

6-2007

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Jankvist, Uffe Thomas and Toldbod, Bjørn (2007) "Mars Exploration Rover: Mathematics and People behind the Mission," *The Mathematics Enthusiast*. Vol. 4 : No. 2 , Article 3.

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Mars Exploration Rover

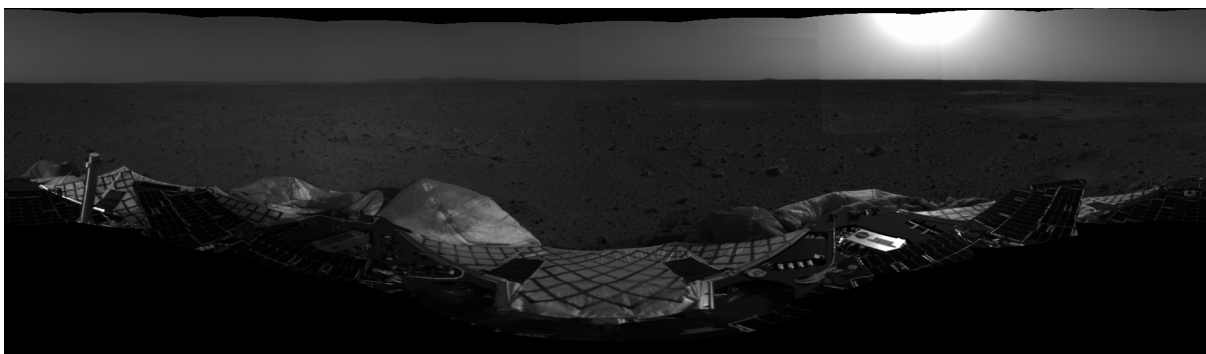
Mathematics and People behind the Mission

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Abstract

This paper is a selective study of the mathematics and people involved with a specific space mission, namely the Mars Exploration Rover (MER) mission launched in 2003. The specific mathematics of the MER mission are sought uncovered through interviews with applied scientists who worked with different aspects of the mission at the Jet Propulsion Laboratory (JPL). Some of the more specific questions attempted answered through this article for instance concerns if the aerospace industry, exemplified by the MER mission, calls for new developments in mathematics or if it mostly relies on well established theories; how much independence the scientists have in their daily work and in their choosing of solutions to problems and whether or not this variate within different areas of the mission; how ready the aerospace industry is for new solutions and new ideas; and to what extent the economics of the missions play a part in this.

Keywords: Mars Exploration Rover (MER) mission, Jet Propulsion Laboratory (JPL), aerospace industry, applied mathematics, applied scientists.¹



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¹ The image on this page is the first picture of the Gusev Crater made public to the press. The picture, which is a mosaic, is taken with the navigation camera onboard the Mars Exploration Rover *Spirit*. http://nssdc.gsfc.nasa.gov/planetary/mars/mars_exploration_rovers/mera_images.html

Consider the recent flight to Mars that put a ‘laboratory vehicle’ on that planet. Whether or not that excites you, you must admit that it is an almost unbelievable technological accomplishment. Now, from start to finish, the Mars shot would have been impossible without a tremendous underlay of mathematics built into chips and software. It would defy the most knowledgeable historian of mathematics to discover and describe all the mathematics that was involved. The public is hardly aware of this; it is not written up in the newspapers. (Davis; 2004)

The adventures of the Mars Exploration Rover Mission truly began on January 4, 2004, when the MER A rover named *Spirit* entered the Martian atmosphere and performed a perfect landing in the Gusev Crater. Later that month, on January 15, the second rover (MER B) known as *Opportunity* landed in the Terra Meridiani. But before these points of time the mission had, of course, been under preparation for years. The place of preparation was the *Jet Propulsion Laboratory* (JPL) in Pasadena, California.

Introduction

In March 2005 we spent a week at NASA’s Jet Propulsion Laboratory as part of our joint master thesis² at the mathematics department of Roskilde University. The purpose of our stay was to conduct a small investigation of the work related to the Mars Exploration Rover (MER) Mission being performed at JPL and in particular the work relying on the use of mathematics.

The basic idea of this investigation was provided to us by Professor Emeritus Philip J. Davis who in the fall of 2004 implicitly had suggested this in an article from which the introductory quote of this paper is taken (Davis; 2004). Even though we were not “the most knowledgeable historians of mathematics” we still decided to go ahead with an investigation of what mathematics was involved in the MER mission. Unfortunately newspapers were not



Figure 1 Visiting Brown University. Left: Professor Emeritus Philip J. Davis. Right: The Brown University’s Division of Applied Mathematics.

² The thesis consists of the texts (Jankvist & Toldbod; 2005a), (Jankvist & Toldbod; 2005b) and (Jankvist & Toldbod; 2005c) and can be found in its original Danish version as IMFUFA-text number 449 at <http://mmf.ruc.dk/imfufatekster/index.htm>

the only place in which this wasn't written up. In fact finding extensive literature on the subject was so difficult that we decided to base the investigation on interviews. Hence the long travel from Roskilde, Denmark to Pasadena, California.

While in the U.S. we decided to take a detour to visit Davis at Brown University in Providence, Rhode Island in order to discuss our pending investigation at JPL. Davis advised us to "be a little bit like journalists rather than teachers of abstracts mathematics" in order to make a more interesting story – perhaps even a story of *general* interest (Davis; 2005). Davis also gave us the following advice:

I think you want to go also for what you might call the human side of the story. When you interview these people try to get something about their background, what were their experiences before they came to the JPL, what their training is [...] how old they are and how they see their professional future [...] Try to find out what each one does, how much independence they have, how much opportunity is there for them to develop new ideas or new things whether it's in mathematics or whether it's in writing algorithms, software and that kind of thing. [...] are they using packages or are they developing their own stuff, how is this going to fit in to future projects that NASA has? [...] I think you want to make a story here of what it means to work in a space program as a mathematician or a computer scientist or whatever these people are. (Davis; 2005)

To a large extent we have tried to follow the advice of Professor Davis. While at the JPL we tried to get an insight into the employees' personal motivations for working in the aerospace industry as well as an insight into the nature of the mathematical work performed at JPL. Also we looked at what one might call external influences on the daily work, such as deadlines and basic economical limitations. This might also be called the basic *work context*.

In our investigation we have tried to combine the human/daily work approach with a selective study of some of the mathematical problems that arise in a space mission like MER.



Figure 2 Jet Propulsion Laboratory in Pasadena, Californien. Left: JPL seen from above. http://ipac.jpl.nasa.gov/media_images/jpl_small.jpg Right: One of JPL's main buildings. <http://www2.jpl.nasa.gov/files//images/browse/jpl18161ac.gif>

As is concluded in the article³ most of the mathematical problems that arise in a space mission are well known and well described problems (at least from a mathematical point of view). Therefore detailed analysis of the problems will be omitted in the article.

Rather we will use the mathematical problems to illustrate some of our observations regarding how the basic work context of the institution influence the typical approaches taken to solving such mathematical problems.

Both the human approach to JPL and the selective study are sought uncovered by letting the JPL scientist speak for themselves to some extent, i.e. by frequently quoting from our interviews⁴. Before getting to this, however, a small presentation of the Jet Propulsion Laboratory is in order.

The research institution ‘the Jet Propulsion Laboratory’ is located an out-of-the-way place on the way up in the mountains, surrounded by forrest a little north of the Los Angeles suburb of Pasadena. After about half an hour of driving from downtown Pasadena you reach the area’s entrance where you are met every day by uniformed guards. As an outsider you get the firsthand impression that this might be an entrance to a ‘secret world of science’. On our first day we were met in the reception by a lady named ‘Bobby’ who informed us about the safety procedures of the place and then telephoned Dr. William Folkner to announce our arrival. Folkner showed up a few minutes later and took us around the areas of JPL. During our stay we met around a dozen of JPL employees, researchers and scientists, people whom we shall introduce in the following section of this article.

People of JPL

As the main focus of our master thesis was coding theory (which includes both image compression and error correcting codes), we interviewed Dr. Aaron Kiely and Dr. Matthew Klimesh who were deeply involved in the making of ICER – the algorithm used by the rovers for compressing images taken on the surface of Mars, as well as Dr. Jon Hamkins, who told us about the use of error correcting codes for reliable transmission in deep space. We also were fortunate to interview Dr. Mark Maimone who worked with the steering mechanisms of the rovers. Finally we had the pleasure of interviewing Dr. Jacob Matijevic, a mathematicians who had been a long time with JPL and Dr. Miguel San Martin (engineer) with whom we discussed various aspects of the mission.

Dr. William Folkner was the person at JPL with whom we had the most contact and therefore also the person we had most opportunities to interview. We talked to Folkner about the general conditions for working with a space mission and the typical problems that needs solving, as for instance orbit calculations for the spacecrafts. Besides that we also had the opportunity to ask him about his carrier and his view of the people working at JPL. Folkner told us that he himself had taken a Ph.D. in physics from the University of Maryland. After completing his Ph.D. he decided to apply for a job at the JPL to become part of the planetary

³ A slightly different version of this article in Danish has also appeared in the Nordic mathematical journal *Normat* (Toldbod & Jankvist; 2006). The *hidden* mathematics of the Mars Exploration Rover mission and anticipated consequences of the mathematics being hidden are described in an article in *The Mathematical Intelligencer* (Jankvist & Toldbod; 2007).

⁴ Transcriptions of these interviews in their full length along side with the Davis conversation can be found in (Jankvist & Toldbod; 2005c).

exploration. This was in 1988. We asked him if he thought his way to becoming an employee at JPL and his following carrier at the institution was a typical one.

Most of the people I know have been here a long time or tend to be here a long time. There aren't many people who come here who just want to be here a year or two and then go away again. People who want to do space work tend to *want* to do space work. I think this is the best place in the world to do space work. The people here are all very, very good. They are all very dedicated and they want to make these things fly. (Folkner; 2005)

According to Folkner the strongest motivation for the majority of JPL's employees is the fascination of the missions themselves – a matter we got confirmed several times during our stay. Mark Maimone told us that he wished to stay at JPL as long as possible. He was educated in engineering and had completed a Ph.D. in Pittsburgh, Pennsylvania followed by a post doc in *space robotics* before receiving a position at JPL in 1998. He explained the following about his motives for wanting to work at JPL:

When I was a kid I saw the Viking missions on Mars and I thought that would be pretty neat, hunched over the terminal looking at these pictures that nobody else would see for a year before it got published. But of course now every picture we take gets published on the Internet the next day so it's not much of an advantage but it's still nice. (Maimone; 2005)

We also discussed the personal motives for wanting to work at JPL with Dr. Aaron Kiely and Dr. Matthew Klimesh. Both of them had studied in Michigan and held Ph.D. degrees. They had come to JPL shortly after completing their studies and both of them wished to stay as long as possible (Kiely; 2005) (Klimesh; 2005).

From what we learned through the interviews a general characterization of JPLs scientists would be people with the highest educational level who join the institution shortly after completing their university studies. They are driven by a desire to be part of the aerospace industry and a passion for planetary exploration. To some extent they were of course also fascinated by the mathematical, physical and engineering problems involved in space exploration, but as a motivating factor this seemed only secondary.



Figure 3 Guided tour at JPL. Left: Uffe and Dr. Albert Haldemann who showed us some of the facilities. Right: Visiting JPL's museum for earlier space missions with Dr. William Folkner.

Work at JPL

One of the first persons we discussed the mathematical aspects of the work at JPL with was Jacob Matijevic. Particularly we discussed the modelling aspects of the work which takes place before the actual mission is set in motion.

A mission like MER is to a large extent about being able to predict how the technology onboard the craft is going to behave in space or in the Martian environment. Once the craft is flying it is impossible to make adjustments which demand more than just a radio signal. Therefore everything must function as expected.

Take, for instance, the Mars environment's influence on the instruments onboard Spirit and Opportunity. You have to have a very precise knowledge about how heat and cold distributes inside the rover and how this affects the instruments. To acquire such a knowledge virtual models of the rovers are built in software so that the thermic conditions can be simulated. Such thermic models are typically based on a number of differential equations which are solved within the programs. The work for the JPL employee consists of building the virtual model of the rover. The exact method of solution which the program implements is to some degree subordinate to the JPL scientist as long as it works and isn't too slow.

According to Matijevic (Matijevic; 2005) you also need to have models of how the environment depends of the seasons on Mars to be able to predict the concrete influence on the instruments in the above mentioned models. Such models are partly based on data from the different Mars orbiters and partly on concrete measurements performed on the Martian surface. The correctness of the surface measurements to a large extent depends on how good the description of the instrument's behaviour in the Martian environment is, and can therefore not be guaranteed. By comparing the data from the orbiters with the surface measurements in question a more accurate picture may arise though and this may then be used to modify the models, so that these slowly become better and better. All of this is done in software. Regarding the models of how the seasons affects the Mars environment it is probably fair to compare the work at JPL with the work performed by an institute of meteorology. Matijevic told the following about this work:

When I first arrived here over twenty years ago there were still efforts to hand-implement certain mathematical models for certain applications. And there were specialist applications here for specialists in the applied mathematical sciences who worked here to make those applications possible. But over time much of that has been incorporated in fairly standard and available simulation and modelling packages – computer packages. Expansions have been introduced slowly over time to these packages and that's basically how the engineers here do their job. Instead of going back to first principles they apply these tools... the foundation theories are from the eighteenth century to a large degree. (Matijevic; 2005)

Hence a lot of work involving modelling and simulation is done at JPL, but all of this is done in software packages. This might lead one to suspect that JPL has its own staff of mathematicians developing such packages, but Matijevic informed us that the packages mostly come from commercial companies.

A few days after our interview with Matijevic we had the opportunity to interview Dr. Miguel San Martin. He told, with great enthusiasm, about the challenges which the scientists must overcome to make the rovers able to figure out their orientation on the Martian surface.

San Martin explained that the navigation on the surface is based on a well known technique which sailors have used for thousands of years on Earth – you look at the Sun. Together with a vector of gravitation, which can be measured by the rover, the position of the Sun on the sky all in all provides the information necessary to figure out the rover’s position on the surface. San Martin himself didn’t think of this problem as being very mathematical. In fact he claimed that the majority of the mathematics involved in his work was very simple of nature and he concluded:

The most important is that you have millions of these little, simple things. And that’s the trick; to make them all work, and talk to each other and make sure that no parameter is tightened too much or too little. The complexity of the space problem is keeping it simple. (San Martin & Folkner; 2005)

Folkner, who was also present during the interview, elaborated on this view:

Well, you hit a lot of mathematics in your descriptions, right, because you need to know the positions of the axes of Mars around the Sun as a function of time and you need to know what the orbits around the Sun and the Earth were. There is a lot of mathematics hidden in what you just said. [...] We’ve worked all that out for us in the tables. So to know where the Sun is now, you just look it up.

Somebody had to figure it out the first time. (San Martin & Folkner; 2005)

Folker’s answer illustrates why the question of what mathematics is used in MER is difficult to answer. Knowledge that mathematics previously have made accessible can over time become such an integrated part of our conception of the world that we no longer connects it with mathematics. The trajectory of Mars around the Sun as a function of time is a good example of this: Not many will consider looking in a table to see the Sun’s position relative to Mars at a given time as being mathematics. The making of such a table on the other hand *is* a mathematical problem. Thus, the mathematics is very much a part of the scientific work at JPL, but is to a large extent disguised as ‘common knowledge’ and may therefore in a sense be hidden to the scientist.

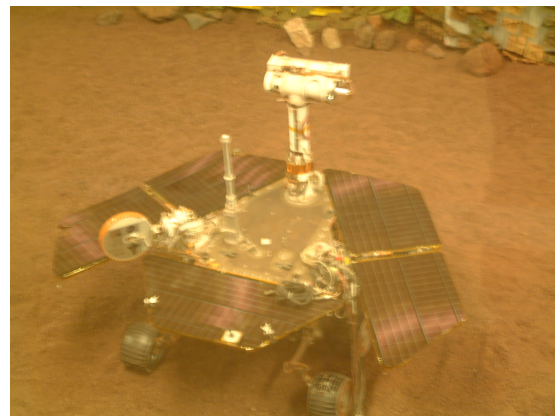


Figure 4 The guided tour takes us by JPL’s “sandbox” where rovers are test driven. Left: Bjørn Toldbod in front of the sandbox. Right: A replicate of a Mars Exploration Rover used for test drives at JPL.

The Demand for Reliability

A project like MER sometimes involves around a thousand people at a time. Such a grand scale project of course calls for a huge amount of planning and bringing up to date between different departments and working groups. Besides this an incredibly high reliability of the work performed is demanded. A single mistake in a piece of technology or an algorithm may have serious consequences and in the worst case may result in several years of wasted work for hundreds of people. All of the work being done at JPL is therefore subject to careful development and testing.

The extent to which such planing and testing is done can be hard to imagine. We asked Jacob Matijevic about the development of the parachutes for the rovers, partly because we thought there would be little interaction between the parachutes and other technological devices. In other words, we thought this would be a ‘simple’ task. Matijevic, however, explained the following about the work with the rovers parachutes:

We did drop tests. We did wind tunnel tests with the parachutes. But even before this time it was through models of the profiles of these devices that we came up with things like what the entry angles would be, what sorts of release points should we be looking at, as well as designing the algorithm that checks for height above the surface and finding out at which time to deploy the parachute and at which time to fire the rockets for slowing the descent. All of this was based on what we expected to be the environmental profile that the vehicle would see as it came down to the surface. So this was all done in simulation. (Matijevic; 2005)

Reliability is paramount for any mission. If the choice stands between two different approaches to a problem, a space scientist will be most inclined to choose a well known, well tested solution instead of a new and perhaps more efficient solution which has not been thoroughly tested at the planning of the mission.

An area of mathematics in which a lot of new solutions to a problem is being developed all the time is *channel coding*. The *problem* here is reliable communication, and the *solutions* are new error correcting codes. In the last 10-15 years a lot of progress has been made in this

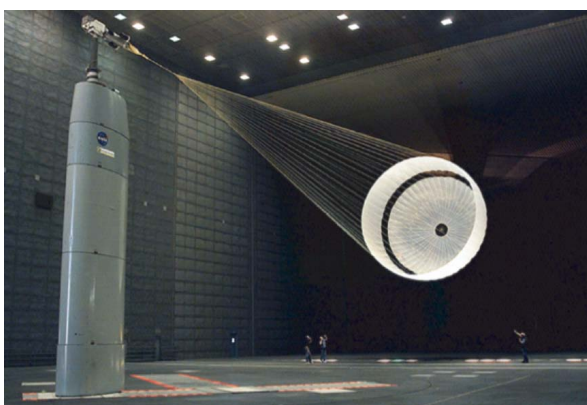


Figure 5 Left: Wind tunnel test of the MER landing module parachute. http://www.nasa.gov/centers/ames/images/content/79641main_picture_2.jpg Right: MER landing module airbags. http://photojournal.jpl.nasa.gov/jpegMod/PIA04999_modest.jpg

field with the introduction of *turbo codes* which offer a considerable improvement over earlier codes (Berrou et al.; 1993). The demand for reliability, however, has kept the turbo codes out of the space missions, and the codes have only recently been introduced to the missions. Our interview with Dr. Jon Hamkins, one of JPL's leading coding theoreticians, confirmed this for the error correcting codes used in MER:

The process of flight qualification is very long actually. ... and you know missions that are signing up for a very complicated space craft... they are out to minimize risks so they want stuff that has been flown in previous missions, they don't want something new. It's kind of contrary to the spirit of exploration. They don't want risks even though we are confident that it works... it is a risk to a mission if it hasn't flown before. (Hamkins; 2005)

Mathematics and technology which has been onboard an earlier mission is considered to be safer and therefore makes a more attractive choice. This approach is taken in all aspects of the missions. Of course some development is taking place from mission to mission but only at a pace that makes extensive testing possible. Jacob Matijevic called this "steady progress" (Matijevic; 2005). New ideas which are introduced into the missions will be at least 5-10 years old at launch time, because they must be laid down already when the missions are planned. In the case of channel coding a lot of the mathematics involved has to be implemented in hardware for speed. Generally hardware is much more expensive to replace than software, so the gain of introducing a new error correcting code has to be considerate in order to balance the expense of the substitution.

Deadlines

The scale of the project also means that the work performed by different departments must be completed at specific deadlines. Not surprisingly deadlines may serve as a stop block for the development of new ideas, since it may be difficult to keep a deadline when working with tasks whose solutions are not always well known. It is inconceivable to take on a working

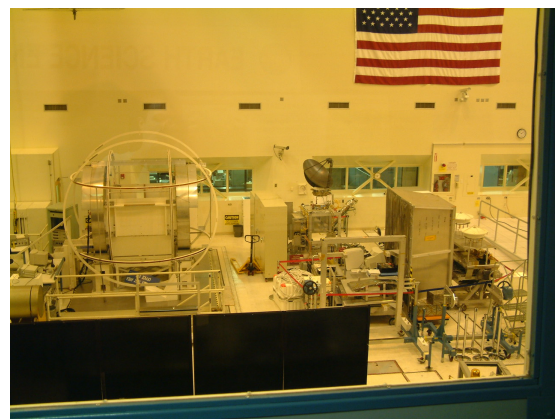
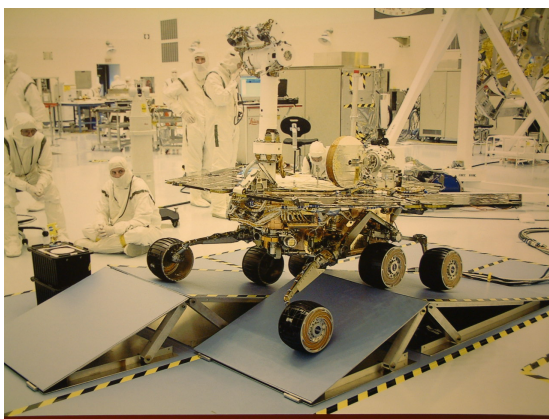


Figure 6 Guided tour at JPL. Left: A JPL photo of a MER rover testing prior to launch. Right: One of JPL's laboratories.

task if you are not sure that there will be enough time to conduct the necessary testing of the work performed. Folkner explained how the constant deadlines affects the composition of JPL employees:

There are five thousand people here. How many post docs are there? I don't know, maybe a hundred and fifty – something like that. We don't have a lot of graduate students and part of that is because almost everything we do here has to be done on a particular time scale. Very little of our budget is spent doing research where if we get the answer next year or the year after that, it doesn't matter. Everything has to be done on a schedule. So it is not a good training environment for graduate students. Graduate students here are not used much, because the people who could supervise them are busy doing other things. (Folkner; 2005)

Small versus big missions

Besides specific deadlines there is another matter which clearly enhances the tendency to opt out the new and more unsafe: In recent years JPL has gone from a small number of large and expensive missions to a large number of small but cheap missions. For instance the Pathfinder mission of 1996 had a total budget of 265 million dollars⁵ whereas the Viking missions of the seventies had a budget of around 8 billion dollars. In this light the Pathfinder mission is most certainly to be considered a 'low cost' mission. The MER mission was more expensive than Pathfinder, but still nowhere near the Viking budget.

If anything from a previous mission can be used again there are huge amounts of money and time to save. Many of the cheaper missions must therefore necessarily rely on reuse from earlier missions. To some extent this reuse issue also applies for the scientists involved in the missions. Folkner explained:

A problem in doing the smaller missions is that you can only do them with reasonably experienced people, and because they are small you don't have the budget to have an experienced person train an inexperienced person. So JPL is getting older on the average, because we don't have a big mission to afford enough people to have senior and junior people. So JPL is short of junior people. That is not a problem yet, but in five years or ten years it will be a disaster. The management here knows that and is trying to deal with it but it is not often you fix problems until they occur. They are trying to get ahead of that, but it is a difficult thing. Because we are trying to do so many cheap missions we depend on experienced people and we are not budgeting training inexperienced people. It's probably true throughout NASA and the aerospace industry. (Folkner; 2005)

So far we have mostly focused on how external factors influence the mathematics of a space mission. We will now turn to examples of concrete mathematics in the MER mission. Still the purpose of our selection will be to illustrate common features of the mathematics in a space mission.

⁵ <http://nssdc.gsfc.nasa.gov/database/MasterCatalog?sc=1996-068A>

Mathematics in MER

When describing the mathematics used in MER, the scientists at JPL often referred to the different phases of the mission. We shall adopt this approach and begin by presenting these phases.

As mentioned above, before the actual mission begins there is a lot of planning, computer simulation, and testing of equipment taking place. All of these activities belong to what you might call the pre-mission. The actual mission begins with the *launch phase*. When the spacecraft is separated from the launch vehicle the *cruise phase* begins. This phase lasts until 45 days before the craft enters the Martian atmosphere where the *approach phase* begins, a phase under which the trajectory of the craft is constantly adjusted. The *entry, descent and landing phase* (EDL) begins when the craft enters the Martian atmosphere and ends when the landing module's airbags are being retracted (see figure 8). After EDL comes a phase called the *post landing phase*. This phase begins when the solar arrays are unfolded and ends with the rover driving onto the surface – the beginning of the *surface operation phase*. It is in this phase that the rover is driving around, examining the surface, taking and transmitting pictures. In the following we shall refer to these phases of the actual mission.

Miguel San Martin told us about several mathematical problems which are involved in the missions. One of these problems which is always of interest for planetary missions consists of finding out the craft's position in space at a given time – the so-called *Lost in Space* problem. This problem is an always recurring element of the cruise phase. The problem is in essence solved by looking at the stars and trying to identify the stars you are looking at. As it turns out this identification is not at all trivial. According to San Martin you cannot identify a star

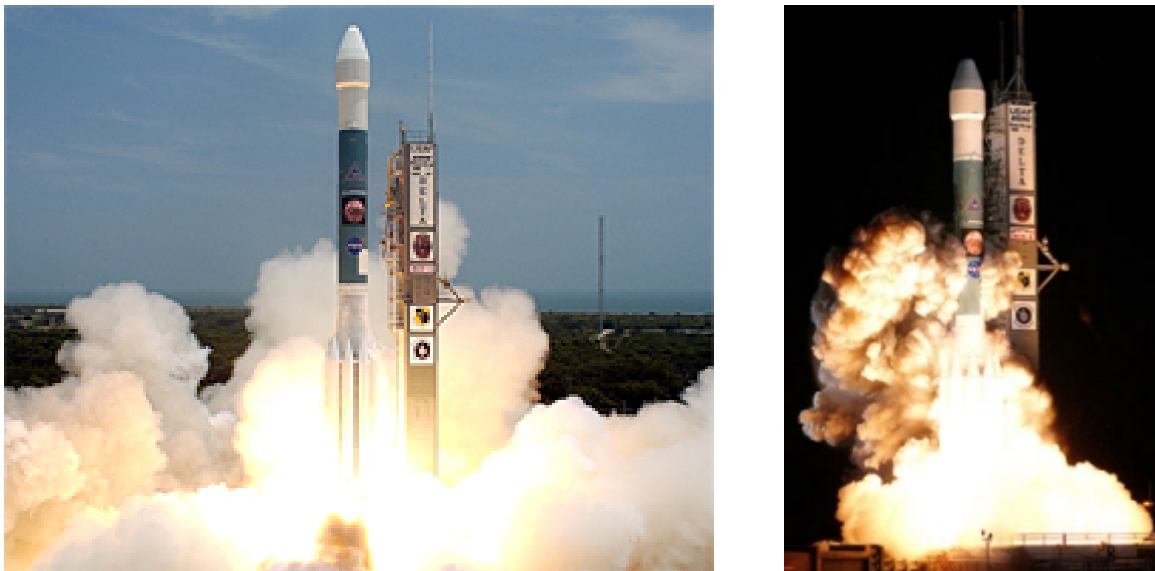


Figure 7 The beginning of the launch phase. Left: Launch of Spirit (MER A) on the 10th of June 2003 from Cape Canaveral Air Force Station. http://www.nasa.gov/lb/missions/solarsystem/merb_covg.html Right: Launch of Opportunity (MER B) on the 8th of July 2003 also from Cape Canaveral. http://en.wikipedia.org/wiki/Opportunity_rover_timeline

by its luminance or position relative to other unidentified stars alone, you have to look at star patterns. San Martin explained:

It's a pattern recognition problem. The way we do it is using the Sun. That gives us two axes. [...] People have been playing with variations of this since the '60s. Some versions are more clever than others. This one for instance cheated because we used the Sun. 'I'm in three dimensions and I don't know where I am. By looking at the Sun I know two dimensions.' Without using the Sun, this problem is called the 'Lost in Space' problem. (San Martin & Folkner; 2005)

So the problem solved in MER was a simplified version of the Lost in Space problem. The Lost in Space problem was both formulated and solved during the 1960s, i.e. in the beginning of the space exploration era. The reason we have begun this tale by pulling the Lost in Space problem out of the hat, is that it has a quality which is typical for the problems in MER – the solution rests upon *well established* applied mathematics.

The solving of the Lost in Space problem requires a number of measurements performed onboard the spacecraft. This gives rise to another problem.

In the cruise we are spinning the spacecraft. We use conservation of angular momentum to keep our spacecraft from turning. The most sophisticated piece of software that we have onboard during that time is a Kalman filter. A Kalman filter is a statistical framework, or algorithm perhaps, to mix information from different sensors. So you have a dynamic model – in this case it's a simple rigid body, which you represent mathematically and then you have some sensors which measure where the Sun is. And you have a statistical noisy model. (San Martin & Folkner; 2005)

The Kalman filter makes it possible to combine data from different sensors. Data from these sensors will often be incomplete, according to the behavior of the measuring equipment in the specific situation, or affected by noise of some kind. The Kalman filter gives an estimate of what the measurements would have been had it not been disturbed by all these elements. San Martin commented on the use of the filter:

It was invented in the '60s ... You can come up with a sequential filter that allows you to optimally combine the information from three things; your dynamic model, your sensors and your star tracking into optimal information about your attitude [position of spacecraft relative to a frame of reference] and the inertial properties of your plant. So that is going on all the time. It's a well known aerospace industry. (San Martin & Folkner; 2005)

The Kalman filter is named after Rudolph Emil Kalman who first published on this matter in 1960 (Kalman; 1960). Thus the Kalman filter is another example of MER's use of a well established mathematical theory also dating back to the '60's.

During our interview with William Folkner we came across another issue that all space missions needs to attend to, that of finding the optimal trajectory between the point of launch and the destination. The trajectory is first corrected during the launch phase by firing rockets on the launch vehicle. During the cruise phase the trajectory is also frequently adjusted and, as mentioned before, especially during the approach phase a lot of adjusting is taking place. The Mars missions always uses the same trajectory, a so called *Hohmann* trajectory. For two bodies (Mars and Earth) circling another body (the Sun) the Hohmann trajectory is the solution of the trajectory problem which calls for the least amount of energy at launch.

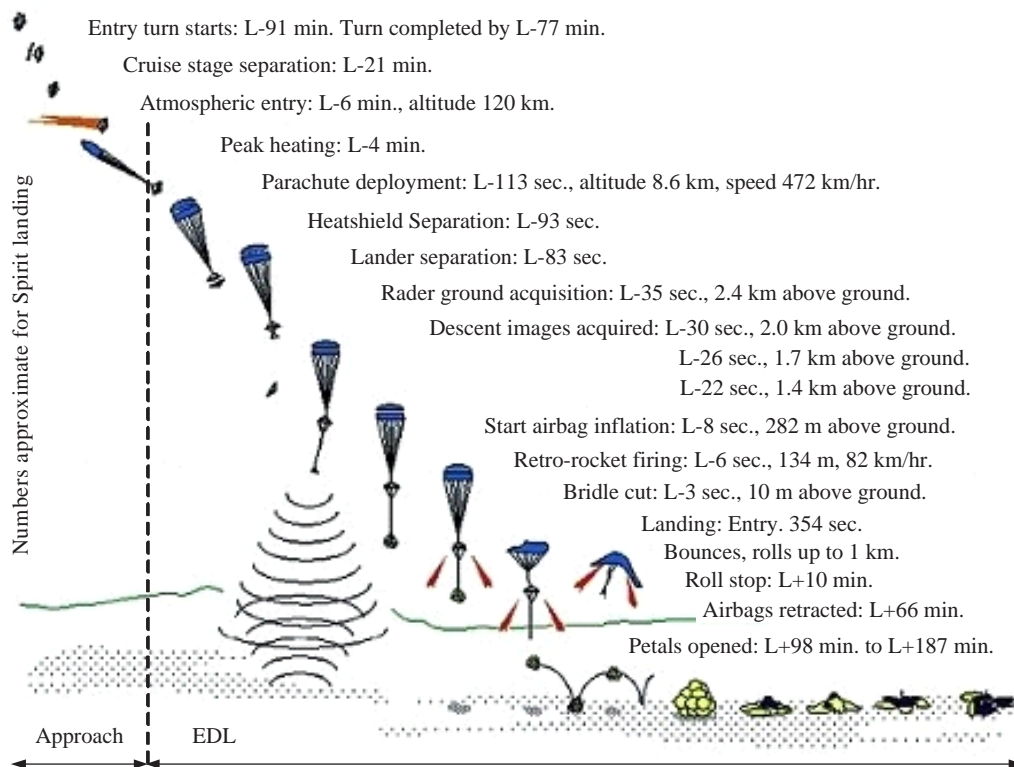


Figure 8 Entry, descent and landing. In the text above 'L' stands for landing. (JPL; 2002)

Because the movements of Mars and Earth around the Sun are not located in the same plane, two optimal solutions (a type 1 and a type 2) exist which calls for the least amount of energy. We asked Folkner if it would be reasonable to call the trajectory problem for going to Mars a standard exercise:

Yes, for Mars it is a very standard exercise. For the other planets it tends to be more complicated because you'll trade flybys of other planets for angular momentum against the mission operations time. [...] For going to Mars it always comes down to: 'Do you want to do type 1 or type 2?' (Folkner; 2005)

The Hohmann trajectory was discovered by the German engineer, Walter Hohmann in 1925⁶ and thus serves as another example of how the aerospace industry relies on well established mathematics. A description of the more complicated trajectories used for flights to other planets of which Folkner speaks can be found in (Marsden & Ross; 2006).

The mathematical problems and disciplines which we have described above are relatively isolated. If you are to mention a larger and more complete theory that played a significant role in MER, *control theory* might be suitable. We asked Mark Maimone about the amount of control theory used for steering the rovers:

We have to drive the wheels so there is software that controls the motors there and all the instruments have motors and have to be controlled so there is some amount of control theory being applied there. [And] there are a lot of motors,

⁶ See for instance <http://vesuvius.jsc.nasa.gov/er/seh/know2.html>

there are motors that drive the wheels, that spin the cameras, that control the arm and when we landed we had a lot of motors that were used simply to get up off the lander; stand up, spread out the wheels, spread out the legs, open up the solar panels, pull up the mast and do all those things. (Maimone; 2005)

From our interview with Matijevic we could understand that the steering might not be so mathematically complex as one might think:

Mainly what we're taking advantage of to at least create the driving pattern, is the fact that each of the individual six wheels is controllable at least for steering and so we can create in essence any kind of arc condition. Because of the individual control we can modulate the speed of the motor turns. That gives us a means for being able to accommodate surface interactions between the wheels and the terrain. The foundation is actually fairly simple. We're using in each case a simple proportional integral derivative; the control algorithm is very linear. (Matijevic; 2005)

From Matijevic's description it seems that most of the control theory used was also well established mathematics that has been known and applied for many years. During our interview with San Martin, he suggested that more advanced control theory might be found in the EDL phase. However, we did not have the time to investigate this any further.

As mentioned early in this article the focus of our study was partly another coherent mathematical theory called *channel coding*, which deals with the notion of *reliable communication*.

The signals transmitted to and from Mars are subject to interference during their travel through deep space. Such interferences of a binary signal may result in bits becoming altered. The communication between Earth and the rovers not being reliable is of course not acceptable, just imagine what consequences this might have for the adjustments of the trajectory. The problem is solved by way of channel codes. The use of coding theory makes it possible to correct altered bits in a message, hence the codes are also called *error correcting codes*. The coding system used in MER depends on two different codes used in combination. Jon Hamkins explained:

The majority of the missions flying now are concatenated. So the data comes in and is Reed-Solomon encoded, then it goes through a block interleaver and then it's convolutionally encoded. (Hamkins; 2005)

Thus MER's coding system consisted of two combined, or concatenated, error correcting codes; a Reed-Solomon code and a convolutional code. Reed-Solomon codes are algebraic codes whose code symbols comes from a Galois Field. Convolutional codes are another type of codes which are not as mathematically well understood as the Reed-Solomon codes, but on the other hand are very efficient and therefore very often used in technology. Convolutional codes are excellent for correcting single bit errors, the kind of errors which most often occurs from interference in deep space. Unfortunately the decoding of convolutional codes⁷ often results in a run of consecutive errors, so-called burst errors. Fortunately Reed-Solomon codes are excellent in correcting exactly burst errors, hence the concatenated system. The reason for first encoding the data with the Reed-Solomon code and then the convolutional code is that the decoding procedure must be the reverse of the encoding procedure. Block interleaving is

⁷ JPL uses the so-called *Viterbi algorithm* in their convolutional decoder.



Figure 9 The Mars rover *Spirit* moves its robot arm over a Martian stone in order to take a series of pictures with its microscope camera. <http://marsrovers.jpl.nasa.gov/gallery/press/spirit>

a technique used to ensure that the burst errors from the convolutional decoding are no more severe than what the Reed-Solomon decoder can handle.

Reed-Solomon codes are named after Irving S. Reed and Gustave Solomon who introduced these in 1960 (Reed & Solomon; 1960). The convolutional codes are due to Peter Elias who published on this matter in 1954 (Elias; 1954). The concatenated system was originally described and tested in the beginning of the '70's (J. P. Oddenwalder et al.; 1972) and used for the first time during the Voyager 2 mission in 1985.

Coding theory was used in the majority of the phases in the actual mission. Every message between Earth and the rovers, whether they were in flight or on the surface of Mars, were subject to the coding system described above. Adjustments of the trajectory was not the only communication occurring during the cruise phase for which reliable communication was of paramount importance: Software uploads of the rover's steering systems took place both during the cruise phase and the surface phase (Maimone; 2005). On the surface another mathematical theory which is also part of coding theory came into play, namely that of *image compression*.

One of the main purposes of the MER mission was to take pictures of Mars. Before these pictures were transmitted to Earth they had to be compressed. The image compression

technique primarily used in MER is called *The ICER Progressive Wavelet Image Compression*, in short just ICER, and was developed at JPL by Kiely and Klimesh. The word ‘progressive’ refers to *progressive fidelity compression*. In such a compression a low quality approximation of the picture is first transmitted. Afterwards bits are transmitted in such a way that the quality of the picture are gradually improved. When all bits are transmitted the reconstructed picture equals the original picture. By stopping the transmission before it is complete lossy compression can be obtained. In this way ICER supports lossless as well as lossy compression even though it was entirely used for lossy compression. For lossless compression MER relied on the commercial compression algorithm *LOCO*. The ICER algorithm, like many other image compression techniques, overall consists of three stages; preprocessing, modelling and entropy encoding of data. Kiely explained:

We got data coming in, an image or whatever it is, and then some sort of preprocessing stage for example a wavelet transform plus quantization or a discrete cosine transform or something. The goal is that it doesn’t perform any compression and in fact it is often a lossy process, it might throw out some of the data but the idea is to process the data in a way that makes it more receptive to compression through the entropy encoder. The entropy encoder is sort of the engine. Given some sort of probabilistic model of the source it compresses data or represents it in a more efficient way through something like a variable length code. That is sort of the big picture of what is going on. So for example for ICER what is going on is mostly a probabilistic transform. For LOCO it is in essential trying to project a probability distribution on the next pixel that it is about to encode based on what it has seen in the nearby neighbors. (Kiely; 2005)

ICER uses a wavelet transform that closely but not exactly resembles a *Haar*-transform, a context model (also known as a Markov model) and the majority of the entropy codes used by the entropy encoder are the so-called *Golomb codes* (Kiely & Klimesh; 2003). The LOCO algorithm is a bit different from ICER since it do not have a preprocessing stage which is typical for lossless compression techniques. It does use context modelling and it uses both Golomb codes and *Huffman codes* (Weinberger et al.; 1996). Alfréd Haar’s transform dates back to 1910 (Haar; 1910). Golomb codes are due to Solomon W. Golomb who described these codes in 1966 (Golomb; 1966). Huffman codes are due to David A. Huffman who discovered these codes in the beginning of the 1950s while still a student at MIT (Huffman; 1952).

From our point of view the most interesting aspect of ICER is that this compressor for the first time introduced wavelets into an image compressor for space applications. Wavelets has only recently found its way into commercial standards like the JPEG 2000 compressor, so this is an area of the mission where a surprisingly new approach to a problem is taken. The reason for this is probably that using a new image compressor is not so much of a risk because it is implemented in software rather than hardware.

The above presentation of mathematics in MER is of course merely scratching the surface. Discovering every little piece of mathematics put to use in the MER mission probably is an impossible task to undertake despite being “the most knowledgeable historians of mathematics” or not.

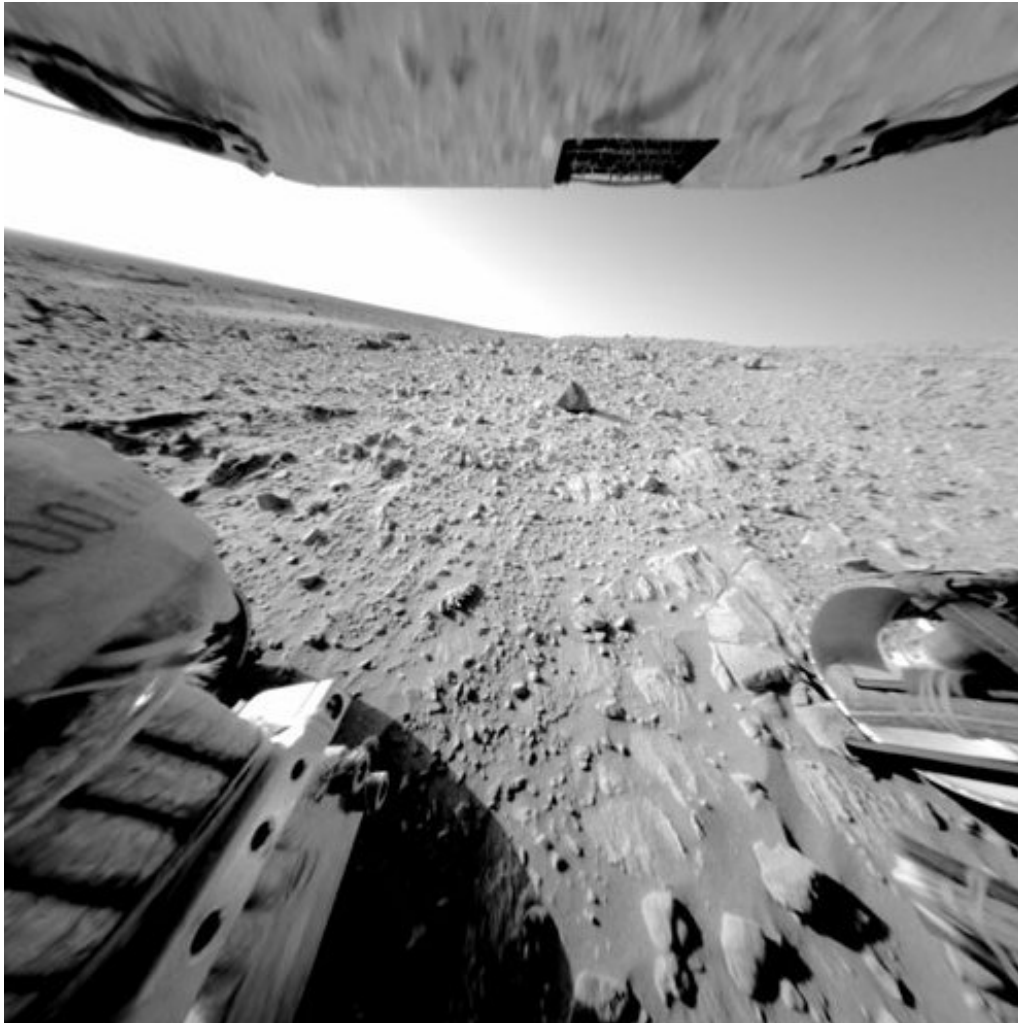


Figure 10 A picture taken from Spirit's rear camera. The rover's left rear wheel can be seen to the right in the picture. <http://marsrovers.jpl.nasa.gov/gallery/press/spirit>

Conclusions

From the mathematics we did discover and investigate we found that the majority of the theories are well known and well established theories of mathematics, like for instance the Hohmann trajectory, Kalman filters, Reed-Solomon codes and convolutional codes. Also in the image compression both Golomb codes and Huffman codes as well as the Markov models are examples of well established mathematics. The newest piece of mathematics we came across was the wavelets in ICER's preprocessing. The mathematical theory of wavelets only dates back to the beginning of the 1990's and is due to Ingrid Daubechies (Daubechies; 1992). However, wavelets has been around in applied mathematics for a long time and as we mentioned the specific transform of ICER is inspired by the Haar-transform from 1910.

Besides the use of all these 'advanced' mathematical theories are of course an enormous amount of more basic mathematics. So Philip Davis was certainly right when he said that "the Mars shot would have been impossible without a tremendous underlay of mathematics".

Folkner said it himself when we discussed the MER mission:

There is mathematics in everything. There is control theory, aerodynamics, orbital dynamics, Newtonian gravity, bodies going around the Sun. We use general relativity, that's mathematics of physics [...] Linear algebra is a field of mathematics we use all the time. Matrices. That's in the control theory all the time. There is Riemannian geometry in the general relativity. Calculus. (San Martin & Folkner; 2005)

Thus, a mission like MER is relying on a vast amount of mathematics. However, the mathematics is often hidden. Not surprisingly the mathematics is hidden to the public as most mathematics in our society is. But more surprisingly, it is to some extent also hidden to the employees of JPL. The main reason for this is that a lot of the work done at JPL involves mathematics embedded in commercial software packages. Another reason is that some of the mathematics is such an integrated part of a mission, that it is not thought of as mathematics (we mentioned earlier looking up planetary positions in a table). Finally, classification contributes to hiding the mathematics – something which we also encountered with a couple of times during our stay at JPL. (For a further discussion of the hidden mathematics in MER, see (Jankvist & Toldbod; 2007).) The hiding of the mathematics in the mission is bound to make it more difficult to discover the mathematics involved.

Due to the extreme nature of a Mars mission one might think that this would call for 'extreme' mathematics, mathematics that would have to be developed for the sole purpose of this mission. This, however, does not seem to be the case. We did not come across any contributions to basic research in mathematics as a result of the MER mission through our investigation. The role of JPL seems to be another, namely that of the consumer of already developed mathematics – applied mathematics.

Due to this the majority of the scientists of JPL are applied scientists, but with a wide variety of educational backgrounds; engineering, computer science, physics, mathematics and so on. The thing these people have in common is that they all have very high levels of education (Ph.D. degrees). For JPL the educational background does not necessarily seem to be the main focus, rather it is a question of whether or not an employee can solve the task that needs solving. Klimesh told:

I didn't start working on data compression until I came here. Actually that is not so unusual in engineering... [...] A lot of the mathematics is the same and when they hire someone, what they really want is someone who is good at solving problems. (Klimesh; 2005)

All the people we talked to seemed to have come to JPL more or less immediately after having completed their studies. And all of them seemed driven by the desire to work within the aerospace industry and none seemed to want to leave JPL again.

The work performed at JPL are subject to a number of conditions all with consequences for the work. The lower budgets of the smaller missions has the effect that the JPL staff is composed by many senior scientists and only few juniors, a matter which in part is also conditioned by the smaller missions' dependence on experienced personnel. The lower budgets also makes it attractive to reuse existing solutions to problems from mission to mission, a matter which is not likely to promote the implementation of new ideas. Also the very strict timetables and deadlines may make it almost impossible to pursue new ideas and solutions to old problems. The long and tortuous process of flight qualification, including the extensive

testing of all equipment also enhances this tendency. However, it seems that for some areas of a space mission new ideas are more welcome than in other areas. The image compression scheme used in MER was the newly developed ICER compressor. ICER could more easily be introduced to the mission, because ‘only’ software needed to be replaced. But on the overall it seems that new ideas must always be weighed against the effort needed to implement them. We shall end our conclusions with a quote by William Folkner in which he nails this point exactly:

Everything is a cost-benefit analysis. The whole space system is a cost-benefit analysis. (Folkner; 2005)

Acknowledgements

We want to thank all the employees at JPL who set aside their duties to talk to us, especially Bill Folkner who arranged most of our meetings. Also we would like to thank Phil Davis for taking such an extraordinary interest in our investigation and for commenting on this article. Thank you also to Tinne Hoff Kjeldsen, Anders Madsen and Bernhelm Booß-Bavnbek (all at Roskilde University) for making suggestions and commenting on the original thesis. And thank you to Jørgen Larsen for L^AT_EX assistance all along the way.

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