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Steve Humble

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Von Neumann and Computers

Steve Humble
New Castle College (England)

The title of this paper should not be Von Neumann and computers, but Von Neumann and the Von Neumann machine. Von Neumann may be famous for many things but humility was not one of them. Yet no one had anything bad to say about 'good time' Johnny Von Neumann; he just was too likeable. He gave massive parties and loved women, fast cars, jokes, noise, Mexican food, fine wine, and, most of all, mathematics. 'Unbelievable', said one of Von Neumann's old friends, 'He knew how to have a good time. His parties were once if not twice a week at 26 Wetcott Road. Waiters came around with drinks all night long. Dancing and loud laughter. With Von Neumann at the centre of it all he was a fantastically witty man.'

Johnny von Neumann (1903 – 1957)

In a way Von Neumann could afford to party, since he had been born lucky - lucky enough to have a great mind, that did not forget. He had a true photographic memory and never forgot a thing. The story goes that you could ask him to quote from anything he had ever read, and the only question he would ask you was, 'When do you want me to stop?' In addition to having a great memory, he was also very fast. Start asking him a question and, before you finished, he would be answering, and suggesting interesting follow-ups that you should consider. It's no wonder, with a super mind like his, that he is credited with inventing the present-day computer.

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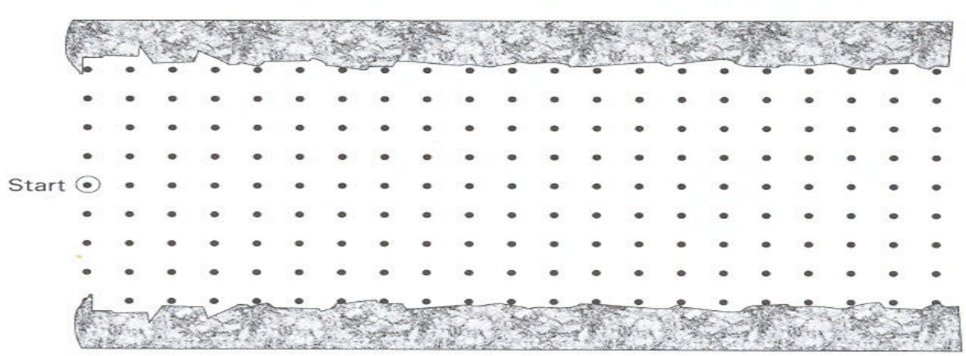
When one of the first computers was to be tested, Von Neumann was on hand to help. The test for the computer was to work out powers of 2, and to find the first number to have 7 as its fourth digit from the right. The computer and Von Neumann started at the same time, and you guessed it, Johnny finished first!
As well as working on the development of the computer, Von Neumann also worked on the atom bomb and created a branch of mathematics called game theory. His work in these areas argued strategies for the Cold War and inspired the movies *Or Strangelove* and *War Games.*

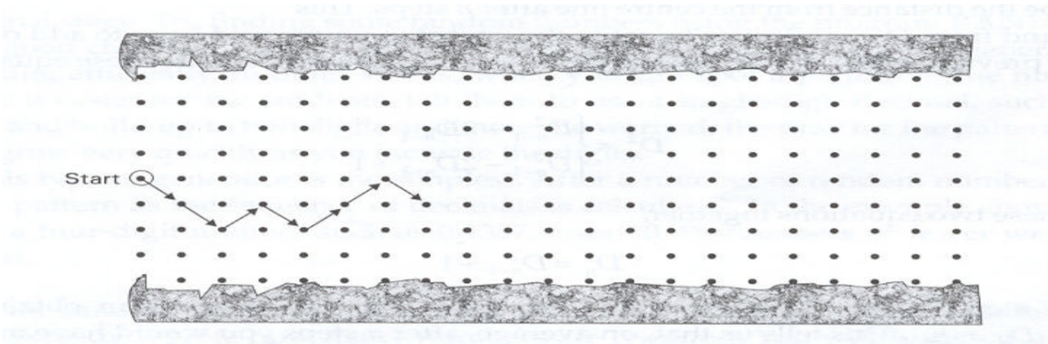
When Von Neumann finished building his computer he had to find a use for it. In his eyes the only useful thing to do was mathematics so he and fellow mathematicans Fermi and Ulam invented a simulation method that they called the Monte Carlo method. This method simulates random events using the computer. In Buffon's needle problem the JAVA applet uses Von Neumann's simulation method to validate the correct value of pi. To simulate the unpredictable event of throwing a needle, the computer has to use something called a pseudo-random number generator. The pattern of numbers generated are deterministic, yet of sufficient complexity to cause the outcome to appear unpredictable (random). In the following experiments we will look in more detail at random number generators.

Experiment Time

There is a problem in mathematics called the random walk. You start with a player at some point. He tosses a coin, and how it falls decides his direction of motion. This randomly makes the choice of which way to go.
To set the scene for students, imagine that you are on a narrow mountain ridge. It is windy and the rain is battering down on you. This makes it difficult to see where you are going. Every time you move forward you get blown randomly to the left or right. What are your chances of making it across the ridge?



Start at the base camp dot, and throw a coin. If it is heads move to the right and if it is tails move to the left. For example, if you throw HHTHTTH then it would look like this:



Keep throwing the coin until you make it to the other side or fall off the edge.

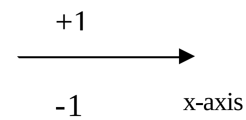
What is the chance that you will:

- fall off the cliff
- make it to the other side
- fall off before the tenth move
- fall off before the fifteenth move
- fall off the right or left side of the ridge?

Before you start this experiment using paper, it is a good idea to let the students try to walk the ridge for 'real' in the classroom. With a pre-defined ridge marked on the floor, you can flip the coin to say move left or right. This works well as a starter to get them thinking about how wide the ridge needs to be if they are to make it to the other side. After this introduction get students to perform this experiment on paper at least five times, then collect the data on the board. Be as detailed in this collection phase as the class's ability allows. For example, you may ask if anyone went over the edge in the first 5 moves, or between 6 and 10 moves, or 11 and 15 moves, and so on. Once you have collected the results the class can then talk about the probability of crossing the ridge.
Here are some extension ideas for this task:

Instead of using a coin use the random number button (RAN#) on your calculator, moving to the right if the number generated is in the range 0-0.5, and to the left for higher numbers. What happens if the wind blows harder from one side? Does this make it more difficult to cross the ridge? Simulate this by moving to the right for random numbers 0-0.3, and to the left otherwise, or something similar.
Include the chance that you will be blown two dots to the right or left.

Using mathematics to predict this random event



One question you can ask is how far on average the ridge walker will move away from the centre line after the start. Let the centre line be the *x-axis*. If you move to the left this is +1, and if you move to the right this is -1. Let *On* be the distance from the centre line after *n* steps. This can be found from *On-I'* and since to get to the next step you would have to add or subtract one to the previous step, $D_n = D_{n-1} + 1 \text{or} D_n + D_{n-1} - 1$ If you then square these equations you obtain

$$D_n^2 = \{ \begin{matrix} D_{n-1}^2 + 2D_{n-1} + 1 \\ D_{n-1}^2 - 2D_{n-1} + 1 \end{matrix}$$

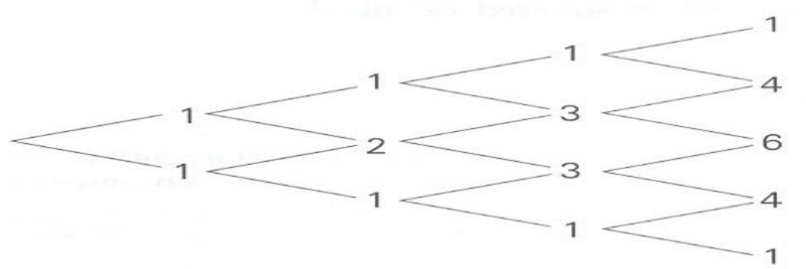
Adding these two equations together,

$$D_n^2 = D_{n-1}^2 + 1 \qquad [1]$$

After one step, $D_1^2 = 1$. Using this, and equation 1 repeatedly, we can obtain $D_n^2 = n$. Therefore $D_n = \sqrt{n}$. This tells us that, on average, after *n* steps you would have moved $\sqrt{n}$ away from the centre line.

Calculating the Probabilities

To calculate the probabilities involved in the random walk, you can use Pascal's triangle to get the chances of following different routes in a random path, for at each step you are making a decision to go left or right. The numbers in the triangle indicate the number of routes from the start position to that point. To find the next line in Pascal's triangle add together the two roots to that point.



Extension Ideas

The 147 random number generator works in the following way. First it selects a decimal between 0 and 1, which it then multiples by 147. Then it takes the fractional part of this result and multiplies it by 10. The integer it produces is the random number.
For example, if the decimal between 0 and 1 that is selected is 0.1357, we have

0.1357 x 147 = 19.9479
The fractional part is 0.9479
10 x 0.9497 = 9.479
So the random number is 9.

If you want a larger random number using this method, just multiply by 100 or 1000 in the final stage. Try finding some random numbers using the program RAN147.

A good challenge for the students is to discover when the random generator is not working efficiently. In other words, when you can spot a pattern in the numbers. To make it easier for the students, tell them to use a single-digit decimal, such as 0.7, at first, and build up to two digits and more. Be warned, the time for the pattern to repeat will grow very quickly as you increase the digits.

This type of generator is the simplest. After a number of random numbers you will see a pattern as the sequence of decimals is calculated. In the example above, starting with a four-digit number such as 0.1357, after 10000 numbers or fewer we will get a repeat.

Add the random numbers generated from your calculator's random button until their total exceeds 1. Note how many numbers are required. You will find that the average number of random numbers required is e = 2.7182818 . . . Changing the total to 3, so that the average becomes 8, makes a good challenge for students.

A similar idea is to find the average random number between 0 and 1. This is a problem that the students can work on using their calculators by finding the average of blocks of ten numbers. The answer comes to

$$\int_0^1 x\,dx = \frac{1}{2}$$

In other words, this is the area under the curve y=x from x = 0 to 1. What about the average of random numbers squared, or cubed. . .?

Further References

Aspray, W. (1990) *John von Neumann and the Origins of Modern Computing.* Cambridge, MA: MIT Press.

Berg, H. C. (1993) *Random Walks in Biology.* Princeton, NJ: Princeton University Press.

Oyson, F. J. (1981) *Disturbing the Universe.* London: Pan.

Gentle, J. E. (1998) *Random Number Generation and Monte Carlo Methods.* New York and London:Springer.

Malkiel, B. G. (1985) *A Random Walk down Wall Street.* New York: Norton.

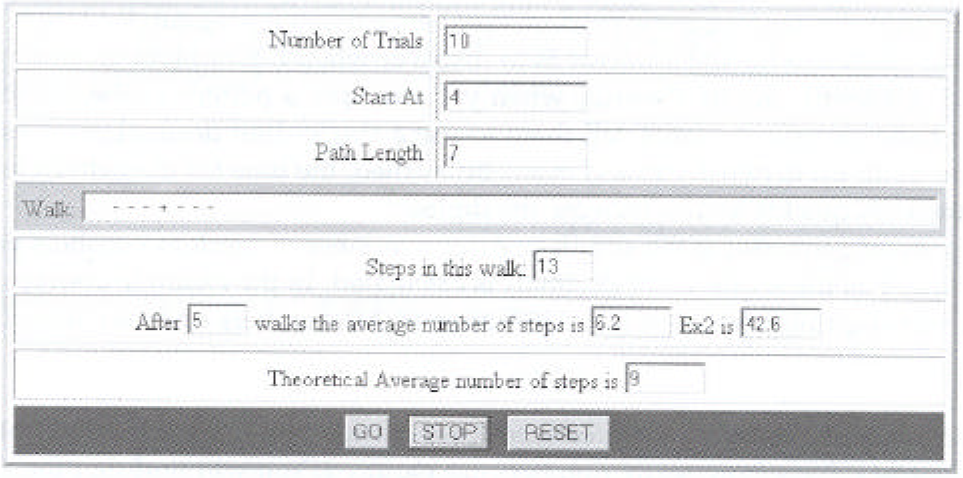
Spitzer, F. (1964) *Principles of Random Walk.* Princeton, NJ: Van Nostrand.

Von Neumann, J. (2000) *The Computer and the Brain,* 2nd edn. New Haven, CT and London: Yale Nota Bene.

For a number of random walk movies, see
http:/ /cic.nist.gov /lipman/ sciviz/ random

See also: www.math.uah.edu/Stat/walk/http:/ /math.furman.edu/ -dcs/java/rw
http:/ /polymer.bu.edu/java/ www.random.org/ essay.html.

For a computer simulation of a random walk along a line see:
www.math.sc.edu/ -sumner /Random Walkhtml. Here is a screen shot from it.


Number of Trials 10
Start At 4
Path Length 7
Walk: - - - + - - -
Steps in this walk: 13
After 5 walks the average number of steps is 6.2 Ex2 is 42.6
Theoretical Average number of steps is 9
GO
STOP
RESET