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Abstract:
The 18th century was a milestone for the incorporation of mathematics into physics. By this time already seen in Newton's work, we know that a great deal of progress had found the light of day concerning the relationship between physics and mathematics, particularly as the latter began to deal with universal gravitation and optics. Based on his theory of monochromatic light, which used mathematics to describe optical phenomena, Newton became interested in the behavior of the various colors which, according to him, compose white light. Furthermore, toward the end of the 17th century, scientists had a remarkable mathematical tool at their disposal, differential and integral calculus, both of which were probably developed independently by Newton in *Principia*, and Leibniz in *Acta Eruditorum*, respectively. What interests us today is the fact that the utilization of these tools—or the lack thereof—was the object of many rich debates, even controversies, the nature of which partly depended on the scientific, cultural and political environment, not to mention the various ways of studying physical phenomena.

Keywords: history of science; historical mathematical methods; Newton; Leibniz; teaching of derivatives

Introduction
First, I will demonstrate the influence scientific and political circles had on the development, popularization and use of this method, followed by the impact of this new mathematical process in light of results obtained both in France and in Great Britain, the latter having taken into account and put forward various forms of theoretical expression for physical phenomena. To this end, I will concentrate on a case involving the phenomenon of astronomical refraction, about which Pierre Bouguer offered an explanation having relied on the use of infinitesimal calculation. I will then highlight the results obtained by Pierre Bouguer through a comparative study between both his, as well as the English approach to the same phenomenon. Finally, I will explore the possibilities of using this historical and epistemological perspective in the modern-day teaching of the notion of derivative function.

Published in 1729—time at which links were made between astronomy and optics—these works initiated the study of the optical aspect of bodies in movement, and later the works of Albert

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Einstein; this is also a time at which the soaring use of the mathematical method can be observed. This is the context in which we intend to pursue our study.

Context

He who wishes to find his way on land—or particularly at sea—by means of heavenly observation is obligated to take into account two domains which are intimately linked: astronomy and optics. This question was of special importance in 1729, year when Pierre Bouguer wrote a thesis, *On the best method of observing the altitude of stars at sea* having won a prize conferred by the French Science Academy, and in which its author addressed a long studied phenomenon, astronomical refraction. We know that the light that reaches us from a star will be more or less purloined depending on the medium through which it travels. This phenomenon—called astronomical refraction consequently prevents us from seeing where a star really is.
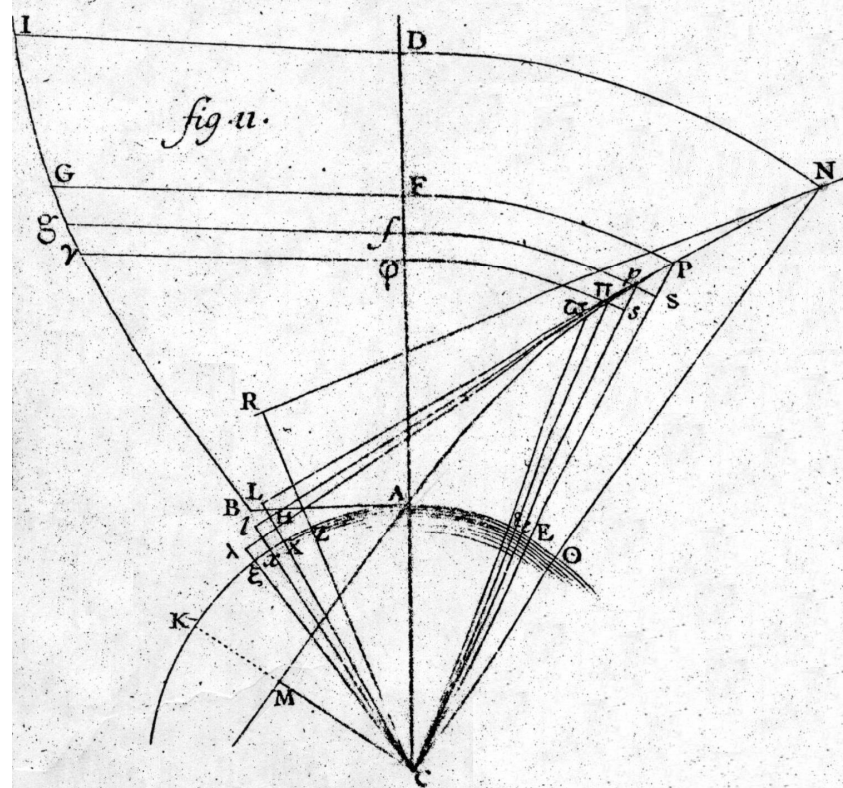
Pierre Bouguer, *De la méthode d'observer exactement sur mer la hauteur des astres, 1729*

Instead of being located in such or such position, it is elsewhere. Therefore, stars are not observed in their actual position. Hence the necessity to make corrections with the help of tables that can be adjusted according to one's location in order to find the real position of the stars. Long studied by Ibn Al Haïtham, Roger Bacon, or Tycho Brahé, the investigation of this phenomenon was resumed by Pierre Bouguer at a time when there were a great deal of breakthroughs bringing both astronomy

and optics into play. Fifty years prior in France, Rømer brought the idea of the finiteness of the speed of light to the fore through his observation of the emergence of Jupiter's moons. From 1725 to 1728 in England, James Bradley continued with some of the astronomical observations made by Jean Picard in 1671 and had noticed some anomalies that he was able to render public. His text was published in the December 1728 issue of Philosophical Transactions.

I am going to expose Bradley's results here, which are interesting because he worked both at the same time, and in the same field as Bouguer—that is to say at the forefront of astronomy and optics—but also because he belonged to a very different school than that of Bouguer, allowing new insight into the uniqueness of his methods, as much in form as in substance, and in particular concerning the use of mathematical analysis mentioned above. Indeed, Bradley exposed his work using a narrative style in a letter addressed to Edmond Halley whereby he presented events in the form of suspenseful riddles to be solved. He focused firstly on curiosity, surprises and questions evoked, and only afterwards on any observations he made. The implementation of experimental procedures can be seen here, and which one may consider having been inspired largely by Francis Bacon. Bradley took care to highlight the importance of questioning oneself about what pitfalls to avoid, if any, the attention to be brought to the handling of instruments of observation, and the necessity to reproduce observations. Nothing has been found, however, regarding approaches to the differential and integral calculus.

From the beginning of his letter, he indicates that his initial project was to determine parallaxes of several different fixed stars. Measuring these parallaxes revealed very interesting for astronomers since it allowed them to determine interstellar distances. There was also another more important reason for this—a lot less technical this time—and that was that the parallax would definitively prove the Copernicus' heliocentric theory. To measure this parallax, one preceded in this way: the letters S', S, E, respectively stand for a star, the Sun and the Earth; α is the angle that makes ES' and SS'
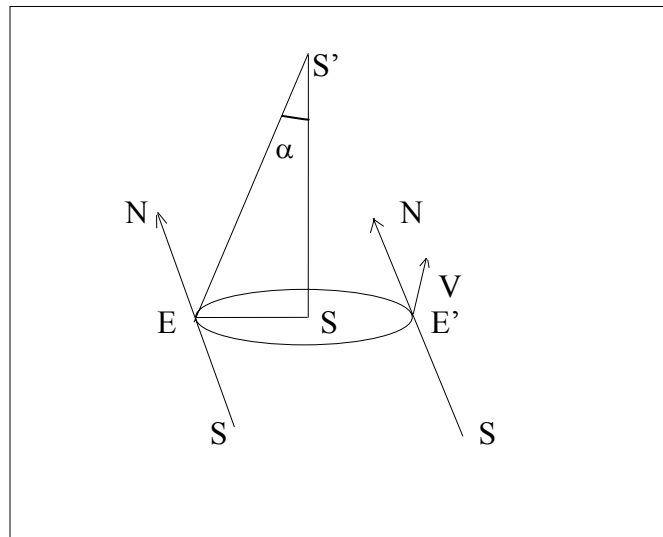
It was a matter of measuring the value of the α angle, which represents the parallax, where E and E' are the positions of Earth at six-month intervals. Knowledge of the α angle permitted the

measurement of the star's distance (SS'), since SS'=a/tg α, where *a* is the distance from the Earth to the Sun.

Bradley carried out his observations with the purpose of finding an eventual parallax, and in so doing, he noticed an anomaly. Indeed, the star under observation moves, but not in the direction expected, especially having only considered spatial relationships, as happened to be the case. From this point of view, the star should, in fact, move continuously in one direction on the ESS' plane. The star seems to move around perpendicularly on the ESS' plane.

This anomaly drove Bradley to implement what Bacon had called "the negative path", according to the framework of induction. Therefore, instruments were not brought into question, since—as indicated by Bradley— a great deal of regularity can be seen in the observations. Besides, the anomaly's regularity was such that it allowed for the prediction of cyclic phenomena which took place as predicted. Bradley successively eliminated, one after another, other causes of the anomaly, such as in the example of nutation. He then took on an affirmative procedure, and concentrated on the results of Rømer's observations, taking into account both the Earth's rotation and the finiteness of the speed of light: "…the phenomenon "proceeded from the progressive motion of light and the earth's annual motion on its orbit".was due to light's continuous movement and to the Earth's yearly orbit around the Sun."[2]

James Bradley is therefore aware of the cyclic phenomenon known as the aberration of fixed stars; this is important, notably because it foreshadows the development of the study of the optics of bodies in movement, relying on both astronomy and optics. Founded on the use of inductive reasoning, his approach was rather influenced by the ideas of Francis Bacon. To this end, he differentiated his approach from that of Bouguer's by studying astronomical refraction at the same time. Furthermore, we shall come to see that even if he had come to this very significant conclusion, he ended up not stating his them, for which he was blamed by Clairaut, because these conclusions would have had practical applications. The fact that Bradley was unable to further elaborate on these useful conclusions by finding practical applications for his discoveries is probably explained by the fact that he doesn't use the differential and integral calculus.

The development of the method

Probably developed double-handedly by both Newton in *Principia* and Liebniz in *Acta Eruditorum* toward the end of the 17th century, the differential and integral calculus elicited varying reactions from the scientific community. It is known that debate concerning its application in France was the source of conflict at the turn of the 18th century at the French Science Academy, where its members were, to say the least, divided and even opposed to its use. La Hire, Abbey Bignon, the Academy's president, Gallois and Rolle all expressed strong opposition to the new method at the time of its publication in France in spite of enthusiasm of such academics as L'Hospital or Varignon, both of whom had already been initiated to methods put forth by Leibniz.

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[2] James Bradley, A Letter from the Reverend Mr. James Bradley Savilian Professor of Astronomy at Oxford, and F.R.S. to Dr.Edmond Halley Astronom. Reg. &c. Giving an Account of a New Discovered Motion of the Fix'd Stars, *Philosophical Transactions of the Royal Society* , N°406, Vol.35, (1729) 637-661 ; 646. For a detailed analyze, see Mayrargue, *L'aberration des étoiles, et l'éther de Fresnel (1729-1851)*, (thèse Université Paris 7, 1991).

The method's dissemination was nevertheless made possible through the publication of several works, such as *L'Analyse démontrée,* published in 1708 by Père Reyneau. Furthermore, it was probably thanks to Reyneau that Bouguer had been initiated into differential and integral calculus.

The English scientific community also seemed to have experienced similar reservations regarding the use of the differential and integral calculus —even in the 1730's—, notably among astronomers working on particular sectors of the celestial sky. Evidence of preferences can be found among scientists who were concerned about both the reach of their findings, as well as how their work was organized, a fact to which supporters of the new method didn't hesitate to bring attention. The works of Bouguer in 1729, or those of Taylor who had preceded him in 1715, testify to this fact, not to mention the works of Clairaut, who, in 1737, and then again in 1746, read two theses at the Academy criticizing the Bradley's aforementioned works by distinguishing his approach from that of the latter's, highlighting the importance of using the differential and integral calculus.

"I believe that the Academy would approve of the goal that I have set for myself, to clarify this Theory in the Thesis that I am currently submitting. I will demonstrate the Methods for which Mr. Bradley hasn't yet given the results and I am adding several new studies on the same subject. I have found two solutions, one by way of this Method, and the other by way of synthesis; I have separated them so that those who only wish to study one may do so without studying the other."

Clairaut also referred to a thesis on the same subject by Manfredi, and offered a simple argument asserting his own results after having used the method: "I believe that [my rules]—in practice— would appear a lot simpler and more precise"

We have highlighted here tensions born between scientists on the basis of their preference for, or against the Method. Here, however, no consensus regarding these points of view can be observed. It can be said that although Taylor, mentioned above, used the differential and integral calculus just like Bouguer did, his style was different. His references were Newtonian analysis, as much in his notation as in his numerous references to *Principia*, not to mention his frequent necessity to exploit the concept of Newtonian attraction in his studies of atmospheric density variations and astronomical refraction. Whereas Bouguer speaks of the movement of light, Taylor reasons in terms of *vis gravitatis*. The former falls back on Newton, while the later—at the very beginning of his thesis—announces his reservations about Newton's preferences regarding the ways in which he studies phenomena, and therefore turns to Leibniz.

The political situation

Through this contextual analysis, it is also necessary to take into account the influence of the political situation. Works dealing with the Method gained popularity in France at a time when the Academy finds itself stuck in a period of decline after the reform of 1699. Its membership dropped sharply, and between 1699 and 1720 there was a 24 % fall, probably due to difficulties in replacing first generation scholars. At the beginning of the 18th century, established institutions seem to continue to demonstrate a sort of ostracism toward scientists who work within this new framework. Research dealing with infinitesimal analysis was actually quite rare. However the situation was divaricatious since at this same time, both Leibniz's and Bernoulli's work elicited numerous debates. Research on the subject was published by L'Hospital (1696), Carré (1700) and Reyneau (1702). While reading a thesis on air density variation, Varignon, in 1716, makes use of differential and integral calculus. The situation unfolds toward the second half of the 1720's, when Bouguer—who

had been working on astronomical refraction since 1727, year in which he supposedly wrote the first draft of *Essai d'optique sur la gradation de la lumière* which was published in 1729—was conferred a prize by the Academy, this time in a different context in view of the fact that there was increasing interest in infinitesimal calculus, and in mathematics, more generally. This political situation is evidenced in relationships that French scientists maintained with their English and German counterparts. After his election to the Royal Society (1728), and his meeting in Basel (1729) with Jean Bernoulli, it is known that Maupertuis began to play a role in the dissemination of infinitesimal calculus in France.

In the thesis defended by Pierre Bouguer at the Paris Academy of Sciences in 1729, the articulation of several ideas originally broached by Leibniz, L'Hospital, and Reyneau can be seen. It is all the more noteworthy that research on astronomical refraction is considered more commonly along the lines of theories originated by La Hire, Cassini, or Maraldi, all of whom Bouguer refers to. Because they ignored expansion of the Leibnizian as much as the Newtonian method, their style is distinct from one another. Apart from the frequent use of differential equations in his calculations, Bouguer was intent on founding his reasoning with the help of serial methods, the Inversion Formula, and the arithmetical triangle in order to establish tables showing astronomic refraction. This particular mathematical practice puts Bouguer in line with scientists such as Taylor in England or Varignon in France, who made use of results produced by Newtonian or Leibnizian mathematical methods. This is what we will examine here.

What path does the light coming from a star follow when it is observed from Earth? This is the question that Bouguer attempted to answer in his quest to establish an equation with a function designed to solve this problem. To this end, he concentrates on an idea introduced by Leibnizian formalism, and later researched by the Marquis de l'Hospital and Father Reyneau: the arithmetical—or differential—triangle, which allowed him to discriminate infinitely minuscule quantities. After having declared, "I call the algorithm of this calculation 'differential'", it is known that Leibniz's 1684 article had substantiated a relationship of proportionality between finite and infinitely small quantities with the help of congruent triangles. He formalized this idea in 1686, "In all these graphs, I imagined […] a triangle that I called 'arithmetical' whose sides were indivisible (rather, infinitely small), that is to say, differential quantities." The Marquis de l'Hospital had addressed this same idea again at the beginning of the second section of his work. Bouguer proceeded in the same way, and could thus, through geometric considerations of congruent triangles, establish a solaire? differential equation, based on which it was possible, we are told, "to easily construct the solaire?" Finally, having relied on these critical results revealed by Leibniz, he was therefore able to connect infinitely small quantities with finite quantities, consequently coming to the desired result after application.

From this example, the sheer muscle of the infinitesimal method in finding solutions to problems can be observed. In France, the concept of "derivative" is first introduced in the sixth form (*première et terminale*). There, the mathematics program asks that preference be given to work on a collection of historical texts linked to a common theme, for example, the notion of "derivative"; the objective of using epistemology and science history is to give insight into the nature of questions at the root of concepts as well as the language in which questions are formulated and discussed. It seems particularly welcome there to call on examples backed by historical accounts to convey notions studied. The example mentioned earlier concerning the phenomenon of astronomical refraction, borrowed from Physics, could be chosen for a sixth-form class, in which a degree of interdisciplinary studies is recommended, the goal being to demonstrate how mathematics has the power to establish patterns.

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