCK)	
	• Present mathematical studies should be separate for student teachers and mathematicians	20
	• More school mathematics needed	12
	Horizon content knowledge (HCK)	
	• Present course contents are not linked with each other	4
	• More skills concerned with teaching students at different levels	1
Pedagogical content knowledge and skills	*Knowledge of content and students (KCS)*	
	• More studies concerning learning difficulties in mathematics	4
	• More skills concerned with teaching students at different levels	3
	Knowledge of content and teaching (KCT)	
	• More courses about didactic mathematics	6
	• More courses about how to differentiate teaching	2
	• More skills to motivate students in mathematics	1
	• More studies about teaching problem solving	1
	Knowledge of content and curriculum (KCC)	
	• More courses about using technology in teaching mathematics	1

Common content knowledge (CCK). The practicing teachers mentioned that the present contents of mathematical studies are not the same as the contents of school mathematics. They were disappointed that they had studied so much mathematics that they had never used in their school teaching. The practicing teachers also mentioned that the present contents did not link properly with school mathematics. Some teachers claimed that presentations at university were either symbolic and theoretical or too complex in comparison with school mathematics, and hence it was hard to see how the course contents were linked with school mathematics. The practicing teachers mentioned that they lacked the competence to teach geometry and financial or statistical mathematics, and so they suggested that MTEP should include more courses in those domains. It is evident that practicing teachers need mathematical content knowledge (Ball et al., 2008), but the opinions of the practicing teachers indicated that to be useful for future teachers, the contents should be linked to school mathematics.

Specialized content knowledge (SCK). The practicing teachers suggested that mathematical studies should be arranged differently for future teachers and for mathematicians. They argued that the current integrated mathematical courses do not support future teachers properly. Some practicing teachers said that at university the focus of mathematics was proving and presenting results, whereas school mathematics consisted of rather more than that. Almost all the respondents suggested that the mathematical representations of the course contents should be modified with respect to teachers' actual work. According to them, in the current situation MTEP includes too much pure mathematics and not enough school mathematics. Most of them recalled that in MTEP there was a course called *School mathematics,* which they found important and useful. The contents of the course were the same as in actual school mathematics, and the implementation of the course resembled the mathematics teaching conducted in schools. In consequence, they argued that they learned the contents of such courses well and that they had been able to use the course contents in their teaching work. All of them argued that there should be more courses of this kind in MTEP. All of these practicing teachers' views are linked to the definition of SCK (Ball et al., 2008): practicing teachers need mathematical knowledge that is particular to the needs of teachers.

Horizontal content knowledge (HCK). The practicing teachers argued that the contents of university mathematics courses were not interconnected or that the links could not be detected during the courses. In their view, courses that were in fact extensions of each other (e.g., calculus 1, calculus 2) were separate courses; alternatively, they were unable to detect the ways in which new mathematical concepts could be constructed on the basis of previously learned concepts. In the view of the respondents, the mathematical knowledge base ought to resemble a network, while, for them, the contents of MTEP did not support the construction of that kind of concept. Mathematical knowledge lacking a proper understanding of the structure of mathematics can be identified as the major challenge in the domain of HCK (see Ball & Bass, 2009).

Knowledge of content and students (KCS). The practicing teachers claimed that pedagogical issues can also be discussed during a mathematics course. They mentioned that they did not develop any clear idea of how students were actually learning mathematics during the mathematics courses. Some practicing teachers mentioned that they had too little competence in the issues concerned with mathematical learning difficulties. Some teachers also argued that they need more skills related to teaching mathematics to both weak and talented students at the same time.

Knowledge of content and teaching (KCT). The practicing teachers argued that issues concerned with teaching mathematics can also be handled in mathematical studies. They mentioned that didactic mathematics and studies about how to differentiate teaching should be included in mathematical studies.

Knowledge of content and curriculum (KCC). One practicing teacher argued that future mathematics teachers needed more enhanced skills concerned with the use of technology in teaching mathematics since teachers were increasingly using technology in schools.

Practicing teachers' views about the contents of pedagogical studies and teaching practice. The categorization of practicing mathematics teachers' views about the contents of pedagogical studies and teaching practice demonstrated that the respondents' views were diverse. Small minorities of the respondents viewed these studies positively (2%) as useful for teachers; or neutrally (9%), often without providing reasons; or negatively (7%), considering the courses useless for teachers; but most of them (67%) consider that there was a need for development in these studies. 3 The categorization of their suggestions is presented in Table 2. These suggestions mainly concerned pedagogical content knowledge and skills (51%), and development of the structure of mathematics teacher education (40%).

[3] 12% of respondents did not answer this question, and the responses of 3% were irrelevant.

Table 2. *Categorization of practicing teachers' (N=68) views on how to develop the content of pedagogical studies and teaching practice. Each respondent was permitted to mention more than one issue.*

Category	Domain of MKT	f
Pedagogical content knowledge and skills	*Knowledge of content and students (KCS)*	
	• More studies of learning difficulties in mathematics	13
	• More skills concerned with how to handle students at different levels	10
	• More skills concerned with evaluating students' knowledge and skills	6
	• More studies concerned with other learning theories	2
	Knowledge of content and teaching (KCT)	
	• More studies of teaching mathematics; didactic mathematics	10
	• More training in the planning and teaching of complete courses	6
	• More studies of how to differentiate teaching	5
	• More skills concerned with motivating students of mathematics	4
	• More courses about functional teaching methods, teaching problem-solving, or visualizing mathematics	3
	• More studies of how to link learning theories to practice	1
	Knowledge of content and curriculum (KCC)	
	• More skills and knowledge to produce teaching materials of their own	4
	• More studies of using technology in teaching mathematics	3
Structure of mathematics teacher education	*Amount of studies*	
	• More teaching practice	21
	• More pedagogical studies	5
	• Compulsory update education after some years of teaching	1
	• More studies of how to teach minor subjects	1
	Quality of studies	
	• Linking theory to practice	25
	• Educators of practice teachers should give more advice about didactic issues	1
	Developing curriculum of MTEP	
	• Contents of pedagogical studies and practice should be better integrated	1
General issues	*The other knowledge and skills*	
	• More studies of teachers' extramural duties	10
	Common issues	
	• Departments' cooperation should be improved	1

Knowledge of content and students (KCS). The practicing teachers suggested that the pedagogical studies and teaching practice should include more courses about the learning difficulties encountered in mathematics. Some teachers said that they were struggling with students who probably had learning difficulties and hence they needed more skills in order to be able to recognize and handle such students. Some of the practicing teachers also mentioned that the skills concerned with teaching students at different levels would be useful for them because student groups were often very heterogeneous. The practicing teachers also mentioned that they needed more knowledge and skills for evaluating student learning and more knowledge concerned with various learning theories, since students learn in different ways.

Knowledge of content and teaching (KCT). The practicing teachers demanded more skills for teaching mathematics. Some of them mentioned that the courses in didactic mathematics were useful for them, but that they needed more knowledge of this kind. The practicing teachers argued that their studies did not include enough courses on planning and teaching complete courses. They stressed that planning was the first thing that new teachers needed to undertake after graduation. The practicing teachers also mentioned that more skills for differentiating teaching and increasing student motivation would be of assistance.

Knowledge of content and curriculum (KCC). The practicing teachers said that textbooks or other printed material did not always fit their ideas about teaching, and so they would need more skills to design their own teaching materials. The practicing teachers also mentioned that they needed more knowledge and skills concerned with using technology in teaching mathematics in a pedagogically reasonable way.

The number of courses. The practicing teachers said that both the teaching practice and the pedagogical studies were very useful and suggested that their number should be increased in MTEP. They felt that the teaching practice was a good place for trying out new teaching methods or for trying to transform pedagogical knowledge into practice. They also told about the use of useful and functional teaching methods learnt during their teaching practice in their actual work. Some of them mentioned encountering similar situations in the classrooms to those that had been discussed in the pedagogical studies, which had helped them to better understand the relevance of the pedagogical studies.

The quality of courses. The practicing teachers argued that the courses in the pedagogical studies and even courses about teaching and learning were too theoretical, which made linking theory with practice difficult. They described a feeling of learning a lot during these studies, but without having the necessary skills to apply this knowledge in classroom situations. Some teachers even felt that the pedagogical studies were useless because they had too few links with real-life teaching situations.

Other knowledge and skills. Some practicing teachers said that they were surprised by the duties that teachers had outside the classroom. They suggested that these issues should be discussed in mathematics teacher education.

Teacher educators' views on the contents of their own courses. One fourth of the teacher educators (26%) viewed their own courses positively and considered that the courses were useful for future teachers (see Table 3). Many of them (42%) viewed their own courses neutrally and regarded the courses as having been only partly useful. Some teacher educators (16%) viewed their own courses negatively and indicated problems in the contents of courses that made them not very useful for teachers.

Table 3. *Teacher educators' (N=19) views about their courses and their suitability for future mathematics teachers. ME = teacher educator in mathematical studies, PTE = teacher educator in pedagogical studies and teaching practice.*

Class and justification	**ME (N=14)**	**PTE (N=5)**
Positive 26% (5)		
• Contents increase pure mathematical knowledge and the teaching methods used teach how to teach mathematics	2	0
• Contents are the same as in school mathematics	1	0
• No justification	0	2
Neutral 42% (8)		
• Contents are not the same as in school mathematics, but studies develop mathematical thinking	2	1
• Only some parts of contents link with school mathematics	2	0
• Some contents go beyond school mathematics or general knowledge for teachers	2	0
• Courses are non-compulsory for student teachers and therefore their contents are not useful for teachers	1	0
Negative 16% (3)		
• Some courses are simply all-around education for teachers and in some courses there is not enough time to teach important issues	1	0
• Students' knowledge is poor at the beginning of courses and therefore they cannot learn the contents	0	1
• Teachers learn contents but they have insufficient skills for using this knowledge in school teaching	1	0
Empty 16%(3)	*2*	*1*

Positive (5). There were three mathematics educators (MEs) and two pedagogical studies and teaching practice educators (PTEs) who considered that the contents of their own course were useful for teachers. The PTEs did not justify their views, but the MEs argued that the contents of their courses increased student teachers' mathematical knowledge and the teaching methods modeled the way to teach mathematics. The MEs underlined the significance of presenting things; they argued that it was important for student learning that the educator demonstrated how things worked. One ME argued that the contents of his courses were the same as in school mathematics and hence the contents were useful for future teachers.

Neutral (8). Seven MEs and one PTE considered the contents of their courses only partly useful for future teachers. One PT and two MEs claimed that, despite the contents not being the same as for school mathematics, the courses nevertheless developed students' mathematical thinking, which was also important for future teachers. Two MEs claimed that only some parts of the contents were linked with school mathematics and therefore these parts were useful for future teachers. Another ME thought that the contents are "good to know", but were unnecessary for teachers. Two MEs argued that the contents of their courses offered teachers only general knowledge since the contents were not specialized for use by teachers or the contents went beyond school mathematics. Both of them justified their views with the argument that future teachers needed a wide knowledge base in mathematics.

Negative (3). Two MEs and one PTE argued that the contents of their courses did not fully support future mathematics teachers. One ME claimed that in some courses s/he had too little time to teach issues that were important for teachers, while in other courses the contents were simply general knowledge for teachers. Another mathematics ME claimed that teachers usually learned the contents of his/her courses, but the course nevertheless did not provide them with the competence to apply this knowledge in their own teaching. One PE claimed that students had acquired insufficient earlier knowledge to learn the contents of his/her courses.

Teacher educators' and practicing teachers' suggestions for developing mathematics teacher education

The second research question was concerned with how practicing mathematics teachers and mathematics teacher educators would develop mathematics teacher education.

The practicing mathematics teachers made numerous suggestions for developing mathematics teacher education. The categorization of the suggestions in Table 4 shows that it would be valuable to develop teacher education both at the general level and also in terms of supporting future mathematics teachers' subject matter knowledge and pedagogical knowledge and skills. More than half of the suggestions (60%) concerned the contents of teacher education that could be categorized with MKT. One third of the suggestions (28%) focused on the quality of the teaching or the quantity of the studies that were categorized as ideas for developing teacher education program. A minority of the suggestions (12%) concerned a number of general issues related to teacher education.

Suggestions for improving the contents of teacher education mostly concerned pedagogical knowledge and skills. Practicing teachers suggested that they would add courses about learning difficulties in mathematics, the evaluation of students' mathematical know-how, and how to teach students with different levels of mathematical knowledge and skills. The practicing teachers also hoped that differentiating mathematics teaching would be discussed during teacher education, since classroom situations required that kind of competence from a teacher. They also suggested that the learning theories courses should be modified so that they would become easily applicable to one's own teaching. The practicing teachers would also add future teachers' knowledge and skills related to using technology in teaching mathematics because technology was assuming a more important role both in the classrooms and in society. Almost all of the suggestions concerning subject matter knowledge dealt with separate mathematics studies programs for future mathematicians and teachers. One common argument was that future teachers needed a different kind of mathematical knowledge from that used by mathematicians.

The ideas that were presented regarding development of the teacher education program concerned both the quality of teaching and the quantity of studies. The practicing teachers thought that the quality of teacher education could be increased by improving students' learning. This could be achieved by modifying present teaching methods as well applying new interactive teaching methods that would include discussions. The practicing teachers also argued that the studies should be modified to be less theoretical because they felt that the present studies were too theoretical, causing the students problems in understanding the course contents to any depth. The practicing teachers also thought that the teaching practice supported their teacher growth. However, they considered that the length of the teaching practice could be increased.

Table 4. *Categorization of practicing mathematics teachers' (N=101) suggestions for developing mathematics teacher education. Each respondent was permitted to mention more than one issue.*

Category	Suggestions for improving education	f
Pedagogical knowledge and skills	*Knowledge of contents and students (KCS)*	
	• Courses about learning difficulties in mathematics	7
	• How to test students' knowledge and skills	3
	• How to teach students at different stages of learning	2
	Knowledge of contents and teaching (KCT)	
	• How to differentiate teaching	5
	• How to bridge the gap between learning theories and practice	4
	• How to produce and use one's own teaching materials	2
	• Functional learning methods	1
	• Learner-centered teaching methods	1
	• Special education in mathematics	1
	• How to plan and teach complete courses	1
	Knowledge of contents and curriculum (KCC)	
	• How to use technology in teaching mathematics	6
	• Curricular knowledge	1
	• Teaching methods based on technology	1
Subject matter knowledge and skills	*Specialized content knowledge (SCK)*	
	• Separate mathematics courses for teachers and mathematicians	16
	• Problem-solving	1
	• Mathematics in different professions	1
	Horizon content knowledge (HCK)	
	• Mathematical concepts at different school levels	1
	• The structure of mathematics	1
Ideas for developing the teacher education program	*Quality of teaching*	
	• New teaching methods (e.g., more discussion)	10
	• Less theory – more practice – linking theory and practice	6
	• Integrating lectures and doing exercises in mathematical courses	1
	Quantity of studies	
	• More teaching practice	7
	• More pedagogical courses	1
	• More teaching practice and less pedagogical studies	1
General issues	*Improving cooperation*	
	• Cooperation between different departments	4
	• Cooperation between university and schools	1
	• Cooperation between students and educators; paying attention to students' suggestions regarding development	1
	Special suggestions	
	• Teachers should be specialized in teaching at different school levels	1
	• Subject teachers' major should be in education	1
	• Compulsory updating of education after some years of work experience	1
	Beyond MKT knowledge	
	• More knowledge about teachers' duties out of class	2
	Uncategorized responses	
	• Irrelevant	4
	• Blank	27

The teacher educators saw less reason for development than did the practicing teachers. Most of the teacher educators suggested developing mathematics teacher education by improving student teachers' subject matter studies (see Table 5).

Table 5. *Categorization of teacher educators' (N=19) suggestions for developing mathematics teacher education*.

Category	Suggestion for improving education	f
Subject matter knowledge and skills	*Specialized content knowledge (SCK)*	
	• Contents of mathematics courses should be revised to be useful for future teachers	4
	• Mathematics courses should be separately designed for teachers and mathematicians	3
	Common content knowledge (CCK)	
	• More courses in mathematics	2
	Horizon content knowledge (HCK)	
	• New course on the structures of mathematics	1
Developing the teacher education program	*Updating structure of studies*	
	• Combining mathematics and pedagogics courses as an integrated unit	1
	• Re-scheduling courses in mathematics, pedagogics, and teaching practice	1
	Uncategorized responses	
	• Irrelevant	2
	• Blank	5

Most of the suggestions concerned modifying future teachers' mathematical studies. Some educators suggested that the contents of present courses should be revised from the viewpoint of teacher's work and current school curricula. Many of the respondents would develop courses to increase future teachers' specialized content knowledge (SCK). Some educators also suggested that mathematics studies should be separately designed for future teachers and mathematicians, an idea that was also put forward by the practicing teachers. However, a few teacher educators argued that pure mathematics was the basis of good teaching and therefore the quantity of pure mathematics studies should be increased. One educator suggested that there was a need for developing a mathematics course whose rationale would be to link together the various domains of mathematics. We categorized this as an example of improving students' horizon content knowledge.

Two teacher educators considered that the structure of the mathematics teacher education should be updated. Another educator suggested that mathematical and pedagogical studies should not be organized separately, since their separation prevented the possibility of linking theory and practice. Another teacher educator argued that studies should be better scheduled to help student teachers to acquire an integrated knowledge of subject matter and pedagogy. It should also be noted that a third of the respondents provided no suggestions for developing mathematics teacher education, and hence it remains unknown whether these respondents were satisfied with the current teacher education or not.

Discussion

This study has investigated teacher educators' and practicing mathematics teachers' views of the contents and the development needs of mathematics teacher education as provided by the University of Eastern Finland. Practicing teachers and teacher educators made various recommendations for improving mathematics teacher education program. We consider that we have been able to identify systematically and in a detailed way the kind of subject matter knowledge and pedagogical content knowledge that these recommendations concern by classifying them in terms of the domains of MKT. Challenges concerning the content of mathematics teacher education seem to become more explicit when subject matter knowledge and pedagogical content knowledge are divided into more detailed components. Markworth, Goodwin and Glisson (2009) found similar benefits when they used MKT to evaluate a single course in mathematics teacher education. The combined results show that a majority of the recommendations concerning the issues that will need to be examined in mathematics teacher education are closely related to four domains of Mathematical Knowledge for Teaching (CCK, SCK, KCS, and KCT). Interestingly, these four domains have been more empirically tested and better conceptualized than the other two domains (Sleep, 2009).

Our results indicate that the majority of practicing mathematics teachers do not regard the present contents of mathematics studies to be fully functional for future mathematics teachers. The practicing teachers suggested, for instance, separate courses for future mathematics teachers and mathematicians, and the possibility of taking school mathematics into account in the teaching of mathematics courses. These ideas were also proposed by some of the teacher educators. These findings are broadly in line with the well-known recommendations by other mathematics educators (e.g., Ball et al., 2008).

Both the practicing teachers and the teacher educators argued that course contents are purely general knowledge for future teachers if there were no explicit links with school mathematics, and hence these contents were not regarded as useful for future teachers. The practicing teachers argued that the links between university and school mathematics were difficult to perceive if the university course contents were set at too high a level compared with the usual school contents. The findings show that the pure mathematical contents, i.e., the common content knowledge (CCK) of the mathematics teacher education, should be carefully examined so that the most relevant mathematical contents for future mathematics teachers could be discovered. It is well known that a weak knowledge of mathematics on the part of teachers has a negative influence on teaching (McDiarmid, Ball & Anderson, 1989), but, on the other hand, a competence solely in mathematics is insufficient enough for good teaching (Hodgen, 2011). It seems that the relevance of mathematics courses depends on how explicitly the link between university and school mathematics is stressed in mathematics courses.

The results suggest that, in addition to pure mathematical contents, mathematics teacher education should include mathematical contents designed specifically for teachers. The practicing teachers argued that they needed mathematical knowledge and skills that were different from the skills and knowledge useful for mathematicians. This reflects the well-known ideas embodied in *pedagogical content knowledge* (Shulman, 1987), or *Specialized content knowledge, SCK* (Ball et al., 2008), i.e., an area of knowledge for teachers that also separates researchers from teachers. The practicing teachers and teacher educators suggested that the mathematical courses should be at least partly separate for future teachers and mathematicians. In practice, this would mean that more resources would be needed for mathematics teacher education, which might be a challenge.

Many of the teacher educators who participated in this study espoused the traditional view of development that emphasizes improving future teachers' subject matter knowledge (SMK) (Ball, 2003). Some educators viewed that good teaching requires knowledge of pure mathematics (CCK) and therefore they suggested that pure mathematical contents should be increased. On the other hand, many educators viewed that future mathematicians and future teachers need different kind of mathematical knowledge (SCK) and therefore they suggested that some of the present contents should be modified to be more suitable for teachers or new courses should be developed for teachers. Some educators viewed that forming integrated knowledge of pedagogy and mathematics (SCK) is one challenge for the mathematics teacher education, and therefore they suggested that the present courses should be re-scheduled or integrated.

On the other hand, a majority of the practicing teachers observed that there was a wider need for development than simply reforming the mathematical contents. The majority of practicing teachers demanded more courses concerned with teaching mathematics, students' learning difficulties in mathematics, and how to differentiate mathematics teaching. These knowledge domains can be identified as *Knowledge of content and teaching* (KCT) and *Knowledge of content and students* (KCS). The practicing teachers pointed out that they needed to alternate the knowledge and skills of teaching and learning in many classroom situations, and they seemed to consider that the KCS and KCT knowledge types were interconnected especially in classroom *actions* (see Fernández, Figueiras, Deulofeu, et al., 2011; Ball et al., 2008). Many practicing teachers considered that they had learned pedagogical and mathematical issues in the course of their teacher education and that they had found teaching practice a very useful experience, but still they had difficulty in forming an integrated understanding of pedagogy and mathematics (see also Korthagen & Kessels, 1999; Sharp, 2004).

This linkage of theory and practice (Carlson, 1999; Tryggvason, 2009) seems to be a major challenge in mathematics teacher education, since it concerns not only the pedagogical and mathematical studies but also the teaching practice. Earlier research work has shown that solving the problem will not be simple. According to Verloop, Driel, and Meijer (2001), it is still difficult to foresee how teacher knowledge can be clarified clear for future teachers in their teaching practice. One of the problems appears to arise from the teacher educators' knowledge: not even experienced educators in the field of teaching practice have a clear grasp of the types of knowledge that teaching procedures involve, which makes it difficult to make the connection between theory and practice visible to student teachers (Verloop et al., 2001; Asikainen, Pehkonen & Hirvonen, 2013).

Filling the gap between theory and practice is a demanding task because teaching practice comprises only a small proportion of the teacher education studies as a whole. Hence, it is almost unrealistic to suggest that the gap could be fulfilled during the teaching practice. Our results suggest that the links between theory and practice should be made visible in all of the components of the teacher education so as to support future teacher development. Numerous suggestions have been made for the solution of this problem, e.g., by approaching it from practice to theory (Carlson, 1999), by developing the pedagogy of teacher education (Korthagen & Kessells, 1999), or by taking problem-solving into account in mathematics teacher education (Leikin & Levav-Waynberg, 2007). There is a possibility that contents, teaching methods, and the learning process may all be involved in the solution.

We have come to the realization that one of the key factors in reforming mathematical studies is the performance of a detailed analysis and comparison of curricula in university and school mathematics. In fact, it would seem obvious that the pure mathematical contents (CCK) should be the same as the topics in school mathematics or, at the very least, explicit links should exist between university and school mathematics. Another challenge is to design and develop special content knowledge (SCK) courses for future teachers. As yet, there is no general consensus about the knowledge and skills included in SCK (see e.g., Carrillo, Climent, Contreras & Muñoz-Catalán, 2013; Flores, Escudero & Carillo, 2013) but there should be no problem in designing new courses for future teachers, since there would be no harm caused if the subject matter and pedagogical contents are mixed. But as far as conceptualizing MKT is concerned, there is still work to be done to reach a consensus about this type of knowledge.

Although teacher education and teachers' knowledge are related (Darling-Hammond, Chung, & Frelow, 2002), more research into the challenges revealed by individual teacher education programs will be needed in order to construct a broader picture of this multifaceted phenomenon. Individual reports may act as an important part in this process by evaluating and improving mathematics teacher education before all of the universal challenges facing mathematics teacher education have been fully recognized. Although the present study has concerned only a single mathematics teacher education program, we would suggest that the following issues may prove to be more general challenges facing all mathematics teacher education programs:

- The connections between university mathematics and school mathematics are not self-evident for student teachers. Teachers need pure mathematical knowledge, e.g. Common Content Knowledge (Ball et al., 2008) and Subject Matter Knowledge (Shulman, 1986;

1987). However, student teachers may find that the mathematics studied at university level is too advanced and has no clearly visible connections to the mathematics taught in school.
- Specific mathematical knowledge is missed from teacher education, while the contents of mathematical courses focus too largely on pure mathematics. In addition to mathematical content knowledge, teachers also need specific mathematical knowledge, e.g. Specialized Content Knowledge (Ball et al., 2008) or School Mathematics (O'Meara, 2010), because they need to carry out a variety of different activities (e.g., producing teaching materials, formulating and marking exams) for which pure mathematical knowledge is insufficient.
- Teachers may have too few tools to be able to teach "good and poor" students at the same time. In the classroom teachers are simultaneously attempting to evaluate their students' starting levels, to recognize their individual learning habits, and also to implement different teaching strategies that will match up to the pertaining situation. The knowledge required in these situations can be referred to as Pedagogical Content Knowledge (Shulman, 1986; 1987) or as both Knowledge of Content and Students and Knowledge of Content and Teaching (Ball et al., 2008).
- Courses in teacher education may be too theoretical (e.g., Carlson, 1999; Korthagen & Kessells, 1999). Student teachers may feel that mathematical and pedagogical courses and also teaching practice are too far removed from teachers' actual work.

The results of this study encourage us in the development work of mathematics teacher education although the circumstances are still difficult at the starting point. The most demanding part has been and will be to evaluate what the personnel in mathematics teacher education teach and what kind of methods they use. We believe that assessment, feedback, and the teacher education personnel themselves and their cooperation are important components in the process of improving teacher education. It is common sense that there is always a possibility of improvement, and therefore the development must begin from critical thinking: what can we do better? With this article, we should like to encourage other researchers to evaluate and develop teacher education, and hence we would close with words that are too frequently dead and buried:

- Without criticism, development dies –

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