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Historical Tidbits, the Shoah, and the Teaching of Mathematics

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Abstract
In this extended essay, I use cultural-historical activity theory to look at the questions Theodore Eisenberg raises about the inclusion of historical facts, both historical tidbits and ethically questionable tendencies and horrific actions (the Shoah), in the teaching of mathematics. I conclude by suggesting that the ultimate answer has to be one that involves a decision, which means that an answer cannot be provided a priori or be *determined* by any antecedent. *Deciding* to include this or that in a mathematical curriculum is an *ethical act*.

> Pardonner le pardonnable, le véniel, l'excusable, ce qu'on peut toujours pardonner, ce n'est pas pardonner. [To forgive the forgivable, the venial, the excusable, that which one can always forgive, is not forgiving at all.] (Derrida, 2005, p. 32)

In his article "Flaws and Idiosyncrasies in Mathematicians: Food for the Classroom," Theodore Eisenberg raises an interesting issue: Should mathematics teaching merely focus on mathematical concepts or should mathematics students (at school and university levels) also know about the lives of the mathematicians who first articulated a theorem or solution, the cultural context within which some mathematicians have worked (Nazism, Russian dictatorship), etc.? Some of the examples he features are those of Einstein wearing shoes without socks—I never wear socks, and always sandals rather than shoes, even during visits to central Canadian cities in the winter—and Alan Turing, often considered to be the father of computer science, being homosexual.

Eisenberg raises other issues that are more serious, concerning, for example, the appropriation and appreciation of the products of labor by anti-Semitic scholars and artists. He has not addressed another situation, that of anti-Semitic philosophers or philosophers who did not declare opposition to the Nazi regime, such as Martin Heidegger. For me, therefore, there are two levels of questions. First, should we use and enjoy the productions of these people—Heidegger's philosophy, Wagner's music, the findings and productions by Nazi scientists and mathematics? Some individuals do not appear to mind, as we can see from the fact that the novel *Seven Years in Tibet*, written by the Heinrich Harrer, a member of Hitler's elite SS, recently was turned into a film for a second time. Here, producers, participants in the making of the film, and audiences willingly contribute to the perpetuation of a part of Harrer's autobiography. More so, the author's subsequent autobiography *Beyond Seven Years in Tibet, My Life Before, During and After* has been released in 2006.

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What should we do about the findings of psychological studies that clearly would no longer pass any human research ethics board? One such study was conducted by Stanley Milgram. In this study, ordinary people began to "punish" other ordinary people with electrical shocks of increasing intensity—and despite increasing expressions of pain—obeyed the experimentalist to punish their non-compliant victims even harder. Many of the participating subjects left the experiment traumatized because they had found themselves committing horrendous violence—they did not know that their "victims" were actually faked—similar to the once committed by Nazi torturers. Nowadays, Milgram's study probably would not pass the human research ethics requirements on "minimal risk," such as those that the Canadian National Council on Ethics in Human Research, representing the Tricouncil (which units the three councils funding research in (a) social sciences and humanities [SSHRC], (b) science and engineering [NSERC], and (c) health research [CIHR]), adheres to. And excuses such as "I was simply following orders" no longer will cut muster.

And what should we do about the studies Nazi doctors did on hypothermia using concentration camp interns from Auschwitz, Birkenau, and Dachau leading to the death of many "research participants" (really, subjects subjected to atrocities)?

And how does the idea of forgiveness play in here?

Eisenberg's paper raises many questions and, fortunately, the author is not subject to the hubris of offering simple answers to these difficult questions. In science education, there is an ongoing debate about the usefulness of teaching not just science content but the nature of science, which means, providing students with opportunities to learn about how science is practiced—including its contingent nature that the science studies literature reported over the past three decades. Surely, what we do in everyday life generally, and how we understand ourselves specifically, mediates what we do professionally. My own activities of intensely gardening (supplying year-round all vegetables we need), cooking (I do the cooking at home), building (I finish the basement, lay tiles and hardwood floors, etc.) have given me an appreciation of the role of the body in knowing; and I have exploited this understanding in the theories of knowing, learning, and meaning with respect to mathematics in the lives of professional scientists. Thus, for example, over 50 percent of research biologists could not interpret a graph that appeared in a first-year university textbook of their own field. Yet some did provide successful interpretations, and these drew on their everyday experiences—for example, going hiking in the local mountains or fertilizing plants and vegetables in their gardens—as resources in their interpretations.

To get a better handle on these issues and questions, I use cultural-historical activity theory, because it makes me look at the systems within which such things as mathematical theorems, technological artifacts (atomic bomb, rockets), scientific knowledge, philosophical masterpieces, musical oeuvres, or paintings and sculptures are produced and reproduced. In the following, I outline the theory and then use it to look at the issues that Eisenberg raises in his article.

A Cultural-historical Activity Theoretic Perspective

The Historical Roots of Cultural-Historical Activity Theory

Cultural-historical activity theory was founded by Russian psychologists (e.g., Leont'ev, 1978) discontent with the way in which most Western psychologist reduced human activity to the intentions and actions of individuals, on the one hand (as apparent in the famous Vygotsky–Piaget debate), or to the determination of human agency by environmental factors, on the other (behaviorism). They proposed, instead, to use entire activities as the unit of analyzing human productions; here, an activity is denoted by a verb such as farming, manufacturing tools, tailoring, hunting/fishing, doing university-based research, and so forth. Different activity systems together allow societies to survive, as the needs of individual human beings are satisfied through the exchange of resources to meet fundamental needs, such as food, clothing, and shelter. Thus, it would be unthinkable today to have a mathematician living like Diogenes in a barrel without doing something in exchange for which he or she would receive food, clothing to live in Canadian climates, and a heated home.

Activity Theory in Its Present-Day Form

Activity theory later was taken up in the West, where, in one of its two main versions (Engeström, 1987), the structural aspects are highlighted in a mediational triangle (Figure 1). Before explaining the figure in its details, I must highlight three important points. First, the triangle has to be thought as consisting of two mutually constitutive layers, one describing the material world, the other describing how the material world is reflected in human consciousness. Thus, as Alexei N. Leont'ev frequently is quoted to have said/written, the object exists twice—once materially, once in the consciousness that reflects the material world. Second, the triangle only represents the *structural* aspects of human activities only, pushing the agency required to mobilize structure into the background. Thus, while looking at Figure 1, readers need to keep in mind that it represents the structure of activity, but that it really requires *agency* to mobilize the resources available in this structure. Third, the triangle constitutes a static representation pushing the historical aspect of the theory into the background. Thus, as its name suggests, cultural-historical activity theory emphasizes the historically and culturally contingent aspects of human consciousness. Therefore, what is possible today in terms of mathematical proofs particularly and mathematical praxis more generally would not have been thinkable 50 or 100 years ago, or, to sharpen this issue, it would not have been possible yesterday. To understand activity systems, such as the one producing new mathematical knowledge, we therefore always need to study mathematical culture in its historical dimensions. The question Eisenberg raises about teaching some of the contingent elements in mathematicians' lives can be answered in the affirmative, for anything that happens in an activity system leaves its mark on the activity system, including, for example, its outcomes (mathematical knowledge) and its subjects (mathematicians as persons).

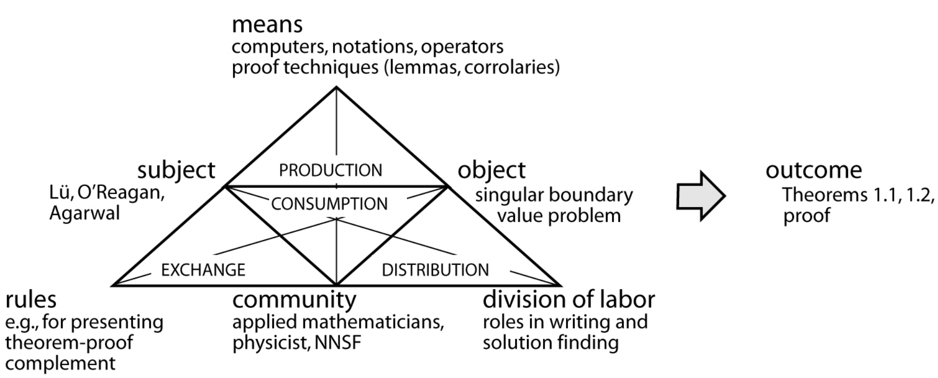

Figure 1. The *structure* of cultural-historical activity theory contains 6 main *moments* that cannot be reduced to each other. Activity as a whole, therefore, is the unit of analysis.

Cultural-historical activity theorists take activity as the minimal unit of analysis. Thus, the triangle in Figure 1 as a whole needs to be considered when we want to know how, for example, new mathematical theorems are produced. Because activity is the minimal unit, none of the terms in the figure denotes an "element" (as some researchers falsely do, even those who self-declare to be practicing cultural-historical activity theory). Rather, these terms denote *moments*, that is, parts that can be articulated on heuristic grounds but cannot be thought independent from other isolable parts because all of those aspects *mutually constitute each other* (Roth & Lee, 2007). Philosophically inclined readers may think of the term *singular plural*, where the whole constitutes the parts and the parts constitute the whole; mathematically inclined individuals know analogous phenomena in systems of coupled differential equations for dynamical systems that cannot be separated in which the current value of certain variables appear as parameters in the evolution of other variables. This then makes it immediately clear that from the chosen theoretical perspective, we cannot think of mathematical theorem production in terms of a mathematician's mental structure and content.

In activity, three levels of events need to be distinguished yet at the same time understood in their mutually constitutive nature: activity, action, and operations (Leont'ev, 1978). An activity—consistent with its origin in the German concept *Tätigkeit* and the Russian concept deyatel'nost'—refers to a form of event at the societal level that contributes to sustaining the life form. Thus, farming, teaching, producing tools, fishing and the likes are activities—doing a mathematical problem in high school is a task. Activities are interconnected, exchange people, products, and money and in so doing, contribute to meet human needs. Activities therefore are oriented toward object-constituted motives. More so, activities contribute to the *sense* of actions (Figure 2), which concretely realize activities. Actions are oriented toward the goals individual and collective subjects set themselves to transform the relevant object into an outcome (product). These last three sentences point us to the dialectical relationship between activities and actions (Roth, 2007a). Actions realize activities, but activities provide the sense for an action: the same action is associated with a different sense in a different activity (showing the middle finger to a teacher who requests silence is different to showing the middle finger when a team mate requests receiving the ball). Actions and the goals they pursue are realized by operations, which are not conscious but determined by the context—we walk to the fridge to get some ice, but the walking itself is realized by steps that we do not think about. But operations are produced only in the service of realizing goal-driven actions. There therefore is another dialectical relationship between conscious, goal-directed actions and contextually determined operations, each presupposing the other. Here, goal-directed actions serve as a *referent* in the unconscious "selection" of operations. Together, the two dialectical relationships between activity and actions, on the one hand, and actions and operations, on the other hand, denote a process that I term *meaning*. As actions may become routinized, they turn into operations; operations also may be "copied" unconsciously while someone participates with others in research or daily activities (by means of a process that has come to be termed *mimesis*). In this way, operations really constitute *crystallized* forms of cultural practices (i.e., patterned actions).

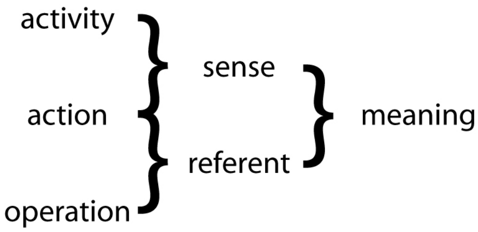

Figure 2. An activity, although it constitutes the *unit of analysis* should be analyzed in terms of three levels that stand in dialectical (mutually constitutive) relations.

Emotions, as I recently showed in an analysis of mathematics in the workplace (Roth, 2007b), are central to events at the conscious and unconscious levels. At the selection of goals, human beings will select those that have a higher valence, that is, that promise some sort of pay-off associated with satisfaction (higher salary, well being). Mathematicians do research and write papers because of the positive emotional valence that comes with innovation and achievement, because publication leads to pay raises, or because of some other reason associated with some pay-off. At the unconscious level, our current emotional states (feeling down, elated) are part of the contexts that shape the production of operations. We know that there are days that we do not feel like doing research or where we do not feel like writing, and no external force driving us will improve the results.

In the following, I use a recently paper published in a journal of applied mathematics (Lü, O'Regan, & Agarwal, 2007) as an exemplary case to explain Figure 1, though not having followed them around with my camera, I am not in a position to write about the emotional aspects in the way I have done it for fish culturists (Roth, 2007b).

An Exemplary Case of an Activity System in Applied Mathematics

The three authors of the paper "Existence to singular boundary value problems with sign changing nonlinearities using an approximation method approach" set out to produce two theorems concerning singular boundary value problems, theorems that—in the words of the authors—constitute the original contribution of the work. In terms of the theory, the three authors constitute the *subject*, the singular boundary value problems the *object*, and the theorems the intended *outcome* of the activity. What they do is mediated by the tools they have available, which may have been some form of electronic *means* to communicate between their institutions located in China, Ireland, and Australia, respectively. That we cannot reduce the different moments also is immediately evident, as the object of activity (boundary value problems) and the outcomes (theorems) define the nature of the subject, applied mathematicians, but the nature of the subject as mathematicians defines the object. To return to the analogy with the coupled differential equations, the temporal evolution of the object and the temporal evolution of the subject cannot be thought (modeled) independently because the state of one at a point in time enters the evolution equation of the other. More so, in a world where difference is required for thinking, the object defines the very nature of the subject. Thus, we would not find everyday folk doing singular boundary value problems: Solving such problems *makes sense* within the community of mathematicians and within activity systems of mathematics; it does not make (immediate) sense in other communities, where this might be considered something outlandish (think about what Einstein's coworkers in the patent office might have thought about him if they knew he was working on what came to be known as relativity theory). Also, we cannot understand what has been produced without the means of production, which mediate between subject and object. Thus, mathematical activity has a mediated nature.

There are further mediations at work to understand the actions of the mathematicians. For example, the *division of labor* that the three authors have chosen mediates the relation between the subject and object—the "flavor" of the solution proposed may depend on who does what and who takes the lead. The community of applied mathematicians also mediates the relationship between subject and object, as it will be the recipient and "consumer" of the outcomes of this activity. Therefore, what constitutes a legitimate object of mathematical activity and who constitutes a legitimate mathematician depends on (is mediated by) the *community* of mathematicians. This also is immediately evident when we think of the first people to read a manuscript: editors and reviewers. The manuscript has to address the concerns of these recipients ("consumers" [Figure 1]) to make it into a scientific journal in the first place. Thus, the three mathematicians do not just develop theorems and proofs, but they do so in a way that they presuppose others to recognize as legitimately mathematical. More so, much of what mathematicians do does not require conscious reflection: Few scholars I have met know, in terms of formal rules, how to write a good paper: they know to write a paper in the same way they know how to walk or in the way children speak grammatically correct without knowing formal grammar. That is, much of what mathematicians do happens at the level of operations, which may have been the result of explicit actions that have crystallized or that they may have appropriated by unconsciously emulating others within the culture. The Chinese funding agency NNSF, acknowledged in the first footnote, also mediated the object, as its grant enabled the pursuit of the solution and the production of the theorems. Finally, there are *rules* that mediate between the mathematicians and their object. Thus, for example, to solve the singular boundary value problem requires a particular procedure, the proposal of the theorem and its proof, including the production of lemmas and corollaries that are required to achieve the *outcome* in the concrete way that it present itself to readers ("consumers") of the article.

Consequences of Activity Theory for Thinking about Tidbits

Two main points need to be made here. First, if human activity is mediated then all moments of activity make their mark on the outcome, including the means of production, the particulars of the (individual, collective) subject, and the community. For example, the arrival of computers on the scene in the 1960s allowed new forms of doing mathematics to emerge, even though mathematical purists do not accept the use of computers as legitimate. More so, what is acceptable mathematics is a function of the current state of the mathematical culture, which is a characteristic of the mathematical community of the day. But so was the theory of the delta function that the physicist Paul Dirac introduced, but which formal mathematicians did not initially accept as a legitimate object of inquiry until a rigorous definition of distributions as functionals was produced a few years later (Balakrishnan, 2003). In a strong sense, therefore, particulars of the individual and collective subject make their mark on the outcomes. Simple, mundane, and everyday experiences may therefore mediate the solution to scientific problems. For example, one story about the discovery of the chemical structure of benzene suggests that Friedrich August Kekulé had a daydream of a snake biting its tail. Other versions of the discovery say that he might have seen a dance with multiple couples joining up in a ring. (The 2005 Nobel Prize in Chemistry was given "for the development of the metathesis method in organic synthesis," a process explained in terms of a "ring dance with partner exchange" between alkene and catalyst pairs.) Quite innocuous events, images, and observations may provide solutions to important scientific and mathematical findings, for which *individuals* are credited, though they *received* rather then intended the insight provoked by their being part of everyday collective and material life.

Now if we were flies on the wall watching mathematicians at work, then to understand what is happening, mathematicians' actions, we would need to look at the activity system as a whole. (I am aware of at least two studies that looked at mathematical activity in real time: Livingston [1987] videotaped the reproduction of Gödel's theorem by two mathematicians, and Mertz and Knorr-Cetina [1997] studied theoretical physicists working out some aspect of string theory, that is, the BRST cohomology of the W-algebra.) We cannot just be concerned with presupposed contents of the mathematicians' minds, but we have to take into account the means they use, the community that they intend the products of their labor for, the (tacit/implicit and explicit) rules they adhere to, the division of labor they enact, and so forth.

Implications of an Activity-Theoretic Perspective

Cultural-historical activity theory allows us to better appreciate the relationship between individual and collective. The individual but realizes a possibility that exists at the collective level. The simultaneous emergence of the verb "to google" in the Anglo-Saxon world is but an example of this fact. Another example is that of language emergence: At the very instance that a (first) human being articulated a first word or phrase, he or she had to presuppose that the listener already understood, and therefore, the first speaker was not the first linguistically competent individual after all given that the recipient of the message (listener) had to be equally competent.

From cultural-historical activity theory we can learn two main things pertinent to the issues that Eisenberg raises. First, the outcomes of activity bear the marks of every single moment that one can identify in the system as a whole. Second, and arising from the first, there are strongly viewed no individual contributions, because individual achievements are the outcomes of historical reconstructions where the system as a whole has been abstracted and made to disappear. Thus, the shoemaker or factory worker producing Einstein's shoes, the tailor who cut and sewed his suit, the farmer producing the wheat for his bread, the architect and construction worker making his home all have been abstracted, though Einstein could not have lived his life without them. Third, cultural-historical activity theory teaches us that we produce and reproduce society at a point in time that is culturally and historically contingent. Had Einstein lived 50 years before, he likely would not have been in the position and would not have had the resources to produce general or specific relativity theory (for which he has become most well known), his paper on the photoelectric effect (for which he received the Nobel Prize), or any of the other contributions that he now is celebrated for. More so, 50 years later, he would not have been in the position to invent these theories, as someone else would have likely invented them because the time was ripe and the resources available for framing and solving these problems. This is so because at the collective cultural level, there are action possibilities; at the time of Einstein, a reformulation of a number of issues in physics could be undertaken. Sooner or later someone else would have realized these possibilities.

Now, we cannot know whether wearing or not wearing socks has contributed in any way to the production of relativity theory or any other of the contribution. But it might have been the case that not wearing socks—like taking walks in ice-cold creek water that the Bavarian priest and hydro-therapist Sebastian Kneipp recommended (Einstein went to school in Munich, the capital of Bavaria)—contributed to a sufficiently healthy condition that allowed him to do the work he did. In this case, if he had been a sickly person, the association between relativity theory or photoelectric effect and Albert Einstein might not have come about. In phenomenology, it is accepted that our bodies constitute what we can know (e.g., Merleau-Ponty, 1945; Henry, 2003). Knowing means knowing to act, not in a reflective way, but in the same way that we know how to walk upright without thinking, in the same way that we talk to our neighbors on the street without having to think about what to say, in the same way we teach mathematics and statistics lectures without having to stop and search for words. This form of knowing leads to the production, in real time, of behaviors that are marked by contingencies: we stumble or stutter during a lecture, we produce incongruencies and malapropisms, we bend the nail rather then getting it into the wood or wall, and so forth. And from such contingencies derive images that produce solutions to the hard problems that exist in the science and mathematics. Take the following examples.

Einstein used the image of an elevator to consider issues concerning relativity. Now this required his knowledge of elevators, and perhaps he had ridden elevators over and over again, such as I had done when I was a child in the hotels near the campground where my family staid during its summer vacations. From a phenomenological perspective, this is entirely intelligible: his experience has changed his way of understanding, and this understanding, intuitive and inarticulate as it may have been, became a resource in his thinking about relativity theory as Kekulé's image of the snake biting its tail, an age-old image existing at the cultural level for a long time, mediated his solution to the benzene structure. Saying that Einstein was fond of riding elevators might be considered a tidbit, but without this experience and the tidbit it gave rise to (if this were to be the case), he would not have been able to think through these issues at all. Does such a tidbit warrant inclusion in the teaching of physics or mathematics of general relativity: yes and no. On a historical level, Einstein may not have been able to produce the principles of general relativity, but someone else might have produced it. On an epistemological level, it would help us understand that experience *is* required for anything like conceptual knowledge—a main point in praxis theories and phenomenological theories of knowing. We do not need to know about his habit—if this were in fact the case—of riding elevators, because someone else would have stated the principles of relativity because they constituted a general possibility. And if someone else had produced them slightly before or after, its statements would be connected to different personal experiences.

Some contingencies and quirks easily can be abstracted from the scientific and mathematical productions; or viewed differently, the marks these contingencies and quirks on the outcomes of scientific and mathematical activity can be considered minor or invisible so that we may disattend to them. Einstein's quirky habit of wearing shoes without socks may be among those. But in the case of Sir Isaac Newton, we have some outcomes of his activities that became contributions to mathematics and physics, leading to celebrations of his outstanding qualities and "genius." But other productions were so much marked by his twisted, tortured, and mystical nature that they did not make it into the annals and history of standard science (White, 1999). This biographer also notes Newton's homosexual tendencies, his ability to hold grudges for decades, and his egomaniac and very petty nature. Thus, Newton's contributions to alchemy and his productions concerning Old Testament prophecies—he thought that the design of Solomon's temple was a code for the entirety of recorded human history—did not become acceptable contributions to any official science and therefore do not feature in today's science (together with his laws) or mathematics textbooks (together with his calculus). That is, to understand Newton's production, we need to understand all these tidbits. To understand Newton as a person, we do need to know about his phantasms, his alchemy, and so forth. These tidbits allow us to understand that Newton was just another person, with all its idiosyncrasies. But to understand the law of gravity, we do not need to know these tidbits. And further, it is not Newton alone who is responsible for the law of gravity or the law relating force and acceleration now bearing his name: $F = ma$ or the calculus in the form he proposed. The scientific community has taken care that the quirks are irrelevant and only those productions come to be recognized as contributions to science that are without the contingencies and particularities that characterized Newton's other productions.

Anti-Semitism

The same will be the case concerning the other main issue that Eisenberg brings forth: Should we accept the productions of anti-Semitic scholars and artists? The answer is not easy and my inclination is to say that the answer and solution must be inherently contradictory to allow us making the choice. If the answer were inherently possible and straightforward, it would not require a choosing and taking a stand, and therefore could be delivered in a mechanical and mechanistic way. It would not take a human being to implement, but could be programmed into a computer, which would produce the pre-determined and pre-programmed solution.

Should we accept the productions made within a society that has anti-Semitic tendencies or made by individuals who also make anti-Semitic statements? That is, should we reject the mathematical and scientific advances made during the Nazi regime, including scientists and mathematicians with declared or undeclared Nazi tendencies or sympathies? History shows that—for pragmatic reasons—such tendencies and sympathies often are neglected and even forgotten. (See also my introductory example of the Heinrich Harrer book and film.) Rockets were developed during the Third Reich, and so was the knowledge and the technology for the atomic bomb, both subsequently further developed in the USA and the USSR, including the collaboration of emigrated and captured German scientists. Is a rocket or an atomic bomb anti-Semitic? It probably is not. Is an atomic bomb anti-Japanese or anti-Nazi? Well, it has been used by the Americans who, like Canadians, interned their citizens of Japanese origins despite their allegiance to the new home country. And it has been used to kill "innocent" Japanese in Hiroshima and Nagasaki, who, as in Germany, may not have adhered to the public ideology but have remained silent for fear of being interned and killed in concentration camps. Are scientists responsible? Most scientists will respond "no," conferring the responsibility for the bomb to politicians. Others will not be so sure and will want to make scientists ethically responsible for their production.

Should we not read the work of the German philosopher Martin Heidegger because of his allegiances with the Nazi regime? Some readers may not want to read him for this reason. Others may claim that his work, such as *Sein und Zeit* (Being and Time) does not bear evident marks of these tendencies and therefore, like the atomic bomb, can be considered as a philosophical achievement acceptable to be discussed in scholarly circles. Do we reject Jean-Paul Sartre because he showed sympathies for the repressive regime of the USSR? Do we reject the productions of those U.S. scholars and artists that were devout Marxists and Soviet friendly (and for that persecuted by McCarthy)? Or should we reject those who assisted McCarthy in the persecution of his fellow citizens? Should we reject the productions by present day Israeli scholars because they live in, and perhaps support, a political system that causes havoc for Palestinian families who have nothing to do with the attacks of militants and suicide bombers? Should we reject the scientific and mathematical findings of U.S. citizens because they live in a country that has the death penalty, that, in the eyes of many people around the world engages in unethical and inhuman interrogation, internment, and repression practices (Abu Graib; Guantanamo Bay; 100,000 civilian "collateral damage" in Iraq as a by-product of "fighting global terrorism")? The US is, after all, one of the countries that Amnesty International cites for human right violations of the kind that individuals from other nations are tried for in the world court at The Hague. This list of questions shows that there are no easy solutions; in fact, any solution may be the possible impossible itself. Personally, it is somewhere along these lines that I would like to place myself for pragmatic purposes. It would force me to make a decision in each and every case, in each and every course I teach, always requiring me to think about the unsolvable mystery of (collective) human consciousness that leads us to these aporetic situations.

Coda: Should We Teach the Tidbits of History?

In the manner of Jacques Derrida, one of my most favorite philosopher, who avoids giving simple answers to complex problems, I make another turn: Though announcing the end (Coda!), I make another beginning. It is a truly Nietzschean (eternal) beginning and renewal. Therefore I make another return concerning the question whether we should be teaching about Einstein's socks: In another area of my research, gesture studies, it is well known that some hand-arm movements are coincidental, that is, without function in the conversation; these are referred to as "grooming" movements, such as scratching one's arm during a conversation. Other hand-arm movements do have a function because they contribute to understanding on the part of the speaker or listener: for example, when the listener scratches her head, the speaker may take this to be an indication that the listener does not understand or has difficulties understanding. How are human beings capable to separate scratching one's from signaling lack of understanding? Pragmatically, we do separate the two forms of hand-arm movements; and if there were a misinterpretation to occur, subsequent speaker- or listener-initiated transactional turns would seek to rectify misalignment. How do we separate the wheat from the chaff, and is the chaff of relevance?

In mathematics (science) education, does it matter for a student to know whether Einstein wore socks or not? On the one hand, it does not matter teaching about it: wearing socks and the outcome of Einstein's thinking processes, e.g., general relativity theory, appear to be unrelated. On the other hand, it does matter: we are less prone to deify, as this often happens, a human being who, after all, is subject to birth, death, and (eating, drinking, defecation, clothing) needs as all other human beings. The emperor has no clothes; and Einstein had no socks when he slipped into his shoes without them. Einstein was special, as we all are; and he was not so special, as we all are. He realized cultural possibilities; as we all do. And he realized some in a way that he became celebrated for; as some of us are when we receive awards for work attributed to us (I have a few of those). But these rewards are from communities that have enabled and accepted the very innovations that we produce—in giving me an award a society actually rewards itself, for I would not have published if the community had not been ready for it. Einstein built on the knowledge produced by others before him, including Albert Michelson and Edward Morley's experiments on the constancy of the speed of light; he knew of the Fitzgerald-Lorentz contraction, and he knew of the transformations that convert the observation of measurements in different systems of reference, which were named by the French mathematician Henri Poincaré after the Dutch physicist and mathematician Hendrik Lorentz (the Lorentz transformations). Knowing about Einstein's habit to wear shoes without socks is but one piece of evidence to recognize that the emperor does not wear clothes.

My answer to the question whether we should teach historical and biographical tidbits has to remain contradictory: I personally like to live in a world where we the emperor has no clothes if this is the case. In my work with graduate students and colleagues, I always make this point clear whenever someone asks me about my scholarly productivity or produces some other laudatory comment, for example, about the number of prizes and awards I have received. I always comment that all these accomplishments would have been impossible without the community that was the very source of the possibilities that I concretely realized, but which someone else could have realized as well. So sometimes I point out that I, too, do not wear clothes, experience pain, suffer, am elated, and so forth.

The question whether we should teach the tidbits of mathematics and science history depends on how we see ourselves. The answer therefore has to remain aporetic, forcing us to *make* choices rather than accepting present conditions that dictate to us whether to include tidbits and the Shoah in mathematics (science) teaching. Are we like dog *trainers*, getting the best to perform whatever we teach? Or are we *educators* interested in more than the mechanical transmission of knowledge and skills? Should high school students know about the context within which mathematical knowledge was produced? Definitely so! Does this mean knowing about the presence or absence of socks on Einstein's feet? Perhaps. Should university mathematics students know about Einstein's socks? Perhaps, especially if they do not continue to pursue graduate studies in mathematics and become professional mathematicians. *Education* means that we know how the world works; training means that we acquire some routine skills without worrying about their epistemological and ontological nature. (As a graduate student in physics, I complained to my professors that they were teaching us *mere* skills, and therefore that university was little different from vocational school. I said that physics had so many epistemological and ontological consequences that we should be discussing. But they responded that training us in certain skills *was* the purpose of university education.) It therefore also means that we live in a world without gods. Einstein's mannerisms concerning his socks is a good way to push a god off the pedestal and to recognize him as but another human being who has done his part to reproduce and produce everyday, mundane, immortal society. Einstein wears no socks in the same way that the emperor does not wear clothes.

Aporia

At the end of his article, Theodore Eisenberg asks the really hard question about what to do with "the Nazi business of Bieberbach and Teichmüller" and other issues surrounding Nazism, anti-Semitism, and the Shoah. Eisenberg states that he "feel[s] uncomfortable in discussing this nasty business." It is not my place to lecture him or anyone else how to deal with this problem, which really is an aporia, a problem without solution, or rather, a problem with contradictory solutions. (This, especially and because of my German origins, and especially and because my parents were only children at the time. These contingencies cannot be excuses, which is a very biblical theme, as we know from the concept of "original sin.") The solution has to be as aporetic as the problem. Let me explain.

In making a decision about whether to include historical facts in the teaching of mathematics, as well as in decisions about whether to include the work of anti-Semitic (pro-Nazi, pro-Serbian nationalist, anti-American) scholars within the community of mathematicians (scientists, artists, culture generally), we must not forget the concept of *forgiveness*. Here I do not mean the simple concept of forgiveness—can I forgive this person, can I not forgive that person—but rather the advanced concept of forgiveness in all its complexity (Derrida, 2005). Derrida points out that we can only and truly forgive the unforgivable, because if we forgive the forgivable, we have not really done anything particular. A computer can forgive the forgivable using an algorithm. And, as the title *Pardonner: L'Impardonnable et L'Inprescriptible* (To Pardon: On the Unpardonable and Imprescriptable) suggests, pardon generally and pardoning the unpardonable specifically *cannot be prescribed* (which is why it is not my place to lecture anyone on how to deal with the issue). If we can forgive Bieberbach, Teichmüller and the likes, forgiveness becomes mechanical or a matter of exchange. If we do not forgive the unforgivable, then we do not make a decision and simply submit to the condition. Forgiving the unforgivable, however, is the most difficult task we face. To make his point, Derrida discusses the case of the Russian-born philosopher Vladimir Jankélévitch (his family emigrated because of the pogroms against Jews), who, in a little book entitled *Le Pardon* (The Pardon), had suggested that pardoning a sin is the greatest challenge to judicial logic. Jankélévitch took a hard-line stance and suggested, in *L'Imprescriptible*, that the Shoah (Holocaust) attained such inexpiable singularity that renders impossible any form of pardon. Derrida also analyzes poem "Todtnauberg," written by the German- and French-speaking poet Paul Celan (born into a Jewish family in Romania) after his visit of Heidegger at his home in Todtnauberg, a poem in which he points to (in his usual oblique style) what he had hoped to hear so much:

. . .	. . .
die in dies Buch	the in this book
geschriebe Zeile von	written line of
einer Hoffnung, heute,	a hope, today,
auf eines Denkenden	for a thinker's
kommendes	coming
Wort	word
im Herzen,	in the hear
. . .	. . .

But his host (Heidegger) did not pronounce it: the request to be pardoned for his allegiance to the Nazi regime. Derrida takes up the complete opposition Jankélévitch showed with respect to any forgiveness of the Nazi crimes and shows that a solution to this problem dignified to be named such has to remain aporetic and contradictory. Derrida suggests that the pardon *has to be asked for*, to be just, for the fact to be just, and because the one asking is just, and, because to be just one has to be unjust (i.e., asking for forgiveness of the unforgivable). But Oswald Teichmüller and Ludwig Bieberbach are no more; they cannot in any way ask for forgiveness. Yet we must be in a position to forgive the unforgivable that they enacted. The upshot is that we may pardon the unpardonable, forgive the unforgivable; and this, too, can become part of our teaching (mathematics, science, philosophy, music, art).

Epilogue

I am glad Theodore Eisenberg took up the challenge to address not only the small problems like Einstein's socks but also the real hard and unsolvable problem of the Shoah (and similar atrocities, the genocides in Rwanda and Serbia, etc.). If there were a simple solution, it would not be a real problem. I like the internal contradiction the author leaves at the end—feeling uncomfortable with the "nasty business," but at the same time, as a step toward forgiving what remains unforgivable, "teaching the strengths and weaknesses of the individuals whose mathematics we teach." I see it as a move toward a better world, hopefully one without atrocities, one in which people of all races and believes resort to mechanisms other than violence to resolve their unavoidable differences—whether they are Catholics and Protestants in Ireland, East and West Germans, North and South Koreans, or the within-Semite differences between Israelis and Palestinians, now living divided on the two sides of an emerging concrete wall.

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