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**SOME OBSTACLES IN MATHEMATICAL COMMUNICATION OF STUDENTS
WHILE LEARNING CONTINUOUS FUNCTIONS LESSON**

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Mathematical Communication is one of the required competencies of the learner in the needs of the real life, and in learning Mathematics, students always have the need to communicate with teacher and other students to solve problems in mathematics, as well as to present the solution or mathematical ideas to the others. However, there are many difficulties with many reasons when they exchange or communicate with the others by mathematical language in classroom. Therefore, the ability for communicating in Math is one of the required competencies of students. In this paper, we try to find out the obstacles which make students have difficulty when they are in mathematical communication while learning continuous functions. The problems set out in this article to assess how difficulties in communication in the Math class, so that we can recommend some possible strategies for teachers to help their students in order to overcome these obstacles in practicing teaching.

Keywords: Mathematical communication, obstacles in mathematical communication.

INTRODUCTION

Developing communication skill is essential in course of mathematical education. When arguing, analyzing systematically, students can reinforce their knowledge and understanding of mathematics more deeply. Through communication, students solve problems more effectively, and the course of mathematical education also helps them to solve practical problems by effectively using mathematical knowledge to understand the meaning of mathematics in practice. Teachers should provide opportunities for students to discuss, debate, and present more in order to create a highly interactive learning environment. In doing so, teachers need to recognize and understand the obstacles students face in the course of mathematical communication and have some methods to help them overcome that.

Many authors have studied mathematical communication and developed mathematical communication skill in students. These studies focus on systematic presentation of the content of mathematical communication and possible methods to develop communication skills in mathematics instruction, such as Hoa Anh Tuong (2014), Pham Thu Ha (2015), Vu Thi Binh (2016), Phan Thi Ai Minh (2017), i.e. However, no author has approached the development of mathematical communication skills from the study of obstacles and how to overcome obstacles in



mathematical communication. In this paper, we have chosen to study the obstacles students face when dealing with mathematics in Continuous Functions in order to serve this need.

1. FRAMEWORK

1.1. Terms used in the paper

Communication: is the process of transferring information between senders and receivers in order to get some targets.

Mathematical communication: is a form of communication which a person tries to determine other people about ideas, thoughts, questions or his mathematical theory in order to share his ideas and make the understanding of mathematic clearly. Through talking and questioning, mathematical ideas can be reflected, discussed and modified. The process of reasoning analytically and systematically can help reinforce and strengthen students' knowledge and understanding of mathematics to a deeper level. Through effective communications students will become efficient in problem solving and be able to explain concepts and mathematical skills (Lim, 2008).

Obstacles of students in mathematics communication: The difficulties and mistakes of students when carrying out mathematical communication activities with students, teachers or with oneself, from which make mathematical communication activities not successfully carried out, students make mistakes or break their mathematical knowledge. These obstacles can also be understood as the obstacles of students when participating in the process of mathematical communication.

Mathematical language: Mathematical language in mathematics teaching in high school is the language of mathematics, including mathematical vocabulary (words, mathematical terms), mathematical representation (such as symbols, drawings, diagrams, graphs, tables, charts, ...) and associated rules they use to express (argue, represent) objects and mathematical relationships while speaking, writing or thinking.

1.2. Tools of mathematical communication

In math class, some tools of mathematical communication can be used, such as:

Mathematical expression: is a description of the relationships between objects and symbols, a bridge to communicate easily with others, maybe it appears on paper, diagrams, diagrams, charts, graphs, geometric outlines, and equations.

Interpret: students give their views about the problem to demonstrate their mathematical comprehension.

Argument: is systematically arrange the argument to present, in order to prove a conclusion on a problem. Students can argue through counter-examples, which may be true or false. Thus, the argument involves knowing what the mathematical proofs are.

Present mathematical proof: A student's expression can be written or spoken to prove a theorem or the authenticity of a certain judgment to persuade and help others understand the problem.

1.3. Research Methods



Based on this research, we used the questionnaire to collect information about common student obstacles in the mathematical communication. Data collection was conducted on 98 students of two high schools in An Giang province, one was in the center of Long Xuyen City, and the other was in Chau Phu district.

Then, the author generates some obstacles in mathematical communication of students, and suggests some strategies for teachers to help students to overcome those obstacles in order to participate in mathematical communication better.

We contribute some educational Maths situations in order to experiment on 82 students (in the two high schools, but from different class) to find some obstacle preventing students to communicate.

The empirical content is focused on the continuous function, so the student's obstacles in mathematical communication and the ways to overcome it are also focused on the content of the continuous function.

2. SOME SITUATIONS ARE DESIGNED TO DETECT OBSTACLES

Problem 1: If $y = f(x) = \begin{cases} x^2 - 2x + 2 & \text{khi } x \geq 1 \\ 2 - x & \text{khi } x < \frac{1}{2} \end{cases}$

a. Calculate $\lim_{x \rightarrow 1^+} f(x)$, $\lim_{x \rightarrow 1^-} f(x)$ and estimate the continuity of function $f(x)$ at $x_0 = 1$

b. Complete the table of value for the function $f(x)$ below:

x			1		
$y = f(x)$			1		

c. Graphing (C) for the function $f(x)$

d. Base on the graph (C) , estimate the continuity of function $f(x)$ when $x_0 = 1$

Comment: This problem has allowed us to verify the obstacles: *The student's difficulties in mathematical communication when performing graphs, drawings, and conclusions about the continuity of the function when $x_0 = 1$ based solely on calculations, without taking into account the characteristic of x_0 .*

Specifically, students may not be interested in domain but based on calculations:

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^2 - 2x + 2) = 1, \quad \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (2 - x) = 1 \quad \text{và} \quad f(1) = 1$$



Conclusion: the function continues when $x_0 = 1$. In addition, the requirements of b and c help students recall knowledge related to the graphical representation and the possible obstacle is that when calculating the values of the value table, students connect the points based on the completed value table into a "solid line" and assumes that it is the graph of the given function. On the other hand, if students draw the graph of the function correctly, then on the basis of d), students can detect their mistake in a). Therefore, this kind of problems can help students to find out their own mistakes and correct mistakes. In cases, if students do not draw accurate graphs, teachers can use the Sketchpad software to draw graphs for students to see and comment.

Problem 2: If $f(x) = \begin{cases} \frac{x^2 - 3x + 2}{x - 1} & \text{khi } x \neq 1 \\ m & \text{khi } x = 1 \end{cases}$

- Is function $g(x) = \frac{x^2 - 3x + 2}{x - 1}$ continuous on \mathbb{R} ? Why?
- Graphing function $f(x)$ and based on the graph determine the value of m to $f(x)$ continue on $(0;2)$.

Comment: This exercise not only reminds students about the theorem “*Function of rational classification (conjugate of two polynomials) and the trigonometric functions continuously on the intervals of their definitions*”, it also requires students to remember the comment “*The graph of the continuous function on the interval is a continuous line "on that interval"*”. Therefore, for a continuous function on the interval, the graph of the function needs to be a continuous line at the point $x = 1$. By that way, when students memorize these theorems, they will support the mathematical communication presented by drawing, or presentation of the evidence will be done effectively.

Problem 3:

If $y = f(x) = 10x^3 - 6x^2 - 24x + 17$

- Calculate the value of the function at the given abscissa points and fill in the following table:

x	0	0.5	1	1.5	2
$y = f(x)$					

- Does the graph of function cut axis on $(0;2)$? Why?

Comment: Problem 3 will help us verify if students apply the value table of question a for answer the question b or not? The question b requires students to explain the reason for their answer which



will reflect the student's grasp of the theorem 3 “ If the function $y = f(x)$ is continuous on $[a; b]$ and $f(a) \cdot f(b) < 0$, does there exist at least 1 point $c \in (a; b)$ such that $f(c) = 0$. ”? From there, we can add missing knowledge for students to help them overcome obstacles in mathematical communication.

Problem 4: Prove that the following function is continuous at $x_0 = 9$

$$f(x) = (\sqrt{81 - x^2} + \sqrt{x - 9})$$

Comment: when solving this exercise, students will calculate $\lim_{x \rightarrow 9} (\sqrt{81 - x^2} + \sqrt{x - 9}) = 0 = f(9)$ and then they will conclude that the function is continuous at $x_0 = 9$.

Actually, the function $f(x) = (\sqrt{81 - x^2} + \sqrt{x - 9})$ has no limit at $x = 9$

Hence set function $\begin{cases} \sqrt{81 - x^2} > 0 \\ \sqrt{x - 9} > 0 \end{cases} \hat{=} x = 9$ which means set function $D = \{9\}$. Therefore, we can

apply the definition of $\lim_{x \rightarrow 9} f(x)$ because there is no $\{x_n\}$ to satisfy the condition of the definition which is " $x_n \in D, x_n \neq 9$ for $\{x_n\} \in \mathbb{R}$ ", so the function has no limit at $x = 9$. This is the reason why we build this exercise to help students find out their mistake and thereby add limited knowledge to them.

Problem 5: Prove that the following function is continuous on \mathbb{R} ?

$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1} & \text{khi } x \neq -1 \\ -2 & \text{khi } x = -1 \end{cases}$$

Comment: when solving this exercise, students may solve as below

We have: $f(x) = \begin{cases} \frac{x^2 - 1}{x + 1} & \text{khi } x \neq -1 \\ -2 & \text{khi } x = -1 \end{cases}$ has set function on $D = \mathbb{R}$



And $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} = \lim_{x \rightarrow -1} (x - 1) = -2 = f(-1)$, so the function is continuous on \mathbb{R} .

They have unknowingly identified that the function is continuous at $x_0 = -1$ with \mathbb{R} . So when we give this problem, we will verify the obstacle "students seem to be misunderstood continuous function on the segment, interval with one point" and promptly remind them to notice that.

Problem 6: A student said "if a function defined on \mathbb{R} , it would be always continuous on \mathbb{R} "

Do you agree with this statement? Why?

Comment: The aim of problem 6 is to test the understanding of students and help them develop critical thinking and mathematical communication. Indeed, to answer this question, they need to present their examples through mathematical language. There will be many different opinions during the answer, they can answer right or wrong, can use words or scripts, drawings to support their answers that are very useful for mathematical communication development and critical thinking.

3. RESEARCH RESULTS

3.1. Student's obstacles in mathematical communication when studying Constant Functions

We recognize that, according to their ability, students use mathematical languages to communicate with their classmates as well as show that when we conduct the survey, and they have some obstacles are as follows:

a. Student's difficulties in mathematical communication when performing graphs, drawings

When solving problems of continuous function, students are less likely to be instructed by the teacher to draw a graph of a function through graph representation, which demonstrates the ability to perform mathematical representations with graphs in their process of mathematical communication. Therefore, when asking the students from the graph, the figure that concludes the continuity of the function at a point or interval, the segment they will face many difficulties. For example, students skip the domain of a function when they prove the function continuously. When drawing a graph of a function, students define only a few discrete points and then joins them together into a solid line without taking into account the characteristics of the specified domain. Or they lack of knowledge about graphs of polynomial functions or rational fraction.



b. A wrong conclusion about the continuity of a function at x_0 mainly because they only based solely on calculations that do not take into account the nature of x_0 .

Students prove the continuity of the function $f(x)$ at x_0 by using $\lim_{x \rightarrow x_0} f(x) = f(x_0)$ without consider the deterministic domain of the function $f(x)$.

c. Students conclude the equation of the function $f(x) = 0$ on $[a; b]$ without proving the continuous function $f(x)$ on $[a; b]$

If $f(a), f(b) > 0$ and student cannot find a pair of integers or special rational m, n belong $(a; b)$ which $f(m), f(n) < 0$, they will conclude $f(x) = 0$ impossible equation. Or students can calculate $f(a), f(b) < 0$ then they will conclude the function $f(x) = 0$ having at least an equation on $[a; b]$ without considering whether the continuous function on $[a; b]$ or not.

d. Students lack of critical thinking in mathematical communication

During communication between students with teachers, students always have the idea or have the thought that everything teachers said are always right without the habit of checking the information that teachers give is really accurate. Thus, communication between students and teachers is always "one way" which is not exchange way. Traditionally, teachers have not had many methods to help students to ask questions in the class, this lead to the situation that students are passive and lack of confidence in communication, especially mathematical communication.

e. The lack of knowledge of students about the limit of functions or series of numbers leads to obstacles in the course of mathematical communication

As the content of the continuous function is closely related to the limited content, thus the shortage of knowledge of the limit in students will affect their course of the mathematical communication in this content.

f. Students use a pocket calculator mechanically in the process of calculating the limit, they just use this way to solve the problem but do not understand why that such result.

The changing the form of constructed-response test into the multiple – choice test also makes students "lazy", they just need to get the right result without paying attention to the presentation of a problem closely and specific for others to understand. Therefore, the methods of using pocket



calculators are always used by students and applied regularly, for that reason the process of mathematical communication will be limited and difficult.

g. The method of learning and recording improperly obstruct the course of mathematical communication of students.

Each student has different learning methods, during teacher lectures, most students will understand what teacher communicates but if they do not record important contents, they only rely on their memory, it is easy to forget what the teachers have taught, "ambiguous" and "vague" remembering will lead to mistake in knowledge of their course of mathematical communication.

h. Students seem to be misunderstood between continuous function on the segment, interval and continuous function at a point.

In the process of solving problems that require students to prove the continuity of a function on a particular domain, students only consider the continuity of the function at x_0 in the specific domain and conclude that the function is continuous over the whole domain.

3.2. Discuss the results

During the process of observing students (in questionnaire), we can comment that:

- Students are more interested in completing multiple – choice exercises than others which they have to give their own opinion so they just give short answer such as yes or no; right or wrong without giving reasons for protecting their opinions as well as not explain why they have that answer. In addition, students do not focus on the mathematical symbols, they introduce them "simply". Based on this situation, we believe that the mathematical communication of students was still a lot of obstacles. We have noticed that students are very hesitant to express their personal opinions as well as they are afraid to use the argument to protect their opinions.

Some obstacles of students were detected by building some specific case study, such as:

- When students use the mathematical communication tool with graphs, drawing and graphing are always difficult, so only 21.62% of students can draw the graph of Problem 1 while 78, 39% of them cannot draw. Also, 56.76% of students solving problem 2 cannot draw graph. Moreover, the observation of the variance table in Problem 7 is only 5.41% of students who can solve it. We thought that students encountered this obstacle due to a number of reasons:

- + Students did not understand the property of the graph;
- + Students had not accurately chosen points in the graph;
- + Students had not identified the value of x in the definite set of graph;
- + Students feel afraid and lazy when drawing the graph.
- + In the current curriculum students are less exposed, familiar with reading and drawing graphs.



- The wrong conclusion about the continuous function at x_0 which based on calculations without regard to the deterministic function $y = f(x)$ is purely understanding because only 2.7% of the students is paying attention to the condition of x when calculating the limit. It shows that the remaining works conclude that the continuity of the function is incorrect. This problem of students can be caused by:

+ Students focus on computational techniques without regard to other factors such as the set of definition of the function;

+ Students only do the same exercises repeatedly like exercises in textbooks, so when they do the strange ones, they do not know how to solve them. In the course of the study, we have found that students have mistakenly commented "the graph of the continuous function on the interval is "a continuous line" on "the straight line". This can also be a mistake in the mathematical communication process with graphs, drawings of students. However, in this study, we have not investigated the causes and solution, we may find them in the further studies.

- When asking students conclude the equation of function $f(x) = 0$ on $(a;b)$ in problem 3;

We found that 43.24% of respondents said this problem is almost accurate when the continuity of the given function is stated. However, the question "Does the graph of function across the X- axis at the point of $(0;2)$? The students almost did not understand so they asked *what did "across the X-axis" mean?* From that, we can see that when reading the graph and linking it with the algebra, students are completely envisioned. In our opinion, the cause of this obstacle is that in the process of learning mathematics, the condition of mathematical communication of students is less and less, mainly because of the lack of instruction about reading graph from teachers.

- The lack of knowledge of limits leads to knowledge obstacles when the mathematical communication in the continuous function showed in problem 4, in particular only 5.41% of students has argument but the knowledge of this work is not accurate, the limit of the problem is not calculated but 94.59% of the work of the students still find the answer.

This problem is caused by one of the following:

+ The knowledge of the limitations of the students is primitive, with little exposure to the concepts of "sequence", "infinity", "limit".

+ During lectures, lecturers instruct fast and not clear enough. Also, exercises are less diverse and comprehensive.

- We have not built lessons that can be discussed or exchanged to express their critical thinking, so we let students apply mathematical communication by showing their opinions in Problem 6, and the result obtained 24.32% of the answer but did not explain, which means they



did not raise their own opinion but agreed with the idea of the article. 21.62% denied comments on the question and raised personal opinions but did not provide accurate examples to negate the opinion. The remaining 54.05% did not reflect the opinions of the question. This proves that they are weak in critical thinking, have not raised their personal opinions and dared not comment on others' opinions as true or false. Possibly for some of the following reasons:

- + Students "afraid" to counter the opinions of others.

- + Students find it difficult to find arguments to protect their opinions, so they have chosen to agree with the opinions of others so that they do not need to explain.

- + Teachers have not built similar teaching situations for students to become familiar with discussion and communication.

- There are more than half of the work done (59,46%) of the students who have confused continuous function at a with continuous function as shown in problem 5, what is more, the proof of the problem is continuous on a which accounted for 29.73% of no answer. This proves that the "students misunderstand between continuous function on the segment, interval with at a point" is probable. We think this is a common problem for a number of reasons:

- + Students misunderstand the problem to solve. Teachers regularly concentrates on exercises to prove continuous functions at a point, so when required to prove continuous functions on a given interval, segment or set, students do the same as at a point.

- + In class, students do not pay attention to the theorems, comments resulting in the lack of knowledge.

- The other thing is "students use the pocket calculator mechanically in the process of calculating the limit, so they just know how to use the pocket calculator without understanding why such results". In fact, when observing, this really happens in the 12th grade, teachers teach students to encourage them to use pocket calculator to speed up the test at the National High School Graduation Examination. Typically, in the questionnaire 100% of student class 12A3 answered correctly and very fast thanks to the pocket calculator.

- Particularly, the obstacle "the method of learning and recording improperly obstruct the process of mathematical communication of student" which we have not really investigated as well as the solution for it because each teacher as well as each student have their own habits, methods of recording, we cannot force them to do in the same framework.

In the obstacles mentioned above, we find that the obstacle of students in mathematical communication when performing graphs, the figure is a strong impediment to students; compared to other obstacles, graphic representation, graph reading, and reading variable table are difficult to draw. The students are not able to observe and visualize the request of the problem or the shape of the graph that they always need the help of friends or instructors. The "lazy" and "afraid" feeling when drawing graph is also one of the manifestation of students not apply mathematical communication in this content well.



4. SUGGESTION AND COMMENTS

4.1. Suggestion

Through the research process, we have learned some disadvantages that students often encounter when the mathematical communication in the continuous functions:

- Feel difficulties in mathematical communication when performing graphs, drawings.
- Make wrong conclusions about the continuity of the function at x_0 due solely to calculations, regardless of the characteristic of the x_0 .
- Students constraint conclude the equation of function $f(x) = 0$ on $[a; b]$ without proving the continuity of function $f(x)$ on $[a; b]$.
- The lack of knowledge of students on the limit of functions or series of numbers leading to obstacles in the course of mathematical communication.
- The problem of critical thinking of students is poor during the mathematical communication.
- Students unknowingly identifies the continuous function on the segment with the continuous function at a point.

Through the case study in teaching continuous function, the success of the research is:

- Students are directly involved in mathematical representations, through observing exercises designed to promote mathematical communication for them.
- In the process of observation and study, students have boldly discussed with friends about their way of thinking, have the opportunity to communicate and discuss with teachers and friends.
- Students have the opportunity to express creative thinking and critical thinking in the process of solving problems with math expression.
- Thanks to the help of teachers and friends as well as students themselves easily find mistake of their own, then change the habit of solving problem, and gain more knowledge and experience from it.

Therefore, if students often apply mathematical communication, students can form positive and sensitive thinking, actively raise their opinions in the process of mathematical exchange.

4.2. Comments

Mathematical communication is a necessary process in every mathematical lesson, but through experiment we find that the learning process of mathematical communication in students still face many obstacles, namely in the continuous function;



Therefore, in the teaching process to overcome the obstacle of students in mathematical communication when performing graph, the drawing teachers need:

- Give more exercises on graphic representation, drawing for students; Instruct them to identify graphs of basic functions.
- Remind students to focus on the definition of function when determining the points in the graph.
- Exercises for students need to be structured in a variety of ways, not stereotyped and only in textbook.
- Encourage students to apply mathematical communication in the learning process so that they have a more comprehensive view.

Regarding to the wrong conclusion on the continuity of the function at x_0 mainly because

students only based on the calculation without taking into account the characteristics of x_0

teachers need:

- Remind students about the importance of set of definition when solving problem; Make assignments more relevant to the problem more often to make the habit of paying attention to this mistake.
- Change the teaching methods to suit each student and each level of exercise.

To overcome the obstacle that students state conclusion of the equation of the function

$f(x) = 0$ on $[a;b]$ without considering the continuity of the function $f(x)$ on $[a;b]$ teachers

need:

- Similar to the above obstacles, teachers should build exercises for students to see their mistakes during the exercise.
- After each exercise teachers need to correct errors for students in detail. Create opportunities for other students to comment and discover mistakes, to create excitement and academic thinking for students.

To overcome the obstacle in the lack of knowledge of the students on the limit of the function or series of numbers leading to the obstacles in the course of mathematical communication,

teachers need:

- Supply the knowledge about limit to the students more, emphasize the important content and often check whether they remember knowledge or lack of knowledge to instruct them timely.
- Taking advantage of the more intensive and self-selected lessons to supply the knowledge that students lack, it is necessary for students to actively interact with regular mathematical representations so that they can learn more about mathematical communication.



In order to overcome obstacle “critical thinking of students is still poor in the course of mathematical communication”, teachers need:

- Encourage students to actively exchange and communicate in class. Create opportunities for students to express their ideas and opinion about problems in mathematics.
- Timely correct mistake of students in terms of words, symbols, mathematical language, ... to form good habits for the course of mathematical communication in students.
- Regularly build exercises showing the course of mathematical communication for students to communicate their ideas and mathematical results, helping students to learn more.
- Create opportunities for students to comment other students work to increase their ability of thinking and criticism. After each issue need to clarify the correct or wrong way to help students deeply remember.

To overcome the obstacle “students indiscriminate between continuous function on the segment, the interval with continuous function at a point, teachers need:

- Do not just give one form of continuous function at a point that should interleave the continuous function on the segment, on the interval, remind students should pay attention to observe the problem requirements to not be wrong.
- After each exercise teachers should remind students how to present symbols to help them have better mathematical communication habits.

For the general course of mathematical communication, teachers need:

- Encouraging students to express their mathematical ideas through mathematical communication will help teachers better understand what students are learning.
- Enhance emotional expression with students through speech, gestures, i.e. (look, laugh and nod with them) to motivate them.
- Actively change teaching methods appropriately and build effective working relationships between math teachers and have positive attitude when receiving feedback from colleagues for improving knowledge, teaching skills of yourself.
- Ask students to express a problem, a solution, i.e. in a variety of ways to give them a broader perspective on math learning than just simply solve a problem.
- Encourage students to answer or ask questions (motivation, praise, i.e.) to create excitement for students to discuss, exchange information and debate.
- It must be recognized that the role of teachers in the course of mathematical communication is extremely important, as the person who controls and coordinates the exchange of students but does not interfere with the exchange between them.
- Do not deny any ideas of students, teachers should give them the freedom to express their ideas and confidently protect their opinions.



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To students, they need:

- Actively interact with friends and teachers in the learning process of math. Be more active in course of mathematical communication with teachers and friends, should not shy, or pushed other students to solve the problem.
- Change the habit of doing the lesson, notice the important content teachers regularly remind when teaching. Students should themselves explore and learn new knowledge in the process of learning math.
- Ask questions and try to answer the questions. Also, willing to ask questions for teachers and friends if students do not know or confuse.
- Accept change of thought when the outcome of the exchange process is not as originally thought.
- Not only know the way to protect personal opinion but also respect the opinions of the opponents.
- Build a good background knowledge before building mathematical communication.

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