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**Abstract**
*The Mathematics Education and Neurosciences (MENS) project is aimed at exploring the development of the mathematical abilities of young (four- to six-year old) children. It is initiated to integrate research from mathematics education with research from educational neuroscience in order to come to a better understanding of how the early skills of young children can best be fostered for supporting the development of mathematical abilities in an educational setting. This paper is specifically focused on the design research that is being conducted from the perspective of mathematics education in which we are investigating the relationship between young children's insight into spatial structures and the development of spatial and number sense. This should result in a series of classroom activities that may stimulate children's development of spatial and number skills.*

**Keywords:** young children, spatial thinking, design research

## 1. Introduction: The Project in Context

It may come as no surprise that several publications support the point that we, the educational researchers, have been failing to properly value the cognitive capacities of young (three- to six-year old) children. A report from the National Research Council (NRC, 2005) concluded that

> early childhood education, in both formal and informal settings, may not be helping all children maximize their cognitive capacities.

It is also clear that there is an increasingly critical attitude towards some of Piaget's work. The aforementioned report concludes that 'modern research describes unexpected competencies in young children and calls into action models of development based on Piaget, which suggested

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that children were unable to carry out sophisticated complex tasks, such as perspective taking' (NRC, 2005). Remarkably, scientists from a different discipline, the education neurosciences, have come to similar conclusions in a report on Numeracy and Literacy of young children (OECD, 2003).

The learning of young children is so intriguing, that it has engaged many different scientific disciplines. What is surprising, then, is that the brain scientists see no references in the educational research literature about the developments they see to be relevant and vice versa. Yet, the tide is changing. The OECD report 'Understanding the Brain. Towards a New Learning Science' (OECD, 2002) suggests trans-disciplinary research to be the way forward. This must bridge the brain sciences (called the 'hard sciences' by brain scientists) and ('more practical') educational research (Jolles et al., 2006).

Reflecting on these issues, a program called *TalentPower* (TalentenKracht) was initiated by van Benthem, Dijkgraaf and de Lange. Several universities and institutions in The Netherlands collaborate in this program to gain a better understanding of what talents, possibilities and qualities young children exhibit as they are engaged in scientific activities, how these talents and qualities may be enhanced, how they may be intertwined, and in what ways they may be connected to language development. Hence the goal of the project is to bring together scientists from various research perspectives, as well as parents and teachers in order to chart the talents of young children and to scientifically fundament how these talents may be used and developed in an optimal way (van Benthem, Dijkgraaf & de Lange, 2005).

As such, apart from the fundamental goal to investigate the possibilities of fostering young children's natural curiosity, an important goal in the methodology of *TalentPower* is the 'trans-disciplinary' approach. Given the abundance of research in the field of mathematics education, the project was designed to try to bridge the gap between the sciences of 'mathematics education' and 'educational neurosciences'. This is how the Mathematics Education and Neurosciences (MENS) project came into being.

The significance of the collaboration between the sciences lies in the grounding of research from the field of mathematics education in cognitive and neuroscientific theory while at the same time providing the research from the field of cognitive psychology and neurosciences with a strong practical basis from which testable predictions can be made. Many recent publications have emphasized how scientists from the disciplines of mathematics education, cognitive psychology and neuropsychology can and should contribute to each others research (Berninger & Corina, 1998; Byrnes & Fox, 1998; Davis, 2004; Griffin & Case, 1997; Jolles et al., 2006; Lester, 2007; Siegler, 2003; Spelke, 2002). As Cobb (2007) points out, comparing and contrasting research from various perspectives has the added benefit of deepening our understanding of the phenomena being studied and of broadening the practicality of the results.

Within the context of the development of mathematical abilities of young children, the authors of this paper are mainly concerned with the mathematics educational perspective. De Haan and Gebuis at Utrecht University are constructing and performing the educational neuroscientific experiments. In time, the results of research from these research perspectives will be compared, contrasted and combined in an effort to contribute to mathematics educational practices that can foster the early talents of young children. Ultimately our findings may help stimulate those children who may be prone to experiencing problems in the development of mathematical thinking.

In the present paper we spiral into the work that is being performed from the perspective of mathematics education. In the first part of the paper, we lay out the theoretical framework of our research. This starts with a rationale for our focus on young children, on the constructs of spatial sense and number sense and on the role of spatial structures in the development of mathematical abilities. Next, we introduce preliminary experimental support that has contributed to the refinement of the research questions, and finally we explain our research methodology. We begin with the primary interest of all mathematics research in *TalentPower*: the importance of attending to how young children develop in their mathematical thinking.

## 2. Young Children Doing Mathematics

The overwhelming scientific attention to the mathematics education of young children can be attributed to seven factors that Clements and Sarama (2007) articulate: that a growing number of children attend early care and education programs, that the importance of mathematics is increasingly being recognized, that differences in performance between nations as well as between socioeconomic groups exist, that researchers are shifting to a perspective that recognizes innate mathematical competencies, that mathematics achievement is strongly predicted by specific quantitative and numerical knowledge, and finally, that knowledge gaps often appear because of poor bridging between informal knowledge and school mathematics.

What repeatedly stands out from studies on development in early childhood is how young children may be characterized by their natural drive to go out and explore the world. This is particularly illustrated in research stemming from Piaget's work. As mentioned in the introduction above, however, Piaget's methodology has strongly been criticized by researchers such as Freudenthal for depending too much on expert-use and interpretation of underlying concepts and on the child's language skills (Freudenthal, 1984, 1991). Freudenthal was greatly concerned about the intertwinement of children's cognitive competencies with their language skills, where relatively underdeveloped language skills could potentially suppress how children may express their understanding. Research methodologies that relied on children's ability to communicate their thinking could, in his view, only assess this language component and nothing more. Yet, Freudenthal's experiences with young children convinced him that children typically do possess remarkable cognitive competencies that develop through early learning processes.

Children's early competencies have been compared to the behavior of scientists in the Theory Theory (Gopnik, 2004; Gopnik, Meltzoff, & Kuhl, 1999). She suggests that children are born with certain theories about the world that they continuously test and amend as they gain new insights from daily experiences. Certain parallels are also drawn between children, scientists and poets who resemble one another in their sense of wonder and in the intense way in which they experience the world (Gopnik et al., 1999). As Dijkgraaf (2007) observes: 'It is often said that young children are ideal scientists. They are curious about the world around them. They ask questions, make up theories, and carry out experiments.' This is what is said to give both scientists as well as children their drive to learn (Gopnik, 2004).

In summary, de Lange emphasizes the 'curious minds of young children' (de Lange, cited in Ros, 2006, p. 9) which 'have to be stimulated'. In this sense it is disconcerting to note that many early elementary mathematics curricula focus mainly on developing curricula that teach number sense (Casey, 2004; Clements & Battista, 1992). Indeed several researchers warn about the gap that has been observed at the start of formal schooling between children's informal, intuitive knowledge and interests, and the formal learning opportunities in school (cf. Griffin & Case, 1997; Hughes, 1986; Murphy, 2006). The key point that we are making, then, is that mathematics education for young children should intertwine with and originate from the natural experiences, the enthusiasm, and the interests of young children as they explore of the world.

Gopnik (2004) put the issue for science in general into the following words:

> If we could put children in touch with their inner scientists, we might be able to bridge the divide between everyday knowledge and the apparently intimidating and elite apparatus of formal science. We might be able to convince them that there is a deep link between the realism of everyday life and scientific realism (p 28).

Through acknowledging the early competencies of young children (concentrating on what the children can already do versus what they cannot yet do; see also Gelman & Gallistel, 1978), we should on the one hand be able to come to a greater understanding about what factors influence the development of mathematical thinking and learning, while, on the other hand, stimulating the child's innate curiosity and eagerness to learn mathematics. We focus our research on spatial sense and number sense, the core of mathematics in the early years (NCTM, 2000), and study whether and, if so, how the development of early spatial sense and emerging number sense may be related. For purposes of our argument, we now clarify what we understand to be number sense and spatial sense.

## 3. Emerging Number Sense

The concept of number sense can broadly be defined as the ease and flexibility with which children operate with numbers (Gersten & Chard, 1999). Berch (1999) compiled an extensive list of components that have been related to the construct of number sense from the literature of mathematical cognition, cognitive development, and mathematics education. As such, he states that

> possessing number sense ostensibly permits one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex math problems; from making simple magnitude comparisons to inventing procedures for conducting numerical operations; and from recognizing gross numerical errors to using quantitative methods for communicating, processing, and interpreting information. (p. 334)

As children progress in their ability to count, they discover easier ways of operating with numbers and they come to understand that numbers can have different representations and can act as different points of reference (Berch, 1999; Griffin & Case, 1997; Van den Heuvel-

Panhuizen, 2001). Given the diversity of the definitions of number sense, we focus our research on the development of awareness of quantities, on learning to give meaning to quantities and on being able to relate the different meanings of numbers to each other. This knowledge can then be applied to determining a quantity, to comparing quantities and to preliminary adding and subtracting. Hence, a well-founded number sense is fundamental to the ease and level of understanding with which children progress to higher order mathematical skills and concepts.

Our focus on young children's ability to determine a quantity and to compare quantities is supported by the Central Conceptual Theory described by Griffin and Case (1997; Griffin, 2004b). This theory is grounded in cognitive research with findings on how children by the age of four can make global quantity comparisons and can count. As Gelman and Gallistel (1978) have shown, children by the age of four can count a set of objects and understand that the last named number word represents the quantity of the set. Much recent cognitive research has supported this finding and has extended it to mathematics operations. Berger, Tzur and Posner (2006), for instance, found that six-month old infants can recognize simple addition errors and that the corresponding brain activity can be compared to that of adults detecting an arithmetic error.

Apart from children's ability to count, research by Starkey (1992), for example, has shown that four-year olds possess numerical knowledge that is not yet numerical, but that allows them to make quantity comparisons. Indeed, more recent cognitive psychological research on children's numerical abilities has provided evidence on how infants as young as six months can differentiate between amounts of objects that differ by a 2.0 ratio (i.e. eight versus sixteen objects; Lipton & Spelke, 2003; Xu & Spelke, 2000). This ability has been seen to improve within months as nine-month old infants can already differentiate sets that differ in number at a 1.5 ratio (i.e. nine versus six objects).

Griffin and Case (1997) describe the ability to compare quantities and the ability to count initially as two separate schemas. At the age of four, children have difficulty integrating these competencies, as if 'the two sets of knowledge were stored in different "files" on a computer, which cannot yet be "merged"' (p. 8). A revolutionary developmental step is said to occur by the age of five or six, in which these two schemas merge into 'a single, super-ordinate conceptual structure for number' (Griffin, 2004a, p. 40) in a manner that is described in the Central Conceptual Structure Theory (Griffin, 2004b; Griffin & Case, 1997). Such a conceptual structure covers 'the intuitive knowledge that appears to underlie successful learning of arithmetic in the early years of formal schooling' (1997, p. 8). It connects an understanding of quantity with number and enables children to use numbers without having to rely on objects that are physically present. Hence, this new conceptual structure provides children with the conceptual foundation for number sense which is believed to fundament all higher-level mathematics (Griffin, 2004a).

The learning of number and operations in early childhood may be the best-developed area in mathematics education research (Baroody, 2004; Clements, 2004; Fuson, 2004; Steffe, 2004). Yet, other research has shown that spatial thinking skills and mathematics achievement of relatively older children are related (Bishop, 1980; Clements, 2004; Guay & McDaniel, 1977; Smith, 1964; Tartre, 1990a, 1990b). For this reason, the NCTM standards (1989, 2000) strongly recommend increasing the emphasis on the development of spatial thinking skills through the teaching of geometry (the mathematics of space; Bishop, 1983) and spatial sense. In the next section we discuss three components of spatial sense that we consider to play an essential role in the development of young children's mathematical abilities.

## 4. Early Spatial Sense

Spatial sense can be defined as the ability to 'grasp the external world' (Freudenthal, in National Council of Teachers of Mathematics [NCTM], 1989, p. 48). In our view, this spatial sense consists of three main components that are most essential for enabling young children to 'grasp the world' and to develop mathematical thinking: spatial visualization, geometry ('shapes' in short), and spatial orientation ('space' in short). These components can be recognized in the foundations of comprehensive mathematics curricula for the middle grades such as Mathematics in Context (1998).

Spatial visualization involves the ability to imagine the movements of objects and spatial forms. In spatial visualization tasks, all or part of a representation may be mentally moved or altered (Bishop, 1980; Clements, 2004; Tartre, 1990a). This has been conceptualized as the ability to make object-based transformations where only the positions of the objects are moved with respect to the environmental frame of reference whereas the frame of reference of the observer stays constant (Zacks, Mires, Tversky & Hazeltine, 2000).

An example of a daily activity in which, already, young children have to apply spatial visualization skills, is when they imagine where in the kitchen it is that they can find their snack before they walk into the kitchen to get it. Recent cognitive research on children's spatial skills has shown how 16-24 month old infants can use the concept of distance to localize objects in a sandbox (Huttenlocher, Newcombe, & Sandberg, 1994). This has suggested an early competence to judge distances that is manifested regardless of the presence of any references in the direct surroundings of the child. Such an ability requires spatial visualization skills for creating a mental picture of the location of the object.

Geometry lessons in school should teach young children about shapes and figures and help them learn to refer to familiar structures such as their own body, to geometrical structures such as mosaics, and to geometrical patterns such as dot configurations on dominoes (cf. Clements & Sarama, 2007). This type of communication may help increase their vocabulary and enrich their imagination (Casey, 2004; Newcombe & Huttenlocher, 2000). Hence, geometric activities can stimulate the children's ability to sharpen and talk about their perceptions, which in turn helps develop children's spatial sense and reasoning skills (Van den Heuvel-Panhuizen & Buys, 2005). Indeed NCTM (1989, p. 48) has described spatial understandings as necessary for interpreting, understanding, and appreciating our inherently geometric world.

The third component that we name in the context of how children may 'grasp the world' is spatial orientation. This is the term that Clements (2004, p. 284) uses to describe how we 'make our way' in space. As children discover their surroundings, they gain experiences that help them to understand the relative positioning and sizes of shapes and figures (Van den Heuvel-

Panhuizen & Buys, 2005). As such, children learn to orientate themselves, to take different perspectives, to describe routes and to understand shapes, figures, proportions and relationships between objects.

Many of the activities in spatial orientation are examples of competencies that are typically manifested even before these children begin their formal schooling. A cognitive study with four and five-year olds, for example, provided evidence that at this age children can already compare proportions and figures (Sophian, 2000). The children in this study were able to match the correctly shrunken picture to the original picture without being distracted by pictures that not only were smaller, but also disproportional to the original picture. Studies such as this one exemplify the remarkably developed spatial sense that many children possess prior to the start of formal schooling.

Now that we have illustrated what we mean by emerging number sense and early spatial sense, we turn to why and how in our research we suspect a relationship to exist between these two constructs.

## 5. Relating Early Spatial Sense to Emerging Number Sense: Spatial Structures

To analyze the development of number and spatial sense of young children, we must first take a step back and find inspiration in how young children learn and think in general. In the process of learning and understanding, young children continuously try to organize new concepts and information about the world (de Lange, 1987; Gopnik, 2004; van den Heuvel-Panhuizen, 2001). Structuring is one fundamental method for children to organize the world (Freudenthal, 1987). In effect, this method of organization contributes to gaining insight into important mathematical concepts such as patterning, algebra, and the recognition of basic shapes and figures (Mulligan, Mitchelmore, & Prescott, 2006; Waters, 2004). Freudenthal even believed that there is no other science in which organization plays such a crucial role as in mathematics (1991). He described mathematics as

> an activity of solving problems, of looking for problems, but it is also an activity of organizing subject matter. This can be matter from reality which has to be organized according to mathematical patterns if problems from reality have to be solved. (1971, p. 413-414)

As children develop through experience, they improve their ability to organize incoming information and they learn to amend their organization schemes accordingly. Piaget regarded knowledge as structures that become increasingly complex through the processes of accommodation, assimilation, and equilibration. When a child with a certain method of thinking experiences something that no longer fits with this method of thinking (cannot assimilate), then it is put off balance until the method of thinking is adjusted (accommodated) and the system is balanced again (equilibrated). In this way, children are believed to reach more sophisticated means of thinking.

Van den Heuvel-Panhuizen (2001) gives an example of a practical mathematical situation in which the learning process illustrated above can be recognized. In this example, four-year old Anita is trying to connect meaning and purpose to the numbers that she is hearing:

> Anita is in a pancake restaurant with her father. They have just chosen a pancake from the menu. "I want pancake twelve," says her father to the waitress. "And pancake seven for this young lady." Anita cries: "But I can't eat *that* many pancakes...!" (p. 29)

Experiences such as these can set young children's thinking off balance and force them to adjust their definitions and frames of reference. Children learn from this, adjust the structure of their mode of thinking and, in doing so, reach a higher level of understanding.

The type of structure discussed thus far is mostly conceptual in nature in the way that it contributes to learning and understanding. Much research has concentrated on such a type of structure in thinking (cf. Dienes, 1960; Sriraman, 2004; Van Hiele, 1997). The particular type of structure that our study is concerned with is analogous to this conceptual structure, and yet it is more concrete. It is structure that fits with children's experiences and current levels of spatial reasoning and it is structure which they may impose on manipulatives to support their mathematical learning and understanding.

To illustrate what we define as structure, we make use of the definition that Battista (1999) gave to describe the act of spatial structuring. In his view, spatial structuring is

> the mental operation of constructing an organization or form for an object or set of objects. It determines the object's nature, shape, or composition by identifying its spatial components, relating and combining these components, and establishing interrelationships between components and the new object. (p. 418)

A spatial structure, then, is a product of this act of organizing space. Such a structure is an important element of a pattern. In line with Papic and Mulligan (2005), we may define a spatial structure in terms of a pattern. A pattern is a numerical or spatial regularity and the relationship between the elements of a pattern, then, is its structure. In particular, we refer to a spatial structure as a configuration of objects in space. This relates to the component 'spatial regularity' in the given definition of a pattern. The component 'numerical regularity' refers to numerical sequences that are not relevant to the mathematical abilities of four- to six-year old children. Examples of spatial structures that children of this age are typically familiar with are dot configurations on dice, finger counting images, rows of five and ten, bead patterns, and block constructions (illustrated in Figure 1).

In reference to the three components of early spatial sense that we elaborated on earlier, we suggest that spatial structures may play a supportive role in the development of number sense. Specifically, the intertwinement of the three components may contribute to children's understanding of quantities and relationships between numbers. We propose that once children can imagine (i.e. spatially visualize) a spatial structure of a certain number of objects (i.e. configuration of objects that makes up a shape) that are to be manipulated (in a space), then learning to understand quantities as well as the process of counting (i.e. emerging number sense) should greatly be simplified. This hypothesized relationship between early spatial sense and emerging number sense is depicted in the figure below.

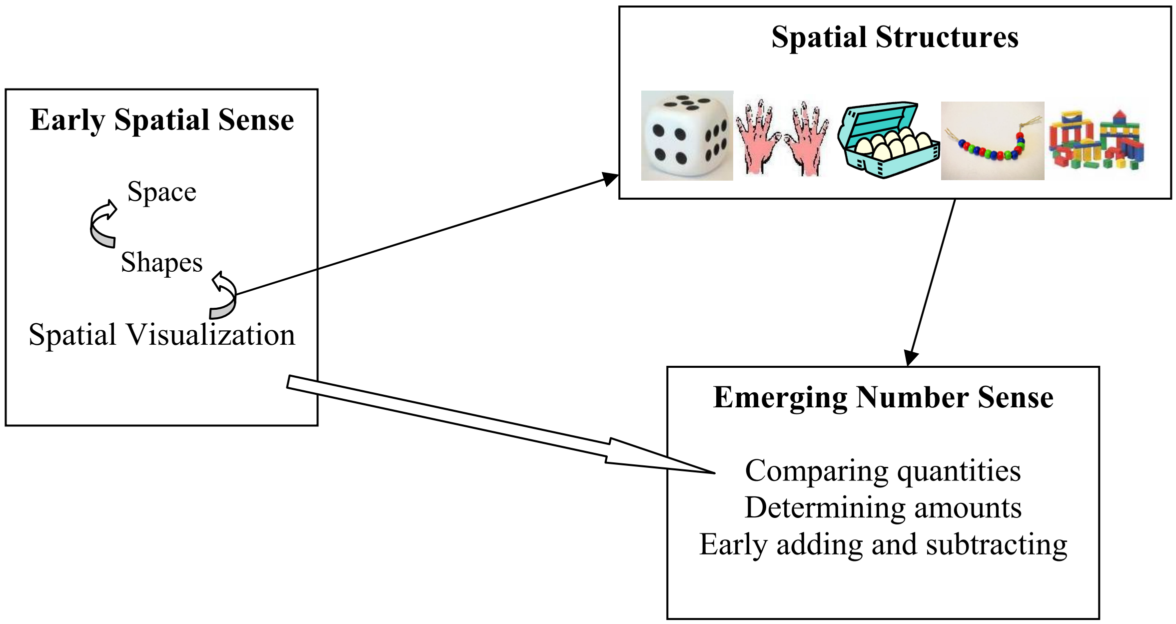


*Figure 1. Spatial structures as a key factor in how early spatial sense may support the development of emerging number sense*

After setting out why we suspect spatial structures to relate early spatial sense to emerging number sense, we continue our argument with illustrations of how spatial structures may play a supporting role in the development of mathematical abilities.

## 6. Spatial Structures in Early Numeracy Problems

To illustrate and support our concern with the role of spatial structures in the development of emerging number sense, we refer to Arcavi (2003) as one researcher who set out to define visualization and to analyze the various different roles that it may play in the learning and teaching of mathematics. Visualization, in his context, requires spatial visualization since it involves the interpretation and reflection upon pictures and images. Arcavi considers visualization to be at the service of problem solving because it may inspire the solution to a problem. In determining how many matches were needed to build an exemplar nxn square, for instance, most students used visual means to solve the problem. These visual means took different forms, one of which was the decomposition into what the students perceived to be easily countable units. This was a first step into changing the 'gestalt' (roughly the whole or the form) of the configuration.

It is the use of the term 'gestalt' in this context that supports our argument and indicates how students can simplify the mathematical problem by spatially visualizing objects into particular shapes in a space. For Arcavi's students, the 'gestalt' could involve 'breaking and rearranging the original whole' or 'imposing an "auxiliary construction" whose role consisted of providing visual "crutches", which in themselves were not counted, but which supported and facilitated the visualization of a pattern that suggested a counting strategy' (Arcavi, 2003, p. 229).

Several studies have related the Gestalt laws to early development. Spelke and colleagues (1993), for example, found that while the perceptions of adults were strongly influenced by the Gestalt relations of color, texture similarity, good continuation, and good form, the perceptions of 5- and 9-month olds were only weakly affected, and the perceptions of 3-month olds were not at all affected. This suggests a developmental course of these particular Gestalt relations (cf. Quinn et al., 1993; 2002). Taken together, these studies highlight how even infants as young as three months are capable of distinguishing particular elements of and establishing crude perceptual coherence.

An anecdote of Richardson (2004) about the children in her preschool classroom illustrates how the extraction of spatial structures may occur in practice. Richardson had her children work with dot cards (showing configurations such as those on dice) so that they could learn to recognize amounts in such arrangements. When, one day, she asked the children to count out a certain number of counters, she was astonished to find that instead of correctly counting out the counters, the children made an 'X' shape to match what the children recognized to be the shape of five dots on a card, and they made a square shape to match what they recognized to be the arrangement of nine dots. Apparently, then, these children extracted a shape from the individual dots on cards and taught themselves that this shape should resemble a particular number.

Richardson (2004) concludes from this experience that teachers must always interact with the children to check whether what they are doing makes sense to them, because performing without understanding interferes with the development of their mathematical abilities. More than that, it is a practical example of how children extract a general shape from individual elements and it adds on to the finding that infants can deploy Gestalt principles to make sense of the real-world and to establish perceptual coherence. The ability to process the gestalt, the whole, is an important requirement for mathematical skill as it is one ability that should help simplify and shorten the children's process of learning to determine quantities (Van Eerde, 1996; Van Parreren, 1988). Such supporting evidence for children's tendencies to organize the world through the use of spatial structures, should encourage mathematics educators to take care to weave spatial abilities into early mathematics curricula.

Children typically begin to formalize their understanding of quantities by connecting a certain quantity with spatial structures such as a number of fingers that are being held up on a hand or dot configurations on a pair of dice. As Smith (1964, as cited in Tartre, 1990a) put it,

> the process of perceiving and assimilating a gestalt...[is] a process of abstraction (abstracting form or structure)... It is possible that any process of abstraction may involve in some degree the perception, retention in memory, recognition and perhaps reproduction of a pattern or structure" (p. 213-214).

These spatial structures require a child to use its spatial visualization skills for organizing and making sense out of visual information. The mental extraction of structures from spatial configurations is also what Arcavi (2003) found to aid the counting process of his students.

Although the students in Arcavi's study were older than the age group in our project, one can imagine how young children can also use 'gestalts' to rearrange objects that are to be counted, for example. The spatial structure that subsequently arises can help the child to oversee the quantity (Van Eerde, 1996; Van Parreren, 1988).

As illustrated in Figure 1, we propose that the spatial visualization abilities help the child to perceive the 'gestalt' or spatial structure, in order to either mentally or physically be able to rearrange the objects in a space. The spatial structure that subsequently arises can simplify early numerical procedures. When young children are asked to determine the quantity of a randomly arranged set of objects, they initially tend to count each object. As the set of objects grows, this procedure eventually confronts them with the difficulties of keeping track of which objects have already been counted and with the time-consuming process that accompanies the counting of larger sets.

The benefit of applying spatial structure to mathematical problems is evident, for instance, when reading off a quantity (i.e. seeing the quantity of six as being three and three), when comparing a number of objects (i.e. one dot in each of four corners is less than the same configuration with a dot in the center), when continuing a pattern (i.e. generalizing the structure) and when building a construction of blocks (i.e. relating the characteristics and orientations of the constituent shapes and figures). Here too, then, children's ability to grasp spatial structure appears essential for developing mathematical abilities such as ordering, comparing, generalizing and classifying (NCTM, 2000; Papic & Mulligan, 2005; Waters, 2004).

More formal mathematical skills require even further insight into and use of spatial structure. This is particularly the case for addition, multiplication and division (i.e. $8 + 6 = 14$ because $5 + 5 = 10$ and $3 + 1 = 4$ so $10 + 4 = 14$; Van Eerde, 1996), for using variables in algebra, for proving, predicting and generalizing, and for determining the structure of a shape in order to subsequently mentally rotate or manipulate it (Kieran, 2004). Various studies have shown that children with serious mathematical problems tend not to use any form of structure and continue to count objects one by one (Mulligan, Mitchelmore, & Prescott, 2005; Van Eerde, 1996). This accentuates the need for children to be familiar with various spatial structures in order to simplify the progression to more formal mathematical concepts and procedures.

## 7. Preliminary Experimental Support

Thus far, we have set out much of the theoretical support for why and how we propose that early spatial sense and emerging number sense may be related. Alongside this are some preliminary outcomes of a previously conducted explorative study (van Nes & de Lange, in press; van Nes & Doorman, 2006) in which we set out to investigate the strategies that four- to six-year old children use to solve various number sense and spatial thinking problems.

One outcome from the explorative study was that four- to six-year old children with relatively stronger mathematical skills seemed to make more use of spatial structures than other children did. These children recognized the spatial structures that were presented and knew to implement these spatial structures for simplifying and speeding up counting procedures. Interestingly, however, there were several low achieving five- and six-year old children who seemed to recognize the spatial structures, and yet who did not proceed to applying the structures to solve the problems. These particular cases triggered our interest into what role insight into spatial structures may play in the development of emerging number sense and, ultimately, in the child's level of mathematical achievement.

The findings from our explorative study complement research of Mulligan, Prescott and Mitchelmore (2004) in which they conducted an analysis of structure present in 103 first graders' representations for various tasks across a range of mathematical domains. They coded the individual profiles as one of four stages of structural development and found that mathematical structure in children's representations generalizes across various mathematical domains. Recently, Mulligan, Mitchelmore and Prescott (2005; 2006) developed a Pattern and Structure Assessment (PASA) interview and a Pattern and Structure Mathematics Awareness Program (PASMAP) to study whether the mathematics of low achieving students can be improved through explicit instruction about structures and patterns in mathematical domains. The preliminary results showed improved mathematical achievement, suggesting that explicit instruction of mathematical pattern and structure can stimulate student's learning and understanding of mathematical concepts and procedures.

Taking the theoretical background and the preliminary findings together, we summarize the research questions of the present study from the perspective of mathematics education as:

1. How are early spatial sense and emerging number sense related and what role may spatial structures play in this development?
2. How can spatial visualization be implemented in educational practices to support the development of number sense?

In order to answer these two research questions we concentrate on designing a teaching experiment in which we may study how the development of spatial sense and number sense may be stimulated in an educational setting. This last issue will be investigated in terms of a design research methodology.

## 8. An Instruction Experiment

In gaining an understanding of how children recognize and apply spatial structures to numerical problems, it is important to decide on a methodology that is appropriate for highlighting the processes that occur in the mind of the child from the perspective of the child. The methodology that appears to be most in line with the principles of TalentPower, is inspired by the main theoretical insights of researchers in mathematics education such as Freudenthal (1984, 1991), Dienes (1960) and Van Parreren (1988). This generally concerns a methodology that is focused on a child's learning processes, that applauds dialogue and interaction, that emphasizes the stimulation of the own actions of the child, and that rejects mechanistic mathematics education (Van Eerde, 1996).

The activities for the instruction experiment stem from the tasks that we developed, tried out and improved in the previous exploratory studies (van Nes & de Lange, in press; van Nes & Doorman, 2006). Next to being based on the abovementioned theoretical insights, these tasks were originally inspired by experimental outcomes and practical experiences as described in related literature (van den Heuvel-Panhuizen, 2001, for example) and developed with input from experts. We also assessed the appropriateness of the tasks in terms of their coherence with the outcomes of the Utrecht Numeracy Test (UNT, van Luit et al., 1994). This is a normed test for assessing the number sense of 4.5- to 7-year old children. We compared the children's scores on this test with their accuracy scores as well as with the level and types of strategies that they used on the tasks. As we were easily able to come to a consensus about the scoring of the tasks, the strategy classifications and their agreement with the UNT scores, we decided that the tasks would be suitable to work out into a series of activities for use in the instruction experiment.

As the methodology is based on the guidelines of 'design research' (Freudenthal, 1978; Gravemeijer, 1994, 2004; Gravemeijer, Bowers, & Stephan, 2003; Streefland, 1988), our theory will cohere with direct experiences from an educational setting. This should keep the findings both theoretical and practical. It will involve an iterative procedure of theory-driven adjustments to the intervention and amendments to the hypotheses that lead to an improved and evidence-based theory (Freudenthal, 1978; Gravemeijer, 1994; Streefland, 1988). Freudenthal (1991) referred to such a research design as an instruction experiment because the activities are meant to broaden the children's insight into spatial visualization, into the perception and application of spatial structures, and, ultimately, into the characteristics of quantities and numbers while, at the same time, providing the researchers with a greater understanding of the children's learning processes. The aim, then, is not necessarily to conclude *that* the series of activities teach the children about spatial structures, but more to come to an analysis about *why* the series of activities may have stimulated the children's thinking (Gravemeijer et al., 2003).

In order to study the children's thinking processes, the series of activities should guide the children along a so-termed conjectured local instruction theory (Gravemeijer, 1994; Simon, 1995). The conjectured local instruction theory is a learning trajectory based on mathematical, psychological, and didactical insights about how we expect that the children will progress from their original way of thinking to our aspired way of thinking. To ensure the practicality of our findings, we must take into account both the cognitive development of the individual students, as well as the social context (i.e. people, setting and type of instruction) in which the instruction experiment is to take place (Cobb & Yackel, 1996).

The cyclical process that characterizes design research is illustrated in the diagram below. In practice this means that we will implement the series of activities in an instruction experiment, perform retrospective analyses on the transcripts from these lessons, adjust our hypotheses accordingly in a thought experiment and improve the activities in line with the amended conjectured local instruction theory. Then we repeat the procedure by implementing the new set of activities in a subsequent cycle, and learning from the class-experiences for, once again, fuelling the next thought experiment. This process will contribute to establishing and refining our conjecture local instruction theory.

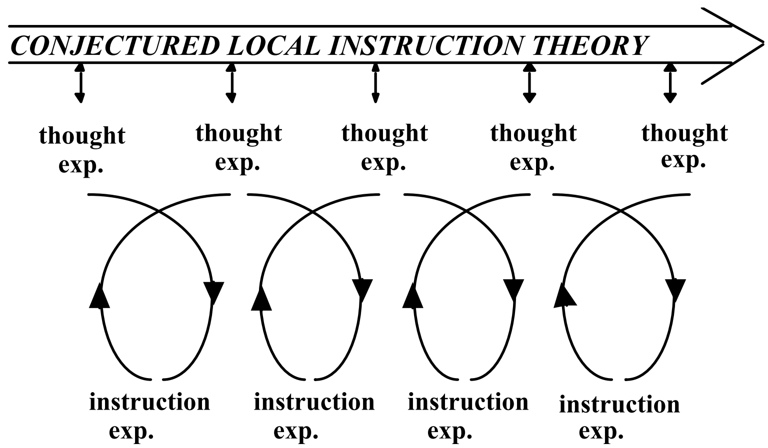


*Figure 2. The cyclical procedure of design research (Gravemeijer, 2004)*

## 9. Summary and Conclusion

After providing a broad overview of the theoretical framework that is propelling the MENS research, it is clear that young children possess spatial and numerical skills that should be cultivated in educational practice. As such, the aim of this research is to bring the spatial sense of young children to the fore and illustrate how spatial skills could function to stimulate the development of more formal mathematical skills that require number sense.

Supported by various fields of research, we consider spatial visualization, insight into shapes and an understanding of space to be three main components that make up young children's early spatial sense. As such, we suggest that children's spatial visualization skills contribute to their ability to organize representations of objects into spatial structures (such as dice configurations and finger images). These spatial structures relate to the children's conceptions of shapes with which they become familiar through exploring their surrounding space. Children's concepts of quantities and number, then, may greatly be stimulated when children are made aware of the simplifying effects of structuring manipulatives.

As soon as we have cycled through enough instruction and thought experiments to fundament our conjectured local instruction theory, we will turn to our colleagues for comparing and contrasting the results of the research perspectives of mathematics education and educational neurosciences. The neuroscientific perspectives may supplement our research with results from studies on brain behavior and neural correlates with respect to early spatial and numerical thinking. Ultimately, in line with the principles of *TalentPower*, the collaboration of these research perspectives should provide a more all-round and in-depth understanding of how education can foster the talents of young children and possibly stimulate those children who may be prone to experiencing problems in the development of mathematical skills.

As Tartre (1990a) stated in a discussion on spatial orientation,

> attempting to understand and discuss something like spatial orientation skill, which is by definition intuitive and nonverbal, is like trying to grab smoke: the very act of reaching out to take hold of it disperses it (p. 228).

She notes that any attempt to verbalize spatial thinking no longer is spatial thinking since spatial thinking is only a mental activity. We recognize that research into spatial sense is always an indirect attempt at trying to understand what is happening in the mind. Nevertheless, by taking into account the three components that we associate with spatial sense, and by relating them to each other in the way that we are, we aim to gain an understanding of how young children's early spatial skills may help them progress in their mathematical development. This is how we intend to better appreciate and more effectively cultivate young children's cognitive capacities that too often are underestimated or even neglected.

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