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THE MONTANA MATH ENTHUSIAST

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Aims and Scope

The Montana Math Enthusiast is the e-journal of the Montana Council of Teachers of Mathematics (MCTM). This biannual e-journal provides its readers with a lively blend of mathematics content, education theory and practice. The journal primarily address the role of teaching and learning at all levels. Thematic issues will focus on grade appropriate mathematics content and innovative pedagogical practices with the hope of stimulating dialogue between pre-service and practicing teachers and university educators. The journal will strive to introduce research based as well as historical and cross-cultural perspectives to mathematics content, its teaching and learning.

THE MONTANA MATH ENTHUSIAST

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Editorial

MCTM is proud to announce the revival of The Montana Math Enthusiast (TMME), as the new e-journal available for its members and all others who enjoy mathematics.The e-journal will consist of 2 thematic issues per year, released in April and October. The first issue addresses the role of Mathematics Education Research and the Standards for best practices in the elementary classroom.

The release of the Principles and Standards for School Mathematics by the National Council of Teachers of Mathematics (NCTM) brought to the forefront the debate of whether research should or should not determine the "validity" of the espoused Standards? Or conversely whether the Standards should influence the research agenda of the mathematics education community? How should university teacher educators address this issue? Should pre-service and practicing teachers blindly accept the Standards as well as the research, or do we cultivate the critical thinking skills that will allow both preparing and practicing teachers to resolve this dilemma. It is often the case that "best" pedagogical approaches suggested by research cannot be practically applied in the daily classroom setting. It is also the case that state driven testing is not completely aligned to either the state or national Standards. Does this mean teachers should just ignore the Standards?

We kick off the e-journal with this first issue containing the idealistic visions of four pre-service elementary teachers. The articles present their attempts to resolve the dilemma of balancing the Standards with the research about the teaching and learning of mathematics with implications for practice in the elementary math classroom.

We hope the articles are of interest and value to you. We welcome submissions from pre-service and practicing teachers, administrators, mathematicians and university educators. Please send an electronic version of your manuscript in APA style to sriramanb@mso.umt.edu

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Teaching Symmetry In the Elementary Curriculum

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Introduction

Symmetry is a fundamental part of geometry, nature, and shapes. It creates patterns that help us organize our world conceptually. We see symmetry every day but often don't realize it. People use concepts of symmetry, including translations, rotations, reflections, and tessellations as part of their careers. Examples of careers that incorporate these ideas are artists, craftspeople, musicians, choreographers, and not to mention, mathematicians.

It is important for students to grasp the concepts of geometry and symmetry while at the elementary level as a means of exposing them to things they see everyday that aren't obviously related to mathematics but have a strong foundation in it. According to the National Council of Teachers of Mathematics grades 3-5 should be able to apply transformations and use symmetry to analyze mathematical situations. This includes predicting and describing the results of sliding, flipping, and turning two-dimensional shapes. They should also be able to describe a motion or a series of motions that will show that two shapes are congruent, and identify and describe line and rotational symmetry in 2 and 3-dimensional shapes and designs. The Montana State Standards for Mathematics are in line with NCTM's standards indicating that by the end of grade 4 students will be able to identify lines of symmetry, congruent and similar shapes, and positional relationships.

Geometry software is a very important tool in developing and testing individual ideas in the classroom. Students can make their own conjectures and "see" them as they are tested, making a connection between what they were thinking and a visual for what is really happening. For a lot of students this is all that is needed to make something "click" inside their heads that tells them "oh, yeah, now I get it".

By showing students that symmetry and it's properties surround us in the world we live in gives them a greater appreciation for the wide-reaching arms of mathematics and how we really do use math throughout our lives beyond the basic adding, subtracting, multiplying, and dividing. We can use dynamic geometry software to let students explore and create the concepts that we are trying to teach. Students will soon be pointing out symmetry and visualizing how shapes move around in our world.

Literature Review

There are several articles in mathematical journals, written by professionals in the field, which expresses the need for teaching symmetry and its properties as a part of the math curriculum at the elementary level. Several articles focus on the different ways of teaching the same concepts and others concentrate more on the tools we have available that link us to technology. All of the articles that I have come across support the understanding that symmetry is all around us and although it doesn't seem to be mathematical it's very roots are buried there.

According to Pumfrey & Beardon (2002), art and math go hand in hand. The connection has been there for a long time as we can trace the inspirations of mathematicians as a "product of the Islamic civilization brought to Europe by the Arab conquests in Spain in the thirteenth century". This relates specifically to tessellations that are a result of rotating, reflecting, and sliding objects in a plane so that there are no gaps or overlaps. Pumfrey & Beardon (2002) sum it up when they state "tessellations are a common feature of decorative art and occur in the natural world all around us."

When the students in Mackrell's (2002) temporary class were given the chance to explore the creation of polygons using geometry software they were creating a lot of abstract patterns that were commonly symmetrical. Mackrell (2002) found that many of the students were concerned with filling in the gaps of their patterns. She concluded that while she was not attempting to teach or emphasize any particular area of mathematics, "ideas regarding size, symmetry, tessellation and representation of 3-D objects were arising spontaneously and, given more time, could have been further developed" (Mackrell, 2002). Mackrell (2002) provides an easy way of introducing the topic of symmetry by taking what students create on their own with no instruction and showing them what they have created and how they did it in mathematical terms.

Gibbon (2001) also likes the idea of using dynamic geometry software as a visual learning tool in the mathematics classroom (Gibbon, 2001). Her first objective was to "first and foremost to enable the students to build on their previous knowledge of rotation and extend their skills in the topic with confidence and enjoyment". She wanted students to be able to do the activity by hand using the computer as an active helper. Her entire article focuses on two lessons in rotations, the second one building on what students learned in the first lesson. The activities presented could easily be expanded on and give rise to other symmetrical concepts such as translations and reflections.

Seidel (1998) applies geometry and symmetry to everyday life for 2nd and 5th graders. Second graders are offered the opportunity to use a program called SuperPaint to create symmetrical flowers and then copy them to make a garden. This lesson gives students a chance to solve their own problems and learn from each other. The fifth graders were introduced to Geometer's Sketchpad and used this program to create symmetrical snowflakes. It requires them to use the concepts of rotation and reflection, and makes it easy to tie in the idea of translations and how all three are related. This article is a prime example of one way in which symmetry is part of our everyday lives. There is a great biographical picture book, called *Snowflake Bentley* that would be a wonderful starting point for this lesson as it explores Wilson Bentley's intense interest in snowflakes and how each one is different.

There is a lot of children's literature available to teach many of the different concepts that math embraces including geometry and symmetry. Harris (1998) focuses a lesson on transformations using the book *A Cloak for the Dreamer* by Aileen Friedman. It contains links to tessellation, tiling, and symmetry and incorporates the importance of predicting, guessing, and thinking of all possible solutions for a problem, ultimately finding the best answer. Using literature as a teaching tool captures the students' attention and engages them in the learning opportunity at hand. It also gives kids a chance to build on their previous knowledge and apply what they know to learn more, coinciding with Mackrell's (2002) idea that these concepts will come out of little instruction and a lot of exploration. Harris (1998) also wants students to be able to create their own designs and then verbalize what they did using mathematical terms giving other students a chance to hear and see the concepts over and over again.

Finally Dolinko (1996) revisits the idea of symmetry in our world once again, this time using flags that represent countries from around the world. She talks about how we can talk about patterns in the arrangement of colors within each individual flag, identify geometric shapes, and compare their sizes. Studying the symmetry in flags from around the world not only gives us an avenue to explore symmetry and tessellation but it also links us to social studies and the similarities among countries when it comes to how they want to be represented by the design of their flags. Using Dolinko's (1996) deas for a math lesson could be incorporated as a wrap up

to symmetrical concepts and a link into a social studies lesson on similarities and differences in countries. It also builds on what students have learned already and gives them a chance to take what they learned using geometry software and putting it on paper in a meaningful way by creating their own flags.

Allowing students to explore and create on their own using the ideas and concepts of geometry and symmetry leads to higher level thinking. Using dynamic geometry software we are giving students a hands-on experience that allows them to visualize and come to an understanding of what is happening in their own minds, and teaching them how to apply that understanding to the concepts of math. We can spark interest in the concepts being explored by using literature to invoke questioning and answering and by discussing what we see in our everyday lives that illustrate symmetry, tessellation, and tiling. Translations, reflections, and rotations give us an idea of how patterns are made and how objects move in space without changing the object itself. The ideas presented in all of the mentioned articles take motion geometry and tessellation to a higher standard and one that should not be passed up in the elementary math curriculum.

Activities

Activity 1

The following activity is based on a lesson in the article by Seidel (1998) mentioned in the literature review. It incorporates using children's literature as an avenue of understanding concepts of symmetry. Geometer's Sketchpad is also used as a manipulative tool allowing students to visually do and see in order to understand the presented topic.

Overview and Purpose: The purpose of this lesson is to use literature as an example of how symmetry occurs in everyday life and to expand on it using a computer program to create snowflakes of our own creation, showing and using the mathematical terms presented.

Objective: Students will be able to apply the terms they learned in a previous lesson, relating to rotations, reflections, and translations, to create symmetrical snowflakes using the program Geometer's Sketchpad.

Materials: One computer for each student that has Sketchpad software available for use, scratch paper for each student, and *Snowflake Bentley* by J.B. Martin.

Instructional Procedure: Start the class out by having students do a quick review of what they already know and what they have learned previously. Answer any questions that may arise and brainstorm with the class things that we see everyday that employs the concepts of symmetry.

Next, read the story Snowflake Bentley to the entire class, focusing on the pictures and the sidebars of information provided beyond the text of the story. By reading this story aloud and using it as a discussion tool many students will become actively involved and want to try some things out on their own. Explain that they will be able to explore creating some snowflakes on their own using Geometer's Sketchpad and give a brief overview of the program, being sure to include instructions on how to rotate, reflect, and translate.

The idea is to let students create snowflakes that are symmetrical but by doing it on their own, learning as they go, and writing down the steps they take to get the figure they constructed. This helps students become more familiar with the mathematical language that they have already been exposed to relating to symmetry.

Assessment: Each student will be responsible for making at least 5 different snowflakes and using the text box tool they will provide a description of their snowflake using mathematical terms to describe how they got their final result. Each student will then share his/her ideas with the rest of the class, exposing everyone to ideas that they may not have thought of on their own.

Standards Addressed with this Activity:
National:
1. Identify and describe line and rotational symmetry in two-dimensional shapes and designs.
2. Predict and describe the results of flipping and turning two-dimensional shapes.
3. Build and draw geometric objects
4. Create and describe mental images of objects, patterns, and paths
5. Describe location and movement using common language and geometric vocabulary

State:
1. Explore properties and transformations of geometric figures
2. Use geometry as a means of describing the physical world

Activity Two

The next activity could be used at the end of a unit on symmetry and tessellation. It will require students to use their knowledge base to create their own designs by applying properties of symmetry. This activity is connected to all of the literature reviews because it examines symmetry and patterns in everyday life and allows for self-exploration in the creation of something that will represent each individual student.

Overview and Purpose: The purpose of this lesson is to connect geometry and symmetry to things involved in our everyday life. Students will actively apply what they have learned during this unit to create a final symmetrical or tessellation project, reflecting the concepts and ideas presented.

Objectives:
1. Students will be able to apply what they have learned to create a flag that is unique and uses many of the types of symmetry they have been exposed to.
2. Students will be able to visualize their flag and transfer that visualization into a meaningful creation on paper.

Materials: Almanac with examples of flags from around the world. Scratch paper for students to get the rough draft of their own flags started. A printed copy of all the things discussed during the unit and a definition for each along with rules that apply symmetry and tessellation, construction paper, markers, crayons, glue, etc. (Students may use any medium they would like to create their flags). Make sure there are computers available for those who would like to use Geometer's Sketchpad as a tool in helping them create their own flags.

Instructional Procedure: Begin class by asking students to look at almanacs in groups of 3-4 and have them write down the shapes, colors, ideas they see that are common and unique to flags from around the world. Students should be able to identify the patterns of the shapes used. After creating a list the class will come together and share their ideas and what they found. Leave these ideas on the board for students to refer to while they create their own flags. Some students

may want to test out the picture in their heads by using Sketchpad and experimenting with that until they are ready to transfer it to paper. Students should be focusing on what shapes they are using, patterns they are making, and using mathematical terms to describe how they created the patterns they have chosen. The flags should be unique and representative of the student.

Assessment: Each student's flag must contain at least 2 types of symmetry. On the back of each student's flag there will be a description of the types of symmetry used and what the colors and shapes represent for that particular student. Students will also write about what they learned from this assignment and in what ways, other than those discussed symmetry, could be used in everyday life.

Standards Addressed with this Activity:
National:
1. Identify and draw a two-dimensional representation of a three-dimensional object
2. Predict and describe the results of sliding, flipping, and turning two-dimensional shapes
3. Identify and describe line and rotational symmetry in two-dimensional shapes and designs
4. Describe a motion or series of motions that will show two shapes are congruent

State:
1. Investigate and predict results of combining, subdividing, and changing shapes
2. Identify lines of symmetry, congruent and similar shapes, and positional relationships
3. Understand and apply geometric properties and relationships
4. Explore properties and transformations of geometrical figures

Conclusions and Implications:

Teaching symmetry in the elementary classroom is very important because it allows children to understand the things they see every day in a different context. It allows children to follow the rules to create their own patterns, and this in turn, allows them to discover what they do and do not like, motivating them to make things better. This area of geometry brings together life and mathematics in a more meaningful way that is not so concrete. Students will often forget while they are studying symmetry and its properties, that they are doing math and it will become a more enriched experience.

I hope that students will gain an understanding of what symmetry and tessellations are and what they mean to us. I would like to see students adding to their previous knowledge base by expanding on what they already know to understand more mathematical properties in greater depth. I think that by applying the properties and types of symmetry to everyday life through literature, snowflakes, and flags students will begin to see math everywhere and as an important part of how we function and see things

Technology can enhance the symmetrical experience by giving students hands-on experience with manipulating objects to create patterns. Students who learn visually will benefit greatly from the use of dynamic geometry software that allows them to "see" what is happening (Gibbon, 2002). For those who learn through auditory using technology gives them a basis for discussions that reflect what they saw. Technology can make math come alive and can be the "aha" for someone who is struggling visualizing ideas in their heads.

The lessons presented in this paper can easily be followed up with additional activities that explore the same concepts and ideas. There are several good picture books that can and should be incorporated into the math curriculum that expands on mathematical concepts (Moore,

2002). A preceding activity that may be done prior to the first activity mentioned would be reading *A Cloak for the Dreamer* which explores the geometric shapes and how they fit together so there are no gaps or overlaps. This is a great way to introduce a unit on symmetry to a class that may or may not have encountered the ideas before. For the second activity it would be easy to focus a future art lesson on tessellations and tiling that will require the students to refer back to what they have learned as a result of the unit on symmetry. It would be very easy to incorporate symmetry into every subject area of the curriculum because it is found everywhere and connects math and our lives in a way other concepts cannot.

Enriching the curriculum for more "talented" kids could be as simple as having them apply more than one transformation of an object to create patterns or prove different ideas. Many students can show that two objects are congruent by moving one object on top of the other, but talented students could delve further and be led to discover that by reflecting an object twice you end up with the original object through a translation. Students who were interested in learning more could have extra computer time using dynamic geometry software to explore and share in a group on their own.

Overall, students of all abilities and levels will benefit from learning about this specific area of geometry often referred to as motion geometry. Math will not seem like math in the traditional sense, but the ideas and concepts behind the activities will give students a real life connection to mathematics and the broad topics that it touches. It is important for students to be interested and excited about math on several different levels because it can relate to their own lives at any point in time. I hope that by studying symmetry students can make a connection between math and the real world with a deeper meaning and understanding, and not just something they "have" to do.

References

Dolinko, L. (1996) . Investigating flags: a multicultural approach. *Teaching Children Mathematics* v3, pp. 186-90

Gibbon, J. (2001) . Some lessons using dynamic geometry software. *Micromath* v17/2. pp. 39-40

Harris, J. (1998). Using literature to investigate transformations. *Teaching Children Mathematics* v4, pp.510-513

Mackrell, K. (2002). Polygons with primary students. *Micromath* v18/3, pp.26-28

Moore, S.D. (2002). Teaching geometry and measurement through literature. *Mathematics Teaching in the Middle School* v8, pp. 78-84

Pumfrey, E. & Beardon, T. (2002). Art and mathematics-mutual enrichment. *Micromath* v18/2, pp. 21-26

Seidel, J. (1998). Symmetry in season." *Teaching Children Mathematics* v4, pp. 244-246

Radius, Diameter, Circumference, π, Geometer's Sketchpad©, and You!

T. Scott Edge

Introduction

I truly believe learning mathematics can be a fun experience for children of all ages. It is up to us, the teachers, to present math as an interesting application. The addition of computers into our ever-changing world has given us an important tool, which can assist us on our journey to teach math in new fun and interesting ways. The Program Geometer's Sketchpad© is one of many mathematic programs we as teachers can use to better help kids understand different geometric concepts. I would like to use Geometer's Sketchpad© in my classroom to help teach my students about circles and the different algorithms that go along with them. Having the students create circles on the program helps them better visual a concept and helps with the learning process in general. It allows them to get away from their desks for a while and explore both computing and mathematics. It actually makes learning math an enjoyable and exciting process.

I want the children to see how an algorithm is made. Creating a circle on sketchpad, finding its center, and then finding the radius will help them better visualize the process. After proving several small algorithms, I hope my students understand algorithms are not just created through trial and error. An algorithm is a systematic procedure used to accomplish an operation. I hope visualizing the algorithmic process helps them better retain the knowledge as well. I also hope the students become more comfortable using this program so we can use it in future geometry problems.

The activities I have chosen to do on Geometer's Sketchpad© addresses several of the National Standards for Mathematics. Out of the 13 mathematics standards for grades 5-8, the lessons I have proposed addresses roughly 9 of them: Standard (1) Mathematics and Problem solving, Standard (2) Mathematics as Communication, Standard (3) Mathematics as Reasoning, Standard (4) Mathematical Connections, Standard (6) Number Systems and Number Theory, Standard (7) Computation and Estimation, Standard (9) Algebra, Standard (12) Geometry, Standard (13) Measurement (NCTM Standards, 1989).

Literature Review

I have such a strong stance on the use of Geometer's sketchpad to help enhance a students learning experience. Therefore, I chose two articles to review that dealt with the integration of the program into the curriculum. The first article I read was A *Classroom Use of the Geometer's Sketchpad© in a Mathematics Pre-service Teacher Education Program* by Medhat H. Rahim. Students can read books until they are blue in the face. Does this mean they understand what they are learning or are they just memorizing what they read or are taught? This article argues that computers can become a valuable learning resource which can help students better understand geometry. Software such as Geometer's Sketchpad© can create an environment where the students explore geometry and make conjectures about different geometric properties. Learning geometry would then be turned into a sequence of part-part, Part-to-whole interrelationships discovery of geometric figures (Rahim, 2002). I agree, and if you look at my lesson plans, I try to follow this approach using geometer's Sketchpad©. This article also includes responses from students and it is clear after reading some of these responses, students would much rather learn geometry using a program such as Sketchpad.

The next article I read concerning the implementation of computer programs in math curriculum was *Do Mathematics with interactive Geometry Software.* By Tingyao Zheng. This article hit's the nail on the head! "Students need many mathematical abilities to solve problems. These abilities include being able to detect a pattern, conjecture, make generalizations and abstractions, perform inductive and deductive reasoning, and make analogies. These skills can be nurtured through solving rich problems, experiencing appropriate classroom discourse, and using technology," (Zheng, 2002). This article simply states that we as teachers need to do more than just generalize mathematics by giving our students algorithms and saying that is they way things are and that is that. We need to give the children an opportunity to prove these theorems and programs like Geometer's Sketchpad© can help them achieve this.

Another article I chose to review is titled *An Angle on Circles. It is an article written by Mary Alice Hatchett* and expresses the importance of including writing in math curriculum. She basically gives an example of a writing activity dealing with circles. It is yet another angle that can be taken to further spice up a math lesson. "In writing, students organize their thoughts and reflect on how to communicate them. Math experiences can thus be "captured" for later recall in making logical connections between strands of mathematics, connections to other content areas, or as a spring board to content extensions," (Hatchett, 1995).

The next two articles I chose to review are about activities dealing with circles done without the use of Geometer's Sketchpad©. The first article is titled *The straight side of sliced circles.* It is an article I found in the Science news. It is an article written about a Hungarian mathematician named Mikolos Laczkovich and his theory that if you cut a circle into a finite number of pieces, you could rearrange them to make a square. I found this article fascinating and would like to challenge some "talented" kids to recreate this on sketchpad.

The next article I have chosen to review about activities dealing with circles done without the use of Geometer's Sketchpad© is titled *The Function Box and Fourth Graders. Squares, Cubes, Circles*. It is an activity having the kids derive rules for figuring the circumference and area of a circle using pairs of measurements. They do this finding various patterns. This is yet another activity that can be easily recreated using the program Geometer's Sketchpad©.

The final two articles I chose to review deal with the ever expanding π. One of the articles, *What is pi, and how did it originate?,* defines pi and gives a brief history of the origin of this seemingly mystical number. I found it amazing that the importance of pi has been recognized for at least 4000 years (Bogart, 1999). I feel this would be an interesting article to present to the kids before they begin their journey on Sketchpad. A possible report could be written about the history of pi and this could be a valuable reference the kids can turn to if needed.

Another article I found interesting and would like to present to my class before they begin their circles quest on Sketchpad is *Finding the Value of Pi.* Although fairly short, this article is rather interesting. It gives a brief summary as to the history behind the notion of Pi (π) and how it was conceived. It also includes a chronological chart of the values of Pi through time beginning with the Babylonians in the year 2000 B.C. and ending with the CDC 6600 in 1967 (Math Forum, 2003). The value of Pi has remained close for nearly 4000 years. It also shows how the notion of pi was discovered and why it is important. It goes on to state:" the significance of this discovery is clear: Circles are everywhere- in the sun, the moon, the pupils of our eyes, the most basic religious rituals and the earliest man-made structures," (Math Forum, 2003). I felt this article was significant because it is a brief summary proving several different proofs of the

formula π=C/D. I want the students to witness proofs before they begin proving algorithms themselves so they can see the significance.

Because the students will work together in groups, the following lessons cover standard 2: Mathematics as Communication. The students will also have an opportunity to explore the World Wide Web and research how algorithms are created. Specifically, I will ask the students to try and find a proof of the algorithms D=2r, C=2πr, and π=C/D. Once they have done some research, students will write a small report about what they found. The students are asked to make a connection between the algorithm and the activity on sketchpad as a way to prove the algorithm D=2πr therefore, this satisfies standard 1: Mathematics as Problem Solving. These are a recreation of activities presented in the article *Circles*. They are doing simple computations and estimations throughout this lesson so standard 7 is satisfied. They measure in this activity, and this is a geometric exercise, so standards 13: Measurement, and 12: Geometry are met respectively.

All in all, this lesson covers most of the mathematical national standards. Using Geometer's Sketchpad© enhances the learning process by giving the kids the opportunity to prove these algorithms visually on a computer. The articles *Do mathematics with Interactive Geometry Software* and *A Classroom Use of the Geometer's Sketchpad in a mathematics Pre-Service Teacher Education Program* stress the importance of this. Teachers can only "preach" so much. We need to get the kids more involved in the proof of mathematical algorithms. The visual stimulation, along with the step-by-step process of proving algorithms, can only help facilitate the learning process. As a future teacher, I am a huge advocate of Howard Gardner's theory of Multiple Intelligence. He says that not all children learn in the same capacity. Programs like Geometer's Sketchpad© can help those Visual Kinesthetic learners that may have problems with math otherwise.

Activities

Activity 1

Objectives

The students will use Geometer Sketchpad© to construct a circle and find its center. When they find the center, they will find the radius. They will then make a conjecture about the diameter, and prove the algorithm D=2r.

Materials:

- ✓ Geometer's Sketchpad©
- ✓ Computer

Go For it!!

- Selecting the point icon, plot any two points (A and B). Next select these two points and go to construct, and click on a segment. Now, double click A and then select the segment. Now, go to construct again and create a circle using the center + radius. A is the center of the circle.
- Now, select the segment and go to measure. Click on length and the two points will now have labels and the measurement should be displayed near the circle.

- We are now going to bisect the circle in order to get a diameter. Select the point on the circle and the center point. Go to construct and select line. This bisects the circle. Create a point on

the circle that includes the circle and the new line and label it C. now click on C and B. Go to Measure and click on distance. This is the measurement of our diameter.

- Next, go to measure and select calculate. Type in 2 and then *(multiply) and select the measurement near the circle. This answer should be the measure of your diameter (D=2r).
- Let's prove this. Let's make another segment connecting the center of our circle and another random point on the circle. Create a segment between the new point and the center. Go to Measure and click on length. Now add these two segments together. The sum of the two segments should equal the diameter.
- Create a new circle and try this again. Does it work? Did we prove the algorithm D=2r?

Activity 2

Objectives:

The students will use Geometer's Sketchpad© to prove the circumference of a circle equals two times pi times the radius ($C=2\pi r$) and, $\pi=C/2r$.

Materials:

- ✓ Computer
- ✓ Geometer's Sketchpad©

Go for it!!

- Plot any two points and label them A and B. Make a segment connecting these two points. We will make B the center of our circle. Double click on B and then select point B and the segment. Next, go to construct and create a circle using center + radius.

- Now we have our radius! First, click on the radius and select measure distance. This will tell us in cm how long our radius is.

- What else do we need? We need to know our circumference. First we will have the computer measure it for us. Select the circle (should be highlighted blue) and go to measure and select circumference. Now we know Two parts to our algorithm…our radius and our circumference. $\pi=C/2r$

- Now we'll calculate Pi. Go to measure and select calculate. Now click on the circumference measurement and divide it by 2 times the radius measurement. What number did you get? Is it around 3.14? This is Pi(π).

- Let us now prove the formula $C=2\pi r$! Go to measure and select calculate. Now type in 2 times your Pi times your radius measurement. Does it equal what the computer measured your circles circumference to be? Is it close?

Repeat this process again with another circle. Did you get the same results? Is this enough information to prove these algorithms?

Conclusions and Implications

In conclusion, there are wonderful tools we as teachers must utilize in our classroom in order to help our students better understand the concept of algorithms and how they are derived. Geometer's Sketchpad© is a terrific and fun way to get the children excited about math! Having students create circles on the program is one of many ways, which can help them visualize a concept and most definitely help the learning process in general. It allows them to get away from their desks for a while and explore both computing and mathematics. It actually makes learning math an enjoyable and exciting process.

After proving several small algorithms, I hope my students understand algorithms are not just created through trial and error. An algorithm is a systematic procedure used to accomplish an operation. I hope visualizing the algorithmic process helps them better retain the knowledge as well. I also hope the students become more comfortable using this program so we can use it while exploring future geometry problems.

References

Bogart, S. (1999). What is Pi and How Did it Originate? Retrieved April 28, 2003 from http://www.sciam.com

http://mathforum.org/isacc/problems/pi1.html . Finding the Value of Pi. (1994-2003).

Hatchett, M. A. (1995). An Angle on Circles [Electronic version]. *Teaching PreK-8*, 25, 58-59.

NCTM (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: Author.

Rahim, M. H.(2002). A Classroom Use of Geometer's Sketchpad in a Mathematics Pre-Service Teacher Education program [Electronic version]. Retrieved from http://math.unipa.it/~grim/Jrahaim

Sulzer, James S. (1998). The Function Box and Fourth Graders. Squares, Cubes, and Circles. *Science News*, vol. 4, 8, 442-446.

Sulzer, J. S. (1989).The Straight Side of Sliced Circles. *Science News*, vol. 136, 2, 31.

Zheng, T. (2002). Do Mathematics with Interactive Geometry Software. *Mathematics Teacher*, 95(7), 492-497.

Exploring Perimeter and Area with 4th Graders

Amber Lieberg Winkler

Introduction

I am going to teach basic introductory geometry skills to 4th graders using Geometry's Sketchpad. At this age, children are only beginning to learn about geometry in their math classes, and I would like for the students to understand these basic concepts using technology. This problem is so important for the students to learn early, and learn correctly. These skills will involve finding the area and perimeter of regular polygons, basic skills using Geometry's Sketchpad, and activities that will apply these introductory concepts; directly correlating within the national geometry standards of mathematics. Children use geometry everyday, even when they don't realize they are using them (actually we all do). But the most important concept for the children is to recognize shapes and structures. If they can identify the use of geometry, then it will automatically trigger the children to think; think in a mathematical state of mind. Thinking mathematically creates questions and also answers questions.

With Geometry's Sketchpad, children can learn these skills much earlier because they can construct, observe, calculate and assess everything independently. Fortunately this contemporary technology is becoming more available in elementary classrooms. These concepts are most likely new to the students at the lower levels; therefore, teaching basic Geometry's Sketchpad skills to 4th graders will be a great introduction to beginning geometry. According to the national geometry standards for fourth graders, children will be able to analyze characteristic and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships. In these activities, I will explain the basic concepts of perimeters and areas of polygons with and without Geometry's Sketchpad. I will then give specific instructions on how to use the Sketchpad, next I will transition into three fun activities where the students can use what they've learned and understand how these skills can be associated with their own lives. These activities will include the national standards by having the children specify locations and describe spatial relationships using coordinate on Geometer's Sketchpad and other representational systems such as the geoboards. This will not only give students a fundamental introduction to geometry, but will provide them with knowledge for future application. Most importantly, students will become aware how mathematics and specifically geometry can be used in their everyday lives. Again incorporating these national standards by using visualization, spatial reasoning, geometric modeling to solve problems and apply the problems to analyze mathematical situations.

Literature Review

Using the geoboard is a great way to introduce the perimeters and areas of polygons to 4th graders. Of course, the children will already have been introduced to the definition of polygons in previous lessons. The geoboard can be used to teach geometric and algebraic concepts informally. A student can find the area of a polygon or perimeter using the each square that can be counted. With geoboard exercises, the students can visually calculate and grasp the basic concepts of straightforward and uncomplicated geometry. The idea of using this technique to teach the students about the perimeters and areas of objects is a great introduction if only whole units were involved because it is hands-on and practical. For more advanced students, use basic fraction units to enhance their learning experience.

This connection is simple because it basically gives me a formula of how to introduce a form of geometry to my students in the classroom for fourth grade students. This is beneficial because there are many facets to geometry, and this is only one of those facets. But I believe that it is a demonstration that the children will understand and learn fairly quickly from. Using the geoboard, the children can work with the rubber bands with their hands; it is not abstract to them. Since this is an age in which children don't know too much about geometry, this will be a great introduction.

Finding the differences and similarities between area and perimeter using Geometer's Sketchpad is the beginning phase using a program for geometry. Geometry is integrally related to social studies and architecture, so if children can understand how geometry is used in everyday life, then we will have accomplished something with the kids. The purpose of Geometer's Sketchpad is to draw figures and calculate area and other measurements quickly and efficiently. Many conjectures and explorations using the computer as a vehicle will only enhance the essential skills for learning the material. Woodward and Byrd suggest activities that should precede the method presented in this article.

1) In early grades activities should involve covering a region with mall sub-regions and determining its area by counting these sub-regions.
2) Using graph paper, dot paper, and geoboard may reinforce the concept of area while differentiating it from the concept of perimeter.

The importance of Geometer's Sketchpad is for students to learn as early as possible. Yet it is also important to use the instruments that are everyday resources such as the geoboard, graph paper, and dots. These are resources that can be used later in their academic careers, however. They can use Geometer's Sketchpad to begin their knowledge base and further continue their education of geometry skills.

Teaching children geometry can be a difficult task for many children. The key is to teaching them with many different approaches. One of those approaches being with square tiles. Similar to geoboards, the square tiles will allow the students to have firsthand experience with geometric shapes physically. Using these shapes, they will be able to determine the perimeters and areas of perimeters of the shapes that they make with the square tiles by counting both the sides and the shapes. This will make the concept of learning about perimeters and areas more visual to the students before they use 'abstract' technology.

Using square tiles allows the children to have a brief introduction to geometry for those individual students who don't feel as comfortable with the computer. Here, the students will learn the same skills, but in a different approach. Teachers who use alternative methods in presenting the information will allow more students to learn.

The use of technology is increasingly important and essential, especially the use of it in the schools. There is decreased emphasis on memorizing geometric vocabulary, facts, and relationships. There are several geometric investigations that were developed in the spirit of the curriculum standards and are appropriate for use at advanced elementary and middle school levels, as well as with older students. They encourage age exploration, creativity, and discovery. Triangle explorations and constructions using geometry's sketchpad in great detail can also be valuable. Technology is extremely important in today's society, so we may as well have children learn as early as possible.

For a weekly geometry lesson plan, teaching geometry's sketchpad is crucial. When I was looking for articles, I found articles that would have helped me as a child in the fourth grade. As a child, I hated traditional handouts, chalkboards, and textbooks. I just wanted to find

something that would motivate the children to learn about geometry, make it fun, and make it real using technology. Anyway, that's why assignments can be extremely effective on the children, so they can not only learn, but also remember and use the information. My plans for assignments are, to intrigue, teach, and have fun in the process and hope the children can take something from the lessons I have taught them.

Mathematics teachers and their students may utilize dynamic geometry software such as Geometer's Sketchpad in designing innovative approaches to proof and justification in geometry. Such software may encourage students to conduct a meaningful justification of ideas, create simple geometric figures and explore the figures' relationships. The software may also promote a better understanding of geometric problems.

Using software will allow students to begin their studies in geometry as well as technology at an early age. Using Geometer's Sketchpad is fun and easy to understand. Children can use this program for calculations, shape making, translations, relationships, etc. There are endless uses to these activities. Students will have a base of knowledge that will also contribute to other classes in their futures.

Applications

The students have already learned about all different kinds of polygons, and in these lesson plans, they will get to use a variety of resources to see how these polygons are used in mathematics. The children will learn using the Multiple Intelligences (Howard Gardner) so that each child has the opportunity to grasp these concepts through listening, watching, analyzing, and of course, "doing" the activities.

Introduction to geometry and review of polygons must come first. We will bring out our shapes and work with them a little, discussing their shape, their sides, their lengths, etc. I will also show pictures of how shapes are used in our everyday world (possibly a slide show). We will also bring out the geoboards and rubber bands to explore the relationship between the area and perimeter of the shapes, such as a square and rectangle. Possibly move into even triangles to challenge them, but it might be too difficult which is okay. Next we will also need to bring about square tiles so that the children can find and determine the relationships between the shapes of polygons and their perimeter and area.

We will move to the computers to have a completely run-through of Geometer's Sketchpad. The students will learn how to use all of the tools as well as measure the distances without having to measure them for themselves. I will let them explore the program, and hopefully show them how to use the technology to show them how to use the computer to perform the same skills that were previously learned.

<u>First Application</u>

Objective: To have children better understand the areas and perimeters of polygons using geoboards, shapes, and square tiles. It is a good idea to spread these different activities out for two or three days. Children will learn how to make shapes on a geoboard (explained orally). Each child will receive rubber bands to make their shapes from their worksheet that is handed out. They are then to figure out the area and perimeters of the objects by counting the squares or the dots of the geoboard. The same process is completed with the square tiles. They will be able to visualize the relationship between the shapes that they construct, and their area and perimeter.

Rubber bands and geoboards, or square tiles, will need to be handed out to each student. Instructions will be given to each child to keep their hands to themselves, as well as their rubber bands. Each child will also receive a handout of different shapes, of different measurements. We will review how to look at each shape, and review vocabulary.

Activities will follow. We will begin the first couple problems as a group. I will explain that each peg or square tile represents one unit, and if we count the pegs or sides around the shape, then we will be able to find the perimeter. The children will be able to do even more complicated shapes once they get the hang of it such as other polygons with many sides. Let the children work in pairs or groups to discuss and compare their findings. Let the more advanced students have more difficult measurements to challenge them. Then we will discuss the area of the shapes. I will explain that if we count all of the squares inside the shape, we will get the area of the shape. Once again, the students will be able to compare with the rest of their peers to complete their worksheet.

Discussion of what was learned is extremely important, as is taking time to answer questions. These activities tie into the national standards as well. Students will be able to visually analyze the properties of two-dimensional geometric shapes and develop mathematical arguments about the geometric relationships (perimeter and area). They will specify locations and describe spatial relationships using coordinate geometry and other representational systems (shapes, geoboard, and square tiles). Students will use visualization of our activities, form spatial reasoning, and geometric modeling to solve problems (through discussion and group work on worksheets).

Second Application

Objective: To have children learn fundamental skills about Geometer's Sketchpad through a series of exercises. These exercises will take the students through step by step how to create shapes, get their distances, etc. They will only have so much information at this point, but at least they will have some kind of experience using the computer, using geometry, and applying both concepts with the sketchpad.

This activity will be a series of exercises that are listed on a handout. The handout will mainly consist of triangles, squares, circles, and other basic shapes that the children have learned about.

- Each child should have his/her own computer, or sharing with one or two other students.
- Mostly, the assignment will consist of using the left tool bar so that they won't have to continually memorize which file to go under.
- After explaining what each tool demonstrates, such as the line segments, circle, text, and pointer tools, they will get to explore the shapes they make through these different tools.
- Once everyone has gotten a good chance to play around, I will show students how to construct segments from the top tool bar, make constructions, and show them how to use the color… because children love to make colorful objects.

The students now get the opportunity to make something, anything they want using the tools they leaned about today. This will allow the children to be creative, and see how geometry is used. Maybe they can also come to the conclusion that technology is important and will play a very important role in their own lives. Let the children use their imaginations and have fun making their own geometric creations.

Students will then explore real-life geometry patterns on Geometer's Sketchpad to create objects that they themselves are familiar with. Children tend to take information in better and more efficiently if what they are learning can apply in some way to their own lives, or relate to something that is real. In this activity, I have chosen to real-life, scenarios in which a person would use geometry to construct something or help them with a problem. The children don't have to know or understand each concept, but it will give them the opportunity to see first hand how geometry can be used.

This activity is going to be fun for the children. They are going to construct pinwheels with the skills they learned from the previous lessons, and from my instructions. This activity called Dueling Pinwheels comes from Cynthia Lanius at http://math.rice.edu/~lanius/misc/rotat.html. This exercise will be fun for the children because most children like to play with pinwheels.

1) We will construct a circle, and two radii and a chord forming an acute triangle
2) We will then select the three vertices of the triangle and construct the polygon interior.
3) We will transform the triangle with my instructions and rotate by 60 degrees. (The children will not understand why we do this but they will get some experience in doing geometry sketches.)
4) We will then add color to each of the triangles, and we can then animate the pinwheel, which in turn shows the pinwheel spinning.

This activity would most likely take most of the time, but if I need a follow up activity, I could briefly show them or explain to them how we can measure the circumference of a broken plate that was found at an archeological dig. They probably won't understand the concept of this, but will be great information for them to take in. This will only add to the reasons for learning geometry.

These activities also require time, but cover the NCTM standards for mathematics. The children will be analyzing the properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships (through exploration and the pinwheels). They will specify locations and describe spatial relationships using coordinate geometry and other representational systems (Geometry's Sketchpad). Last, children will apply transformations and use symmetry to analyze mathematical situations.

Conclusions and Implications

The articles and lesson plans that were used in this project are wonderful for students of this age group who are learning the fundamental skills of geometry with technology. Using technology at an early age is extremely important for students who are growing up in a 'world of technology'. It is great because students can begin more difficult concepts at an earlier age with this technology. Geometry's Sketchpad is just one technology program that students will not only learn beginning to more advanced geometry skills, but they will benefit from using the program at their own leisure. Technology can make learning fun.

Accommodating for each student's needs requires extra fine-tuning of the lesson plans. For student's whom are above the average, lessons take a lot of time and preparation to organize. I was very surprised at all of the work and organization involve in this process. Teachers today need to accommodate for every student; therefore, having alternative lesson plans for those who need to be slightly more challenged with their studies is required. Adding in extra activities and using more difficult scenarios, can alter these needs. One example might be with the geoboards and the square tiles, more sophisticated areas and perimeters could be used. With the geoboard,

shapes with triangular edges would create for a more difficult problem to solve the area and perimeter. Geometer's Sketchpad can be as simple or as difficult and complicated as you want to make it. If there are gifted children who need more of a challenge, extend the activities by adding in other translations, calculations, and shapes.

The students will get a lot out of these assignments. I really think that the students will learn significant skills, and have fun learning. Geometry can be really monotonous and boring, or it can be challenging and enjoyable for children. I think that when applying the curriculum to real-life experiences, learning is a much more interesting as well. Children like challenges, they like to move around and actively participate in many activities. Opting to incorporate lesson plans that students will really enjoy and partake in is so important to engage all students. Most of these lesson plans require a great deal of preparation, time, and even money (computers and program software). But when it comes down to it, it's imperative to love what you're doing and strive to meet the needs of each and every student. I know that I would never love what I was doing if the children weren't learning, having fun, and being challenged. That's what teaching is all about!

References

Discovering perimeter & area with logo. September 1988. Arithmetic Teacher 36. pp.18.

Galindo, E. January 1998. *Assessing justification and proof in geometry classes taught using dynamic software.* Mathematics Teacher 91 pp.76-77.

Lanius, C. (2002). *Duelling Pinwheels.* http://math.rice.edu/~lanius/misc/rotat.html.

Olson, M., J. December 2001. *Responses to the tiles and sides problems*. Teaching Children Mathematics 8 pp.219-223.

Smith, L. R. May 1990. *Areas and perimeters of geoboard polygons*. Mathematic Teacher 83 pp. 392-397.

Stone, M. November 1994. *Teaching relationships between area and perimeter with The Geometer's Sketchpad.* The Mathematical Teacher 87 pp. 590-594.

Taylor, L. November 1992. *Teaching mathematics with technology.* Arithmetic Teacher. pp. 187-191.

The National Council of Teachers of Mathematics. (1989). *Curriculum and evaluations standards for school mathematics*. Reston, Virginia: Library of Congress Cataloging-in-Publication Data.

Weeks, A., Howell, M., McMullin, L. January 2001. *Technology tips.* Mathematics Teacher 94. pp. 66-69.

Understanding Polygons and Polyhedrons Using Flexagons

Aaron Tekulve

Introduction

The goal of this paper is to help students understand simple polygons and simple polyhedrons. First the project within this paper involves having students look at polygons. Though much of this information should have been learned in the fourth grade it is still important to review this material. Having the students define certain shapes illustrates their true understanding of the subject. The second part to the project within this paper is to use the student's knowledge of polygons and build polyhedrons. In this paper the students will only have to concern themselves with squares and equilateral triangles. Here they will have to build constructions known as "flexagons." A flexagon in short is a polyhedron constructed by using tag or poster board and cutting out patterns with wings and rubber banding them together. The third part to the project within this paper is to take all of the polygons and flexagons/polyhedrons and make them into a mobile. The mobile works as a nice way to end the project by displaying the knowledge these students have attained in a creative manner.

The Geometry Standards for Grades 3-5 from the NCTM Principles and Standards for School Mathematics web site states; students in grades three through five should be expected to know how to identify, compare, and analyze attributes of two- and three-dimensional shapes and develop a vocabulary to describe the attributes of these figures. Students should also be able to classify these assorted shapes in accordance to their properties and build a distinction between the various classes of shapes. Not only should students understand the vast attributes of two- and three-dimensional shapes they should also be able to create them. Students should be able to create these shapes by drawing them as well as building them. Also students should understand that three-dimensional objects are constructed from two-dimensional shapes.

Montana Mathematical Standards for Elementary Students

Mathematical Standard:

- 3.1.5 Identify and classify triangles, quadrilaterals, pentagons, hexagons, and octagons according to their attributes.
- 3.1.10 Identify and describe solid figures-cubes, prisms, pyramids, cylinders, cones, and spheres.
- 4.2.1 Determine the perimeter, area, and volume of shapes and solids by counting segments, square units, or cubic units.

Mathematical Objectives:

- To physically investigate and analyze the attributes of geometric 3-dimensional figures.
- Students will recognize and be able to classify various polygons and polyhedrons.
- Students will understand how to measure perimeter, area, and volume of shapes.

Both the NCTM and the Montana state math standards and principles run coherently with each other. Though the Montana state (school district 6) math standards and principles are not as detailed as the NCTM's standards and principles they both cover the same key points. The concepts in this paper run parallel with the standards given by these two sources for educating third through fifth graders in geometry.

I believe the project within this paper demonstrates a logical and creative way of learning polygons and polyhedrons. Not only does this project teach the students about polygons and polyhedrons but it gives them a hands-on-approach with the subject. This project serves as a visual, verbal, and a kinesthetic form of learning while including and allowing multiple intelligence learning styles. Not only can this project apply to students of various learning styles it also allows the student to be creative and use their imaginations making math more appealing and fun.

Before I discuss the various sources from where I gathered information I would like to preface it by saying that one of the key factors to the project discussed in this paper is the idea of it working with visual, verbal, and a kinesthetic forms of learning while allowing multiple intelligence learning styles. Since this aspect is highly valued in this paper I have chosen certain pieces of literature discussing learning styles in the classroom.

Literature Review

Being able to understand and work with various learning styles students have in a classroom is a very valuable tool. *Teaching Math Effectively to Elementary Students* is a journal primarily discussing various learning types. It discusses the stress many students have in the classroom setting. Many elementary students feel ashamed, guilty, and depressed when they are not able to grasp a concept conveyed to them by a teacher. The problem usually lies in the method the teacher is using to teach the subject. This article addresses the basic forms of learning, which are visual, verbal, and kinesthetic. Once illustrated how frustrations in the classroom can occur this journal discusses an assortment of ways to integrate these different learning styles in the classroom. I found this journal to be a very valid source to gather information from because of its discussion on different learning styles. One of the greatest aspects to the project in this paper is its ability to incorporate each of the learning styles into one session of learning. A very productive style of teaching is to have the students physically create the objects they are studying, though it is important to first give them a strong vocabulary and understanding of what makes up the shapes they are creating.

Oberdorf & Cox (1999) presented many valuable pieces of information on the importance of teaching geometry. One of the main themes discussed in this article was the idea of misconceptions in geometry made by elementary students. Some of the reasons cited dealt with limited experience, false information from parents, and the lack of exposure to the proper vocabulary. This article then explains ways to properly teach geometrical concepts to elementary kids. The idea given that I found most useful dealt with square, rectangles, trapezoids, triangles, and three-dimensional objects. Given this project works primarily with polygons and polyhedrons, the information in this journal became very effective. Many of the ideas on how to identify these shapes dealt with constructing them or finding them in the classroom. This allows the students to internalize what they are being taught and see how it links to the real world. "Geometry helps us represent and describe in an orderly manner the world in which we live" (NCTM, 1989, 48). In the end the goal is to have a student look a cereal box and

say this is a rectangular prism made up of quadrilaterals, or to be more specific rectangles. This connection to the physical world is very important because students are able to see the rationale behind studying this subject.

Battista (2002) shared a great amount of information dealing with teaching geometry using computer software to ways of educating elementary students in geometry and having them internalize what they are being taught. In this article the van Hiele theory is discussed quite thoroughly. The van Hiele theory deals with a students moving through several qualitatively different levels of geometric thinking. These levels consist of recognizing shapes, understanding properties of shapes, being able to classify shapes with their various definitions, and being able to prove certain shapes by using proofs. This article also deals with the idea of kinesthetic learning. One example deals with a hinged-rod parallelogram. By moving the hinged-rod parallelogram around a student is able to see the various equilateral parallelograms that can be made. Also there is much discussion in this article about the use of technology in a classroom using a program similar to Geometer Sketchpad. This article gave a detailed discussion on how students can visual and create various objects. Being able to create these objects with this computer program also gives the students a chance to see the different possibilities of certain shapes such as trapezoids, parallelograms, rhombuses, etc. All of this is very powerful when teaching geometry because of the direct connection students can play in their own learning process. Plus students are given the chance to interact with geometry on a new level making geometry more appealing while still educating the students. This article served as a great tool because of the many ideas focusing on the teaching of geometry. Various ways of teaching geometrical concepts is useful to a teacher as well, for it allows them one more way to educate their students without having to rely purely on lecture.

Koester (2003) described many interesting ways of teaching third to fifth graders about geometric figures, primarily polyhedrons. The Van Hiele Model of Geometric Thinking is used in this article as well to stress the development of geometric thinking is more dependent on instruction than on age. This article thoroughly focuses on using various activities to teach geometry and have the students understand geometry. One of the activities used in this article dealt with the connection of straws. One of the major concepts behind the activity is the idea these polyhedrons are being constructed from polygons. This article was quite detailed in its overall explanations of how polyhedrons are constructed and accompanying ideas on how to teach these figures to elementary students. Also this article covered all of the various figures from cylinders to prisms giving a thorough background on how they are constructed and where they can be seen in real life. The usefulness of this article is quite understandable for this paper stresses the understanding of geometric figures and the need for the ability to think abstractly about geometric figures. This article also gives many encouraging thoughts on how to teach geometry to elementary students and the need to teach the correct application of geometry.

Pickreign and Capps (2000) presented very interesting explanations on how elementary school text books do not always line up with the National Council of Teachers of Mathematics' (NCTM) standards and principles for what elementary students should be learning. This article thoroughly stressed how children learn geometry and the need for the abilities to comprehend geometric concepts. This article also discussed the importance of geometry and how text books emphasize too much of one area while not enough of another. Also there is discussion on how geometry is presented more towards the end of most text books. This presents a problem for not all elementary classes are able to move through an entire text book in a single school year. From all of this though comes the idea of how geometry is taught and the need for certain aspects of

geometry to be included in every classroom. These aspects of geometry are ones the NCTM says are in the teaching of geometry but are not found in the text book. One of those aspects is the idea of spatial geometry and how it is not discussed enough in elementary text books. There is also discussion on methods of how to teach and reinforce these ideas that may not be presented in every text book as well. This article serves as great reinforcement for the idea of teaching spatial geometry. The many concepts and ideas behind teaching geometry worked excellently for the need of projects similar to the one's discussed in this paper. Teacher's must remember not all that is needed to be taught is found in a text book and it is important to stay knowledgeable on what the NCTM has established as important educational values.

Being able to identify various shapes is one of the most primary goals a teacher hopes their students will attain while studying geometry. Swarthout's (2003) article deals with the recognition of shapes in a picture, geoboard, on dot paper, etc. The project discussed in this article deals with having the students in a class create shapes using dot paper, geoboards, etc. to create the shapes they see. Then the students are to dissect their shapes and see what their shapes are made up of. It also encourages students to use different colors to identify the shapes they find within the shape they have created. This allows clear distinction between the new shapes making up the original figure, while adding an artistic element to the project encouraging the students even more too fully participate. While the students are working on identifying shapes they are also applying the knowledge they have previously learned making this project very useful in the full understanding of the makeup of various shapes. It is very important for students to understand and recognize what makes up a hexagon or a trapezoid. This article illustrates some very key elements to geometry which tie in very well with this paper. The most prevalent of those key elements to geometry is the ability to identify and understand the make up of geometric figures. This concept served as a great emphasis on the thorough education of geometry.

Kaufmann et al. (2000) stress ideas mainly concerning spatial abilities and their importance. First this article discusses the value of understanding the five components relating to the term spatial abilities. Those are: spatial perception, spatial visualization, mental rotations, spatial relations and spatial orientation. These five components to the term spatial abilities are very important in the education of geometry and the development of spatial skills. Though this article focuses much of its attention on 3D technology and virtual reality the overall concern for the understanding of spatial figures is always present. This article gives a great deal of detailed information on the use of technology in geometry. Programs such as Geometers' Sketchpad are a perfect example of the use of technology while studying geometry. Though Geometers' Sketchpad works with two-dimensional shapes only there are programs dealing with spatial shapes as well. The integration of technology in education is a great tool for students are always willing to "play" on the computer. Plus these computer programs allow a great deal of imagination when reshaping the figures and allow a student to see the various kinds of pyramids there are and their possibilities.

Overall these articles serve as a great background for the information discussed in this paper. The articles concern all the major areas this paper contains, such as learning techniques, learning geometric definitions, and understanding two-dimensional and three-dimensional geometry.

The following are three lesson plans that can be used in any classroom to help students understand polygons and polyhedrons.

Lessons that balance research with standards

Lesson 1

1. Assessing knowledge of plane figures.

A. Define plane figures.

1. What is a quadrilateral? Ask students to describe and draw a four sided shape. Many students will draw rectangles. Ask them to differentiate between a square and a rectangle. Also ask students to define a rhombus and a trapezoid. Make sure to show the differences and similarities between a square, a rectangle, a rhombus, and a trapezoid.

2. Define triangles. What is the basic component of a triangle? Name triangles by their sides and by their angles. Reinforce the idea that an equilateral triangle has three equal sides and all acute angles. This is review information as fifth grade students should have developed this information in fourth grade. (3.1.1: Students will identify, compare, define and classify polygons and circles.)

3. Review and reinforce the definition of additional plane figures.
   a. pentagon
   b. octagon
   c. hexagon
4. Stress and review the difference between regular and irregular polygons.
a. Remember that all regular polygons are equilateral and equiangular.

B. Have students carefully measure, cut out, and label 8 shapes using colored tag board.

1. These shapes should be as large or larger then a three inch by three inch square.

Lesson 2

A. Review and introduce polyhedrons by their sides and bases.

1. Polyhedrons discussed should be: Triangular prisms, triangular pyramids, cubes, and rectangular prisms.

2. Review and recognize the differences between prisms and pyramids.

Using models of both a prism and a pyramid have each student be able to differentiate between the two.

Review and discuss naming polyhedrons as described in the fifth grade text.

3. Have each student be able to recognize the differences between polyhedrons. i.e. how a cube differs from a triangular pyramid.

Begin cutting flexagon sides.

Form groups with the students and pass out multi-colored tag/poster board.
Pass out flexagon patterns to each group. Have them trace the patterns on to their tag/poster board and have them cut them out.
Stress the accuracy in cutting out the flexagons and how the corner wedges must be cut out correctly.

Assign cutting of simple shapes as homework.

a. Those shapes should consist of triangles and squares.
b. Each student should have a decent amount of both square and triangular flexagons.

Lesson 3

A. Review polyhedron shapes.

1. Have the students be able to clearly point out the various polyhedron shapes discussed in previous lessons.

B. Model flexagon construction.

1. Demonstrate to the class the basic flexagon constructions. i.e. cubes, rectangular prism, triangular prism, and a triangular pyramid.

a. Show correct method of constructing a flexagon.

C. Have the students create and label their constructions.

1. Each student should be able to define their construction and explain its key features.

D. Creating a mobile with our polygons and polyhedrons.

1. Using a whole punch each student will need to make a whole in each of their polygons and polyhedrons.

2. Then each student should cut a piece of string for each of their polygons and polyhedrons, having each piece of string vary in size by a little.

3. Then take the coat hanger and tie the other end of the string to it.

4. Now each student should have a completed mobile and with the polygons and polyhedrons that they have created.

a. Display the mobiles by hanging them over the desk of the student to whom they belong (see Figure 1)

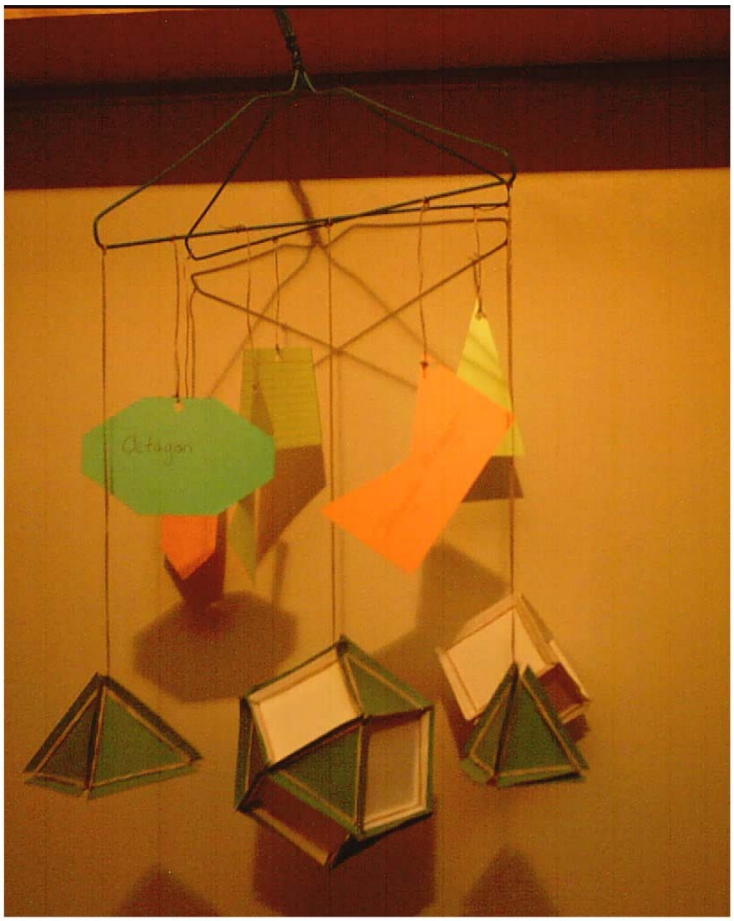

Figure 1. Example of a Mobile with student created Flexagons

The lessons in this project thoroughly address the Montana state and national math standards. The standards from both sources state students in grades three through five need to know how to identify, compare, and analyze attributes of two- and three-dimensional shapes and develop a vocabulary to describe the attributes of these figures. Students also need to be able to classify these assorted shapes in accordance to their properties and build a distinction between the various classes of shapes. Also students need to understand the vast attributes of two- and three-dimensional shapes and be also to create them. Students should be able to create these shapes by drawing them as well as building them. Also students should understand three-dimensional objects are constructed from two-dimensional shapes.

The project displayed in this paper illustrates an excellent way to educate elementary students in geometry. This project also has the ability to speak to those students who are considered the "talented" or most exceptional in a classroom. The flexagons can range from an object as simple as a cube to a hexagonal prism. Also the arrangement of triangles and squares can produce very interesting shapes as well. Since there will always be students in a class who excel, it is important for a teacher to understand how to expand any project they work with. This project is a perfect example to work with because it can be expanded to incorporate all levels of intelligence. If a student believes or it seems what is being presented is too simple for him or her challenge them with a shape to build. Having the ability to increase the difficulty of a project is a great tool because the possibility to challenge any student in a classroom keeps them motivated.

I believe this paper demonstrates a logical and creative way of learning polygons and polyhedrons. Not only does this paper teach the students about polygons and polyhedrons but it makes them become involved. The project discussed in this article serves as a visual, verbal, and a kinesthetic form of learning while including and allowing multiple intelligence learning styles. For those students who learn best when presented with a problem on a page this problem fulfills those needs while also fulfilling the needs of those students who learn best with hands on projects. Not only can this project apply to students of various learning styles it also allows the students to be creative and use their imaginations making math more appealing and fun.

My hope with the project discussed in this article is to have students understand and learn how polygons and polyhedrons are constructed. This paper applies to strongly to teachers as well, and I hope teachers will benefit from the new knowledge of being able to present their students with a project that can include each student and allow them to explore geometry. I also hope students will see objects like polyhedrons are not limited to cubes and square pyramids, opening the student's eyes to the vast and beautiful world of geometry.

References

Battista, M. (2002). Learning Geometry in a Dynamic Computer Environment. Reston, VA: The National Council of Teachers of Mathematics, 333-339.

Burke, K. and Dunn, R. (2002). Teaching math effectively to elementary students. *Academic Exchange Quarterly*. vol6, no.1, p91.

Kaufmann, H., Schmalstieg, D., and Wagner, M. Construct 3D: A Virtual Reality for Mathematics and Geometry Education (2000). *Education and Information Technologies,* p.263-267.

Koester, B. (2003). Prisms and Pyramids*:* Constructing Three-Dimensional Models to Build Understanding. *Teaching Children Mathematics*, pp. 436-442.

Oberdorf, C. and Taylor-Cox, J. (1999).Shape Up! *Teaching Children Mathematics* p340-346..

Pickreign, J. and Capps, L. (2000). Alignment of Elementary Geometry Curriculum with Current Standards. *School Science and Mathematics*, vol. 100, no. 5, 243.

Swarthout, M. (2003). Shape Search. *Teaching Children Mathematics*. vol.9, no.8, p455- 457.

TMME, Vol1, No.1: AUTHOR INFORMATION

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