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What Can Education Learn from Real-World Communication of Risk and Uncertainty?

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Abstract: Probability is a difficult topic to teach, not least because it is rather unclear what it actually means. Modern risk communication has tackled general public incomprehension of probability statements by using the metaphor of ‘expected frequencies’ – for example, “of 100 people like you, we would expect 10 to have a heart attack or stroke in the next 10 years.” We show how these ideas can be taken into the classroom as the basis for teaching probability, using frequency tree diagrams as the fundamental representation. Empirical frequency trees can be used to summarise a series of classroom experiments, and then expected frequency trees naturally provide a basis for deriving the rules of probability, and make complex conditional probability calculations reasonably straightforward.

Keywords: natural frequencies, expected frequency, frequency trees.

Introduction

Risk-communication is a hot topic, whether it concerns the benefits and harms of screening or the chance of a catastrophic earthquake. It is challenging to explain both unpredictability and uncertain knowledge to the public, and yet these are also essential elements in education in probability and statistics. I shall argue that current approaches in communicating risk and uncertainty can contribute substantially to educational practice.

In particular, Gerd Gigerenzer’s recommendation for ‘natural frequencies’ – whole-number outcomes starting from a defined population of cases - can be adapted to teaching probability based on a natural sequence of stages: empirical multiple narratives from experimentation represented as 2-way tables and frequency trees, to expected outcomes in multiple future experiments, and finally to probability trees. Issues of relative and absolute risk continually arise in topical stories, and representations that make these transparent are as relevant in the classroom as in the news.

I will first confront a fundamental question:

Why do people find probability and statistics unintuitive and difficult?

I’ve been working in this area for around 35 years, and after all this time have finally arrived at an answer.

Because probability and statistics are unintuitive and difficult.

I think it is important to remember this, and have sympathy with the struggle for comprehension. When someone asks me a school-level probability question, it’s not like some algebra when I know the steps and just have to slog through to the answer. Probability questions always require some careful thought, and usually checking via at least two different solution methods. And the wording is so crucial (and often unclear).

I think that part of this difficulty is that it is unclear (to me at least) what probability actually *is*. It seems to be ‘virtual’ number – it is not directly measurable as is distance or weight. In fact I come from the subjectivist Bayesian school, founded by Italian statistician Bruno de Finetti, that

holds that *probability does not exist* as a property of the external world (except maybe at a sub-atomic level), and so any number we use is a construct based on our current (imperfect) knowledge.

So any description of probability is a metaphor, and we can expect that different ways of describing probability may be interpreted in very different ways.

Communicating Probabilities

I often ask medical audiences a question like:

If you had to tell a person that around 20% of cases with their condition would suffer a certain bad outcome within 10 years, would you use the terms -

1. *20% chance*
2. *0.2 probability*
3. *1 in 5 chance*
4. *20 out of 100 cases like you*
5. *None of the above*
6. *Wouldn't use numbers*
7. *Depends on person*

They tend to prefer option 4, which fortunately turns out to be the one supported by psychological research.

In particular, using many '1 in X' statements is not recommended. In a recent population survey by telephone, Galesic and Garcia-Retamero (2010) asked

Which of the following numbers represents the biggest risk of getting a disease:

1 in 100, 1 in 1000, 1 in 10 ?

In Germany, 28% of responses were incorrect, and in US 25% got it wrong. The crucial issue is using larger numbers to communicate smaller risks, so a difficult inversion must be done: this is one reason why flood risk maps expressed in terms of '1 in 100 year event' etc are difficult to read and potentially misleading.

But even if using whole frequencies to express risk, such as 20 out of 100, it must be kept in mind that 200 out of 1000 tends to appear bigger (Denes-Raj, Epstein & Cole, 1995) – this is known as 'ratio bias' in which focus is on the size of the numerator. The extreme version, in which the denominator is ignored completely, is known as "denominator neglect": the media does this every time they concentrate on a single accident without, for example, mentioning the millions of children that go to school safely each day. Remember: Newspapers like Narratives using Numerators.

Absolute and Relative Risks

A US direct-to-consumer advert for a statin declares in large font '36% reduction' in the risk of heart attack. In very much smaller font it clarifies that this is a reduction, in absolute percentage terms, from 3% to 2% over 5 years. So 100 such people would have to take the drug every day for 5 years to prevent one heart attack. This does not sound so impressive.

Gerd Gigerenzer has popularised the idea of "natural frequencies": translating probability problems into expectations for, say, 100 or 1000 people (Gigerenzer & Edwards, 2003). Take a recent headline saying that eating 50 grams of processed meat each day (eg a bacon sandwich) was associated with a 20% increased risk of pancreatic cancer (Willey, 2012). It turns out that this very

serious disease fortunately affects only 1 in 80 people. So we want to calculate a 20% increase on a 1-in-80 chance, which is tricky to do.

However, if we imagine 400 people who have a nice healthy breakfast, we can easily calculate that we would expect 5 to get pancreatic cancer. If 400 other people all stuff themselves with a greasy bacon sandwich every day of their lives, this 5 would increase by 20% - to 6. Again, this 1-in-400 increase does not seem so important. Note the trick in identifying 400 as the denominator that will just give an impact of one case.

Expected Frequency Trees

In the previous example, the denominator of 400 was deliberately chosen so that the crucial difference comprised a single person. A similar exercise was conducted for the recent revision of the advice leaflets for breast cancer screening in the UK. I was on the panel that worked on this controversial topic, using evidence from a review of an independent panel (Breast screening benefits and harms, 2010), and with the approach that the leaflets would present the potential benefits and harms of screening, but would not make an explicit recommendation.

We drew up the ‘expected frequency tree’ shown in Figure 1, comparing the expected experience of 200 women with and without screening.

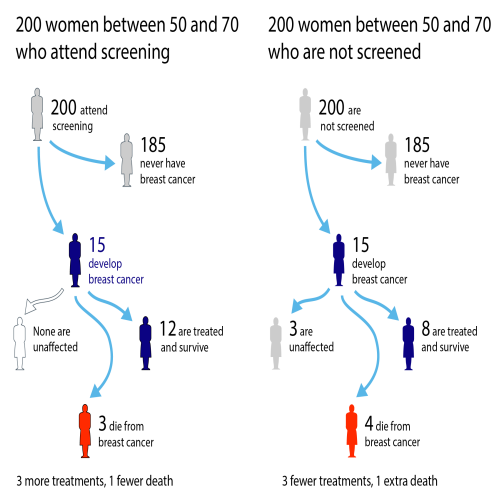


Figure 1. Expectations for 200 women attending or not attending breast screening every 3 years between the ages of 50 and 70

A website (NHS breast screening, n.d.) that incorporated a similar infographic recently won a 2014 UK Association of Medical Research Charities Science Communication Award.

Our ‘Manifesto’ for Teaching Probability

With Dr Jenny Gage of the Millennium Mathematics Project in Cambridge, we have developed a ‘manifesto’ for teaching probability that exploits the ideas of narratives, multiple representations, natural frequencies, expectation trees and so on. This can all be viewed on the Nrich website (Great expectations: probability through problems, n.d.).

Put simply, the stages are:

- Start with a problem (necessarily simplified to some extent)
- Model physically (using simple equipment, such as a die with different coloured faces or small coloured cubes)

- Do experiments (in groups, recording outcomes)
- Pool empirical data to represent multiple ‘narratives’ as
 - 2 x 2 tables
 - Frequency tree
 - Venn diagram
- Spot patterns (classroom discussion to see what hypotheses emerge from the data, rather than asking students to predict in advance what will happen when they have no experience on which to base an opinion)
- If available, conduct large numbers of experiments using animations
- Ask questions about *expectation* in multiple future experiments
- Construct expected frequency tree
- Change to fractions on branches to get probability tree

Note that probability comes in as the *final* step.

Example: The Dog Ate My Homework!

A certain teacher, Mr L I Detector, claims he can tell when students are lying about their homework. This is true. Unfortunately, he also accuses some students who are telling the truth. So what are the chances that someone will be wrongly accused?

Note the deliberate ambiguity of the final question – different possible interpretations will arise naturally.

We use dice and ‘lego-like’ cubes to carry out multiple experiments of situations in which a student claims that their homework has been eaten by a dog. The first throw identifies whether a student is telling the truth or not: a ‘six’ means they are lying, and they get a red cube, otherwise a blue. Then the ‘honest’ ones throw again to decide the teacher’s opinion: if a ‘one’ then he accuses them (yellow cube added to blue cube), otherwise a green cube (added to blue cube). The dishonest ones always get accused, in this version of the story, and so get a yellow cube to add to their red cube.

The pairs of coloured cubes summarise independent narratives and can be accumulated as a 2x2 table – this idea was derived from previous experiments by Martignon and Kurz-Milcke (2006) and Kurz-Milcke, Gigerenzer and Martignon (2008).



Figure 2. 2x2 table arising from multiple experiments of the ‘dog’ question

The data can then also be represented as an empirical frequency tree as in Figure 3.

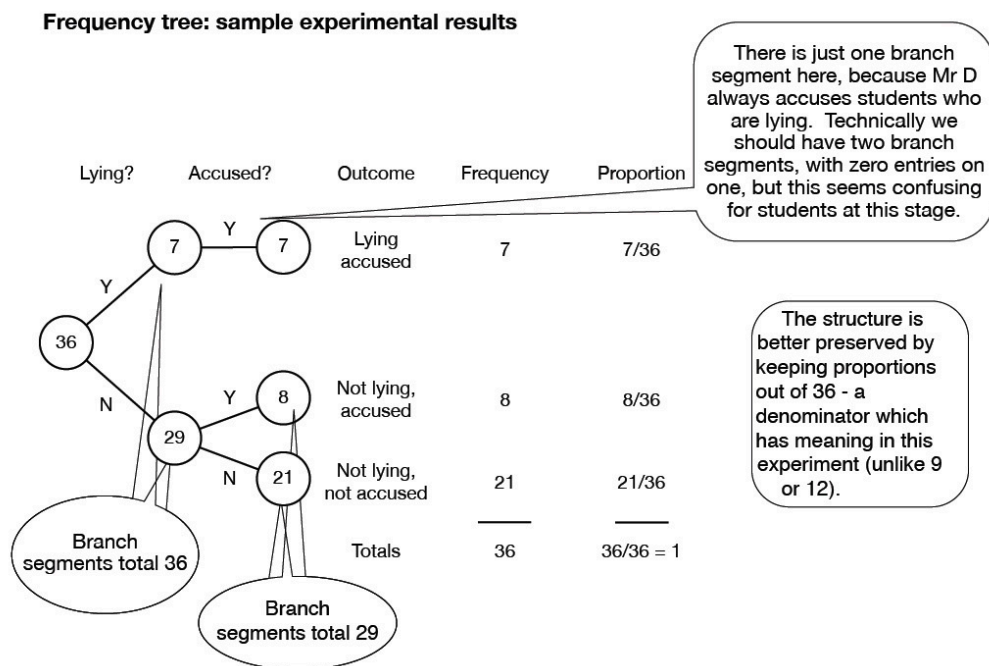


Figure 3. Empirical frequency tree of 36 independent experiments

Students should now have grasped the long-run pattern that would be expected in very large numbers of repetitions, helped by simulation software if available. They are then asked to consider what they would ‘expect’ to happen in 36 further experiments, as in Figure 4.

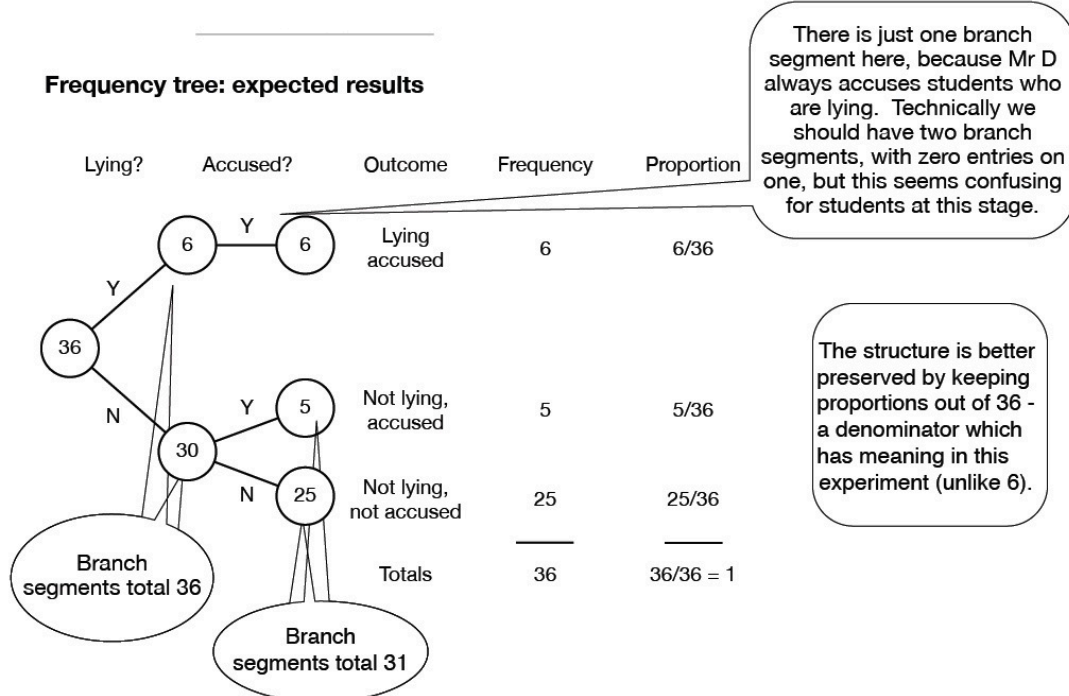


Figure 4. Expected frequency tree for a future 36 independent experiments

For more advanced students, ‘Bayesian’ questions can be answered by examining the expected frequency tree. For example, what proportion of accused are expected to be innocent? It is straightforward to see that, out of 11 accused, 5 will be innocent, a proportion of 5/11 or 45%. This is a difficult and unintuitive calculation, made straightforward by this representation. Gigerenzer’s

team has shown that doctors and other professionals can be taught to accurately carry out such Bayesian tasks using expected frequency trees (Gigerenzer, Gaissmaier, Kurz-Milcke, Schwartz & Woloshin, 2007). These have been adopted in animations such as out screening tool on Understanding Uncertainty (Screening tests, n.d.).

The ambiguity of the question ‘*what’s the chance of being falsely accused?*’ can then be explored. There are 5 people in the tree being falsely accused. Do we consider these as a fraction of the whole 36, of the 11 accused, or of the 30 who were not lying? This reveals the great care needed in specifying the ‘reference class’, which provides the denominator, for the probability statement, and the almost universal ambiguity of questions starting ‘*what’s the chance...*’

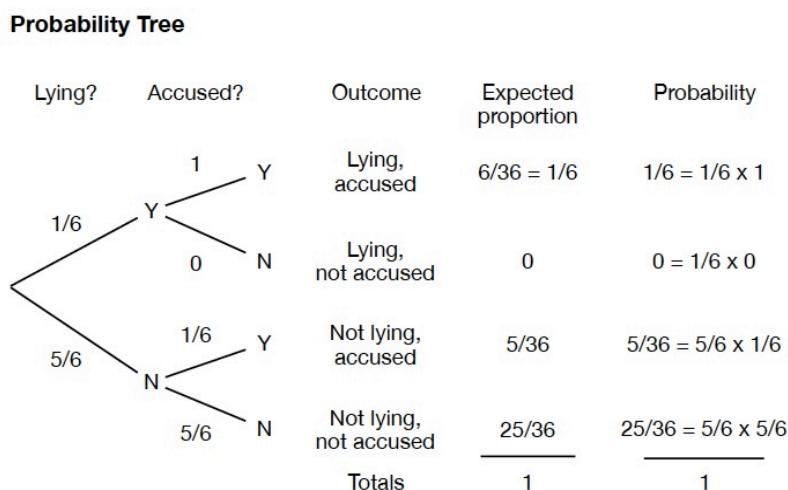


Figure 5. Probability tree obtained from expected frequency tree by examining fractions at each branch. The multiplication and addition rule of probability arise naturally.

As the final stage, the fractions at each branch can be interpreted as ‘probabilities’. The multiplication rule down the branches, and the addition rule across branches, arises naturally by considering the frequencies.

Conclusions

Probability is tricky, and it is fine to think metaphorically. Recent psychology research suggests that people can be helped to solve risk problems using frequency, narrative, and multiple representations. A suggestion for teaching ‘problem-solving’ maths is to adopt our ‘manifesto’ for developing ideas through empirical frequency trees, to expected future events, and finally to probability trees. Thus probability comes at the end of the discussion, after expectation, which I personally believe that is a more fundamental concept. Further discussion and some more examples of this process can be found on the blog post (Spiegelhalter, 2014).

However, this all requires further empirical testing.

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