

# Short-term power load probability density forecasting method using kernel-based support vector quantile regression and Copula theory

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1           **Short-term power load probability density forecasting**  
2           **method using kernel-based support vector quantile**  
3           **regression and Copula theory**

4           Yaoyao He<sup>a,b\*</sup>, Rui Liu<sup>a,b</sup>, Haiyan Li<sup>a,b</sup>, Shuo Wang<sup>c</sup>, Xiaofen Lu<sup>c</sup>

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11       **Abstract**

12 Penetration of smart grid prominently increases the complexity and uncertainty in  
13 scheduling and operation of power systems. Probability density forecasting methods  
14 can effectively quantify the uncertainty of power load forecasting. The paper proposes  
15 a short-term power load probability density forecasting method using kernel-based  
16 support vector quantile regression (KSVQR) and Copula theory. As the kernel  
17 function can influence the prediction performance, three kernel functions are  
18 compared in this work to select the best one for the learning target. The paper  
19 evaluates the accuracy of the prediction intervals considering two criteria, prediction  
20 interval coverage probability (PICP) and prediction interval normalized average width  
21 (PINAW). Considering uncertainty factors and the correlation of explanatory  
22 variables for power load prediction accuracy are of great importance. A probability  
23 density forecasting method based on Copula theory is proposed in order to achieve the  
24 relational diagram of electrical load and real-time price. The electrical load forecast  
25 accuracy of the proposed method is assessed by means of real datasets from Singapore.  
26 The simulation results show that the proposed method has great potential for power  
27 load forecasting by selecting appropriate kernel function for KSVQR model.

28 **Key words:** Short-term power load probability density forecasting; Support vector  
29 quantile regression; PI coverage probability; PI normalized average width; Copula  
30 theory; real-time price

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# 1. Introduction

Load forecasting is a fundamental and vital task for economically efficient operation and controlling of power systems. It is used for energy management, unit commitment and load dispatch. The high accuracy of load forecasting guarantees the safe and stable operation of power systems. Therefore, it is necessary to improve the reliability and the forecasting accuracy of power systems. Reliable power load forecasting can decrease energy consumption and reduce environmental pollution. From different points of view, load forecasting can be divided into different categories. For instance, load forecasting has been classified into short-term, medium-term, and long-term forecasts depending on the forecast horizon [1]. The short term electric load forecasting (STLF) has attracted substantial attentions because of its definitive impact on the daily scheduling and operations of a power utility [2]. So, STLF is the focus of this paper.

In general, load forecasting contains two common types, which are point forecasts and interval predictions. Point forecasts only provide the values of predicted points, but convey no information about the prediction uncertainty [3]. As to interval predictions, a prediction interval (PI) is composed by lower and upper bounds that include a future unknown observation with a certain probability  $(1-\alpha)\%$  named the confidence level [4]. Different from these two types of forecasts, probability density forecasting is able to offer much useful information by constructing probability density functions of forecasting results. Meanwhile, probability density forecasting is advantageous to the prediction accuracy [5]. This types of forecasts can provide estimates of the full probability distributions of the future load demand [6].

To improve the performance of the STLF methods, many studies have been carried out in recent decades. Various hybrid models have been used for load forecasting [7], including wavelet transform and grey model improved by PSO algorithm [8] and some hybrid methods for STLF [9,10]. Considering the complexity and potential nonlinearity of power load, support vector regression (SVR) [11] has been proposed to deal with this problem, which has become one of the most promising and effective techniques due to their attractive features and profound empirical performance in practical applications [12]. As a kernel-based method, SVR is capable of mapping the input data from a low dimensional space to a high-dimensional feature space, which can flexibly convert nonlinear regression into linear regression without assuming

1 particular functional forms [13]. Due to the strong generalization ability of Gaussian  
2 kernel function, the SVR method based on Gaussian kernel [14,15] has been widely  
3 utilized in the field of power load forecasting.

4 In order to fully discover kernel-based SVR, Zhou et.al compared the performance  
5 of three kernel-based SVR in terms of forecasting accuracy [16]. To determine the  
6 ideal kernel, Che and Wang have proposed a multiply kernels model based on a  
7 combination selection algorithm for STLF [17]. Authors in [17] showed that the  
8 optimal combination is more effective than simple kernel-based SVR models and  
9 other multiply kernels combination models. However, these kernel SVR models can't  
10 completely measure the uncertainty of future power load, and only provide the  
11 accurate point prediction results.

12 Different from SVR, Quantile regression (QR) is a popular statistical method for  
13 estimating the quantiles of a conditional distribution on the values of covariates. QR is  
14 capable of explaining the relationships among random variables regardless of the type  
15 of the distribution function [18,19]. It is suitable to the problem with multi  
16 independent variables. If a probability density function is defined, the any shape of the  
17 predictive distribution can be determined by means of the estimated quantiles.  
18 Therefore, QR methods have been used in power load and electricity price forecasting  
19 in recent years [20-22]. However, the shortcoming of traditional linear QR is the  
20 difficulty in solving the complex nonlinear optimization problem. The difficulty in  
21 nonlinear QR lies in how to find the appropriate form of nonlinear function [23].

22 The accuracy of the power load forecasting is also influenced by some other factors,  
23 such as, economy, environment, historical data and real-time price. These factors  
24 make load forecasting become a complicated task [24]. Particularly, real-time price is  
25 one of the uncertain factors for smart grid. Thus, real-time price is considered as an  
26 important factor. Real-time price forecasting has become the core process of the  
27 power load system at the operational level [25-29]. With the emergence and  
28 development of smart grid, people can adjust the electricity expenditures mode  
29 according to the electrical load demand and real-time price. In other words, the  
30 consumers' activities are likely to be influenced by real-time price in smart grid. From  
31 the point of view of consumers, electricity cost can be decreased when the real-time  
32 price is referenced [30]. Although there are a few load forecasting methods, which  
33 considers the effect of real-time price for power load forecasting models [31-35], it

1 may lose some valuable information for power load forecasting without considering  
2 the correlation between real-time price and power load. Renewable energy sources  
3 and distributed generation are integrated into power systems, which increases  
4 uncertainties in both generation and demand sides. These uncertainties of load  
5 forecasting are urgent to be addressed. The probability density forecasting method is  
6 considered as a powerful tool to quantify uncertainties associated with forecasts.

7 To measure the prediction uncertainty, support vector quantile regression (SVQR)  
8 models have been proposed, which incorporate SVR into the QR [36, 37] to construct  
9 a nonlinear QR method. By applying SVR with a check function instead of an  
10 e-insensitive loss function into the QR model, SVQR tends to quantify more uncertain  
11 information. It is estimated by settling a Lagrangian dual problem of quadratic  
12 programming. In practical applications, SVQR provides an effective way to acquire  
13 the nonlinear QR structure by introducing a kernel function, which has shown good  
14 performance of estimating multi-period value at risk [38-40].

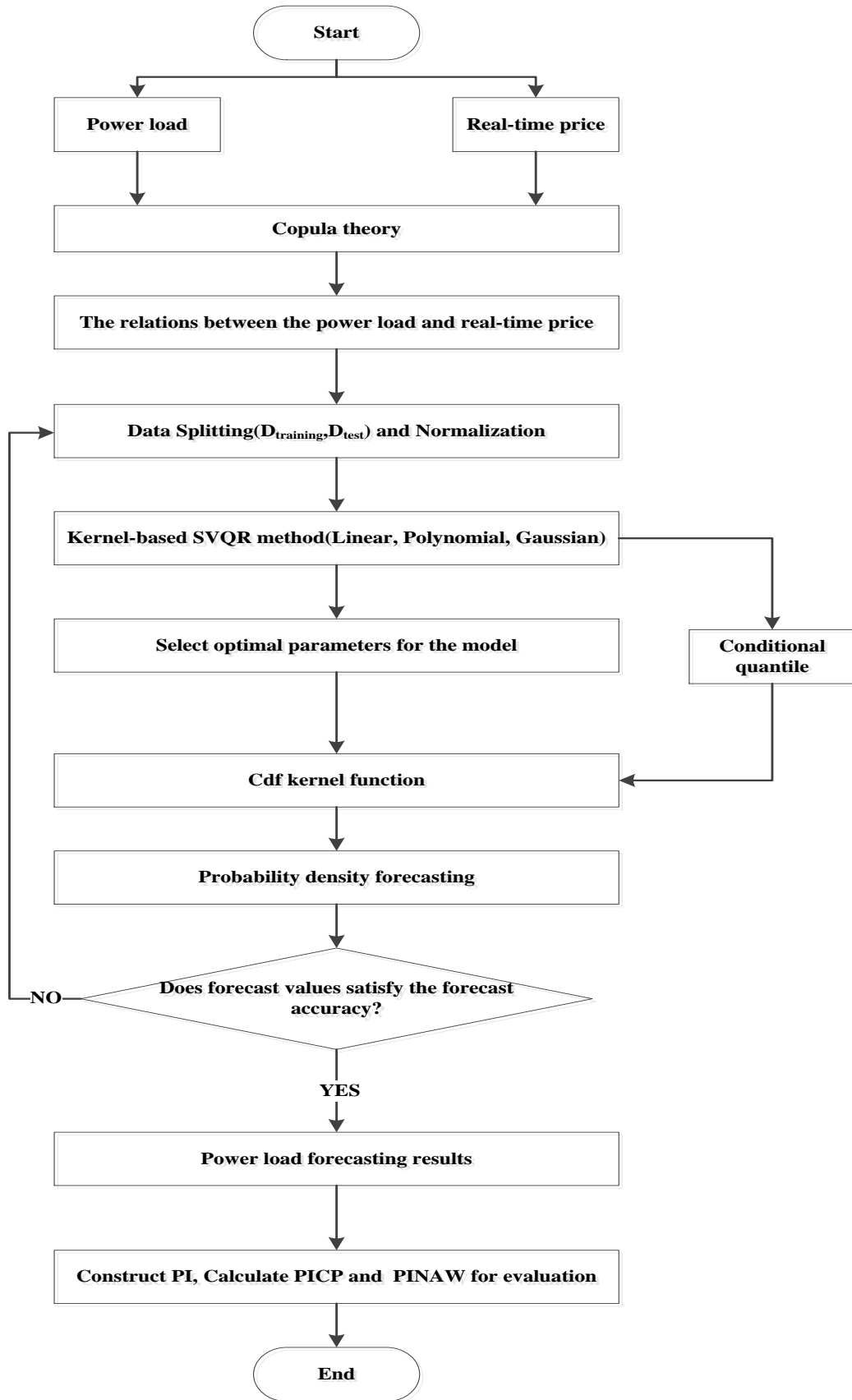
15 SVQR methods are based on kernel functions. Their better prediction performance  
16 is demonstrated based on the selection of an appropriate kernel function that fits the  
17 learning target. However, current SVQR methods only adopt the Gaussian kernel  
18 function and do not consider the performance of other kernel functions. This paper  
19 proposes a kernel-based support vector quantile regression (KSVQR) model, which  
20 chooses most appropriate kernel function from three commonly used kernels, namely  
21 linear, polynomial and Gaussian kernels. The proposed KSVQR method is applied to  
22 the probability density forecasting, which can generate complete probability  
23 distribution of the future value. The real value of power load and real-time price are  
24 employed to probability density forecasting based on Copula theory, which is used to  
25 analyze the correlation between power load and real-time price.

26 The contributions of this article include: 1) This paper proposes KSVQR model,  
27 which compares different kernel functions and selects the optimal kernel function for  
28 power load probability density forecasting. The proposed KSVQR model can  
29 quantify the uncertainty between power load and real-price, and provide much useful  
30 information than existing kernel SVR models. 2) Two criteria of interval prediction  
31 are adopted to evaluate the performance of probability density forecasting method  
32 considering real-time price, namely, PI coverage probability (PICP) and PI  
33 normalized average (PINAW). 3) A short-term power load probability density

1 forecasting method based on Copula theory is presented to verify the importance of  
2 real-price in the smart grid. The  $t$ -Copula function is adopted to draw the relational  
3 diagram and explain the nonlinear correlation between the power load and real-time  
4 price. 4) The accuracy of power load forecasting is assessed by three cases of  
5 Singapore. Moreover, the comparison of prediction results from SVQR, SVR and  
6 Back propagation (BP) method exhibits that the proposed method can achieve better  
7 prediction performance. The main structure of this paper is shown in Fig.1, which  
8 provides the main ideas so the reader can see a roadmap before moving on to the rest  
9 of the paper.

10 The organization of this paper is as follows. Section 2 introduces the mathematical  
11 formulation of SVR, QR and KSVQR models. This section also introduces probability  
12 density prediction based on Copula theory. Section 3 introduces two point forecasting  
13 metrics and two PI assessment metrics to measure the errors and uncertainty of the  
14 power load forecasting. Practical cases of Singapore are used to evaluate the  
15 performance of the proposed KSVQR model in Section 4. Finally, the conclusions and  
16 future work are summarized in Section 5.

17



1

2

Fig.1 The flowchart of the paper structure

## 1 2. A kernel-based support vector quantile regression

### 2 2.1. SVR method

3 SVM is proposed by Vapnik which is based on statistical learning theory and  
4 structural risk minimization principle [41]. It is called SVR when SVM is applied to  
5 regression problems. Given a data set  $T = \{x_i, y_i\}_{i=1}^n$ , where  $x_i \in R^d$  and  $y_i \in R$ , the main  
6 aim of SVR is to obtain a regression model which has good forecasting performance  
7 on future cases. When the data set  $T$  is nonlinearly dependent,  $m(x)$  can be regarded  
8 as a nonlinear function of the input vector  $x$ . In order to solve this problem, a feasible  
9 way for  $m(x)$  estimation is to perform the locally polynomial regression in  
10 parametric form. To implement the nonlinear mean regression, the paper projects the  
11 input vector  $x$  into a higher dimensional feature space using a nonlinear mapping  
12 function  $\phi(\cdot)$ , which is defined by a kernel function.  $m(x)$  of SVR can be obtained by  
13 the following linear functional form.

$$14 \quad f(x) = m(x) = w \cdot \phi(x) + b \quad (1)$$

15 where  $w$  is the weight vector,  $b$  represents the threshold and  $\phi(\cdot)$  denotes a  
16 nonlinear mapping function. The optimal parameter  $(w, b)$  of the model can be  
17 solved by the following formula.

$$18 \quad \min_{\omega, b} \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^k |y_i - f(x_i)| \quad (2)$$

19 where  $C$  denotes penalty parameter,  $k$  is sample size.

### 20 2.2. QR method

21 QR is introduced to replace the classical mean regression [19]. It provides a  
22 comprehensive strategy for the entire conditional distribution of a response variable  $y$   
23 when  $x$  is a explanatory variable instead of the conditional mean only. The idea behind  
24 QR can be ascended to the loss functions in advance. The check function (the  
25 asymmetric loss function) was proposed by Koenker [18]. It can obtain the optimal  
26 parameters through the check function minimization. The check function  $\rho_\tau(\mu)$  is  
27 defined as follows.

$$28 \quad \rho_\tau(\mu) = \mu(\tau - I(\mu)) \quad (3)$$

29 The indicator function  $I(\mu)$  is shown as following.

$$30 \quad I(u) = \begin{cases} 1, & u < 0 \\ 0, & u \geq 0 \end{cases} \quad (4)$$



1 where  $\tau \in (0,1)$  is able to generate quantiles.

## 2 **2.3.KSVQR model**

3 The regression variables of QR are capable of being adopted to provide information  
4 for estimating conditional quantile of response variables. It has been found that  
5 explanatory variables have influence on the response variable under different  
6 quantiles [18]. However, QR is based on the linear regression, which is difficult to  
7 solve complex nonlinear problems. Takeuchi and Furuhashi [36] first utilized SVR to  
8 study quantile regression problems, which are better in solving the nonlinear structure  
9 of the economic system and the heterogeneity of economic behavior. Li et al [37]  
10 proposed a SVQR method and deduced a formula for the effective dimension of the  
11 proposed model, which allows suitable selection of the hyperparameters. Shim  
12 introduced a semiparametric method which combines SVR with QR to construct  
13 SVQR model [39]. Without loss of generality,  $u_i = x_i = (h_i, p_i)^T$  are used as input  
14 variables, in which  $h_i$  represents historical load, and  $p_i$  stands for real-time price.  
15 Therefore, SVQR model can be obtained by applying a check function of QR in  
16 formula (3) instead of penalty function in formula (2) as follows.

$$17 \quad \min_{\omega_\tau, b_\tau} \frac{1}{2} \|\omega_\tau\|^2 + C \sum_{i=1}^k \rho_\tau(y_i - b_\tau - \beta_\tau^T u_i - \omega_\tau^T \phi(x_i)) \quad (5)$$

18 where  $C$  denotes penalty parameters and  $\phi(\cdot)$  denotes the nonlinear mapping  
19 function. We can rewrite (5) to the quadratic programming by formulation as follows.

$$20 \quad \min \frac{1}{2} w^T w + C \sum_{i=1}^k (\tau \xi_i + (1-\tau) \xi_i^*) \quad (6)$$

21

22

$$23 \quad \text{s.t.} \quad \begin{cases} y_i - b - \beta^T z_i - w^T \phi(x_i) \leq \xi_i \\ -y_i + b + \beta^T z_i + w^T \phi(x_i) \leq \xi_i^* \\ \xi_i, \xi_i^* \geq 0 \end{cases} \quad (7)$$

24 To solve the optimization problem, the slack variables are introduced to construct the  
25 Lagrange function, and the estimators of SVQR are calculated by the following  
26 equation:

$$\begin{cases}
\omega_\tau = \sum_{t=1}^k (\alpha_t - \alpha_t^*) \phi(x_t) \\
(b_\tau, \beta_\tau)^T = (U^T U)^{-1} U^T (y - K(\alpha - \alpha^*)) \\
Q_{y_t}(\tau | u_t, x_t, \eta) = b_\tau + \beta_\tau^T u_t + K_t(\alpha - \alpha^*)
\end{cases} \quad (8)$$

The meaning of the above parameters are expressed respectively as:  $k$  is sample size,  $\alpha, \alpha^*$  denote the optimal Lagrange multipliers. Following principles in SVM, the index set of support vectors are acquired in the SVQR model.  $I_{SV} = \{t=1, 2, \dots, k | 0 < \alpha_t < \tau C, 0 < \alpha_t^* < (1-\tau)C\}$  is obtained by exploiting Karush-Kuhn-Tucker conditions [42],  $U$  is the matrix consisting of  $U = (1, u_t^T)$ ,  $y = \{y_t | t \in I_{SV}\}$ .  $K_t$  is a kernel function in the input space, which is equal to the inner product of vector  $x_s$  and  $x_t$  in the feature space, that is  $K(x_s, x_t) = \phi(x_s)^T \phi(x_t), t \in I_{SV}$  ( $s = 1, 2, \dots, T$ ).

How to select regularization parameter  $C$  and the parameters of kernel function play a vital role in the performance of the KSVQR approach. The researchers need to select in advance the type of kernel function and the associated parameters for KSVQR. In this study, we choose three types of kernel functions, namely, linear, polynomial, and Gaussian kernels, which are commonly employed in the related area [17]. The linear kernel is

$$K(x, z) = x^T z, \quad (9)$$

the expression of polynomial kernel is

$$K(x, z) = (x^T z + c)^d \quad (10)$$

and the formula of Gaussian kernel is

$$K(x, z) = \exp\left(\frac{-(x-z)^2}{2 \times \delta^2}\right) \quad (11)$$

where  $c$  is the offset of polynomial,  $d$  is the degree of the polynomial kernel,  $\delta$  is the width of Gauss kernel. The selection of the parameters of the kernel function presents a considerable impact on performance of the KSVQR approach.

#### 2.4. Probability density prediction based on Copula theory

Considering uncertainty factors and the correlation of input variables are of great

1 importance for accurate power load prediction. The renewable energy power  
 2 generation such as wind power has strong randomness and volatility, which brings  
 3 more uncertainty for the planning and operation of power systems. The randomness  
 4 and volatility are brought by output power of wind and photovoltaic units in micro  
 5 grids (MGs) [30]. MGs have the indispensable infrastructure in smart grid, which  
 6 consist of distributed energy resources, customers and energy storage units [43]. In the  
 7 environment of smart grid, renewable energy sources and distributed power are  
 8 applied to MGs, which become an indispensable important segment in the  
 9 development of smart grid. However, the volatility and intermittency of renewable  
 10 energy have significant impact on electricity market under the real-time price guidance,  
 11 which increases uncertainties in both generation and demand sides [44,45]. In addition,  
 12 there exists correlation between many random variables in the power system. If the  
 13 correlation of diverse factors is ignored, it may cause calculation error, which can have  
 14 a direct effect on the safety of power system and economic operation. Hence, in order  
 15 to gain accurate prediction results, power load forecasting should consider correlation  
 16 factors. Copula theory is introduced to build correlation of input variables probability  
 17 density prediction. The theory not only describes the correlation between input  
 18 variables in detail, but also has a certain influence for power load forecasting results.

19 Copula theory is proposed firstly by Sklar [46], which describes accurately the  
 20 correlation of input variables for nonlinear and asymmetric variable analysis. It is  
 21 flexible and important to analyze the tail correlation between input variables which is  
 22 based on Copula theory. In this article, the  $t$  - Copula function is used to describe the  
 23 correlation between the input variables. Power load and real-time price are the input of  
 24 the Copula function, which provides the correlation diagram of input variables.  
 25 Gaussian kernel function is adopted to conduct probability density prediction. The  
 26 kernel density estimation is defined as follows:

27  $X_1, X_2, \dots, X_n$  are taken from one-dimensional continuous total samples, and the kernel  
 28 density estimation of the overall density function  $f(x)$  at any point  $x$  is defined as:

$$29 \quad \hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - X_i}{h}\right) \quad (12)$$

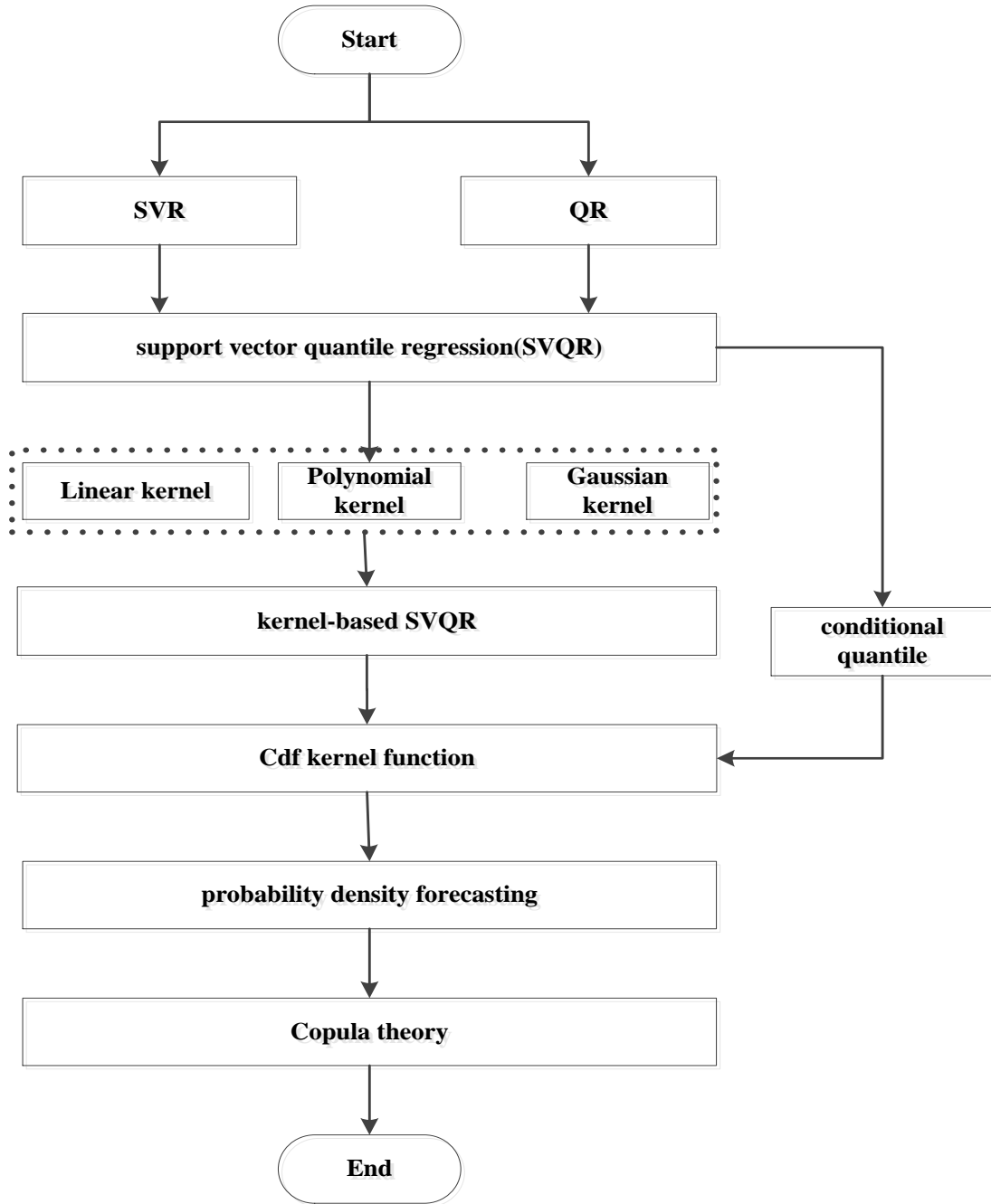
30 where  $K(\cdot)$  denotes kernel function,  $h$  is bandwidth, Gaussian kernel function is  
 31 adopted as the kernel density estimation function.

32  $N$ -dimensional  $t$  - copula density function is defined as follows:

$$c(u_1, u_2, \dots, u_N; \rho, k) = |\rho|^{-\frac{1}{2}} \frac{\Gamma\left(\frac{k+N}{2}\right) \left[\Gamma\left(\frac{k}{2}\right)\right]^{N-1} \left(1 + \frac{1}{k} \zeta' \rho^{-1} \zeta\right)^{-\frac{k+N}{2}}}{\left[\Gamma\left(\frac{k+1}{2}\right)\right]^N \prod_{i=1}^N \left(1 + \frac{\zeta_i^2}{k}\right)^{\frac{k+1}{2}}} \quad (13)$$

2 where the  $\rho$  is  $N$  order symmetric positive definite matrix for all the elements of 1 on  
3 the diagonal,  $|\rho|$  denotes determinant of square matrix  $\rho$ .  $k$  indicates the degrees of  
4 freedom.  $\zeta' = [\mathbf{t}_k^{-1}(u_1), \mathbf{t}_k^{-1}(u_2), \dots, \mathbf{t}_k^{-1}(u_N)]$ , in which  $\mathbf{t}_k^{-1}$  denotes inverse function of  
5 one-dimensional  $t$  distribution when the degree of freedom is  $k$ .  $\Gamma(\cdot)$  indicates a  
6 Gamma function.  $u_i$  ( $i=1, 2, \dots, N$ ) is input variable.

7 In addition, Pearson correlation coefficient is considered as measurement index for  
8 linear correlation of random variable. Fig.2 represents clearly the structure of the  
9 KSVQR model.



1

2

Fig. 2 The flowchart of KSVQR probability density forecasting model

3

### 3. Evaluation metrics

4

#### 3.1. Evaluating the prediction error

5

Many measures have been proposed to evaluate the errors of the power load

6

forecasting for point prediction, including MAPE (mean absolute percentage error)

7

and the MAE (mean absolute error). MAPE and MAE are defined as follows:

$$1 \quad MAPE = \frac{1}{n} \left( \sum_{i=1}^n \left| \frac{P_i - L_i}{L_i} \right| \right) \times 100\%, i = 1, 2, \dots, n \quad (14)$$

$$2 \quad MAE = \frac{1}{n} \left( \sum_{i=1}^n |P_i - L_i| \right) \quad (15)$$

3 where  $i$  denotes the hour and  $n$  indicates the total number of the hour over  
 4 forecasting period.  $P_i$  and  $L_i$  represent the  $i$ -th predicted value and actual value,  
 5 respectively.

### 6 **3.2.PICP (PI coverage probability) criterion**

7 PICP (PI coverage probability) and PINAW (PI normalized average width) are  
 8 usually considered as the criterion for assessing the accuracy of the prediction interval.  
 9 PICP is defined as the cardinal feature of the PIs (prediction intervals), which  
 10 demonstrates the percentage of targets that will be covered by the upper and lower  
 11 bounds. A larger PICP means more targets are located in the constructed PIs. PICP is  
 12 defined as follows [47]:

$$13 \quad PICP = \frac{1}{N} \sum_{i=1}^N c_i \quad (16)$$

14 in which  $N$  is the total number of predictions and  $c_i$  is a Boolean variable, which  
 15 demonstrates the coverage of PIs. From the perspective of mathematics,  $c_i$  is defined  
 16 as follows:

$$17 \quad c_i = \begin{cases} 1, & \text{if } y_i \in [L_i, U_i]; \\ 0, & \text{if } y_i \notin [L_i, U_i]. \end{cases} \quad (17)$$

18 where  $L_i$  and  $U_i$  are the lower and upper bounds of target  $y_i$ , respectively. To obtain  
 19 effective PIs, PICP needs to be more than the confidence level of PIs. Otherwise, PIs  
 20 are invalid and unreliable. The ideal value of PICP is equal to 100%, which indicates  
 21 all the target values are covered completely (namely, 100% coverage).

### 22 **3.3.PINAW (PI normalized average width) criterion**

23 To evaluate the quality of PIs, researchers are more interested in PICP rather than the  
 24 width of PIs [48]. On one hand, if the width of the interval is large enough, the request  
 25 for high PICP can be easily met. However, on the other hand, too wide intervals  
 26 transmit little information about the target values, which are useless for decision  
 27 making. Width of PIs should be as small as possible, and determines the  
 28 informativeness of PIs. In the literature, PINAW has been introduced, which is an  
 29 important quantitative measure. PINAW is defined as follows [3]:

$$PINAW = \frac{1}{NR} \sum_{i=1}^N (U_i - L_i) \quad (18)$$

in which  $R$  denotes the maximum minus minimum of the target values. The aim of using  $R$  is the normalization of the PI average width in percentage. Thus, PINAW can be adopted for performance comparisons.

#### 4. Case studies

In this section, a comprehensive experimental analysis is given. Three KSVQR models are compared with BP and SVR. The software MATLAB 7.14 is used for all the models. All the programs were run on a 3.20-GHz-based Intel dual-core processor (i5-3470) with 4 GB of random access memory.

The real-world datasets are used from Singapore network [49] to demonstrate the effectiveness of the proposed KSVQR model. It has a tropical rainforest climate with no distinctive seasons, and uniform temperature and pressure. Throughout the year, the climate is hot and humid, with temperatures in the range of 23 to 32 °C. So, the power load mode in Singapore is fixed on account of the climate and regional reasons. In addition, Singapore lacks of land and resources. Hydropower and wind power generation are infeasible, and solar energy can not be utilized in the large area widely. At present, 80% power load of Singapore comes from power generation of the natural gas. There are also a small amount of photovoltaic power stations which incorporate into power grid. With the development of opening up policy in Singapore electricity market, most of customers demand more competitive price. Under the condition of the environment and social background of Singapore, the power load forecasting is affected by electricity price, which is the main factor. So, real-time price is considered as an important factor for Singapore power load forecasting. The power load and real-time price data of Singapore are chosen as the input variables for the proposed model.

In case studies, section 4.1 carries out a correlation analysis between power load and real-time price based on the Copula theory. Real-time price is proved to influence the consumption of electricity. In section 4.2, a small size dataset and a medium size dataset in 2014 are selected to compare with existing point prediction methods. To further evaluate the generalization capability of SVQR, a smaller training sample and longer test sample example in 2016 is chosen in section 4.3. Section 4.4 summarizes the observations for the experiments.

#### 1 **4.1.The correlation analysis between power load and real-time price based on the** 2 **Copula theory**

3 Many uncertainty factors exist in power systems, such as equipment failure, power  
4 load fluctuations and so on. Especially, the rapid development of the smart grid and  
5 wide application of renewable energy increase uncertainty, which bring a certain  
6 impact on the operation and control of power systems. These uncertainty factors are  
7 considered as input variables to influence the power load forecasting. However, the  
8 correlation of these input variables has usually been ignored, which may cause  
9 calculation error and even have a direct effect on safe and economic operation of the  
10 power system. The section mainly analyzes the correlation between power load and  
11 real-time price based on the copula theory. This theory can deal with the correlation  
12 between random variables of normal and non-normal distributions.

13 This paper adopts Jarque-Bera (J-B) test, Kolmogorov-Smirnov test (K-S) and  
14 Lilliefors (L) test for real-time price and power load normality test, respectively. The  
15 level of significance is set as 0.01. The real-time price and power load datasets are  
16 chosen from November 2014 to December 2014 in Singapore, with 48 points in each  
17 day. The results of the case show that the  $h$  value of three kinds of the test is 1. The  
18  $p$ -value results are summarized in Table 1, which are all smaller than 0.01. Therefore,  
19 real-time price and power load do not follow the normal distribution. By the  
20 calculation, the value of Pearson linear correlation coefficient is 0.4112, which shows  
21 that the linear correlation of real-time price and power load is not significant. Copula  
22 theory is applicable to any distribution. The paper adopts  $t$  - copula function to explain  
23 the correlation between real-time price and power load.

24 Correlation analysis has drawn more and more attentions in many fields. Fig.3  
25 gives Binary  $t$  - Copula density function of correlation diagram between real-time  
26 price and power load.  $U$  is real-time price, and  $V$  represents power load. It clearly  
27 demonstrates the distributed situation and heavy tail of real-time price and power load.  
28 In addition, it shows strong correlation between the point (0, 0) and (1, 1) in the tail.  
29 In other words, power load consumption has a great influence on the customers when  
30 real-time price suddenly becomes high or low. The fluctuation of real-time price is  
31 able to cause the change of the power load. Similarly, the change of the power load  
32 tends to cause real-time price fluctuations.

33

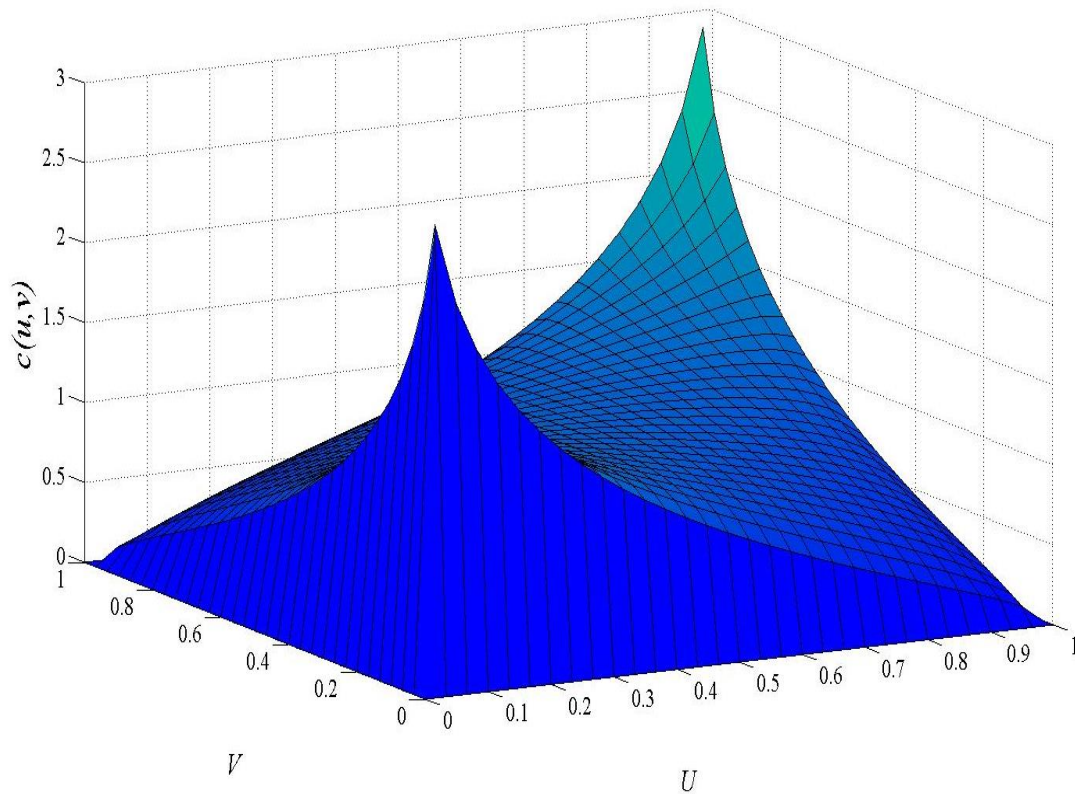


1 **Table 1**

2 The results of J-B, K-S and L test

	<i>p</i> -value of real-time price			<i>p</i> -value of power load		
	J-B test	K-S test	L test	J-B test	K-S test	L test
For the case of Singapore	1.0000e-03	1.5810e-170	1.0000e-03	1.0000e-03	4.8325e-18	1.0000e-03

3



4

5 Fig. 3 Binary *t* - Copula density function of correlation diagram between real-time  
6 price and power load for the case of Singapore

7 **4.2. Empirical results and analysis of Singapore in 2014**

8 In this subsection, the practical historical load and real-time price data of  
9 Singapore include a small size dataset and a medium size dataset in the winter of 2014.  
10 Namely, a small size dataset from November, 2014 is selected to predict the future  
11 load of 1 day, and a medium size dataset from December, 2014 is selected to predict  
12 the future load of 4 days. Without considering real-time price, the power load dataset  
13 from November 8, 2014 to November 19, 2014 are selected as training datasets, and  
14 the power load dataset on November 20, 2014 is chosen as testing data subset. Under

1 the condition of considering real-time price, the power load and real-time price  
2 datasets from November 8, 2014 to November 19, 2014 are chosen as training data  
3 subset, and the power load and real-time price datasets on November 20, 2014 are  
4 chosen as testing data subset. The case forecasts the power load of 1 day with 48 load  
5 points.

6 Moreover, in order to further demonstrate the satisfactory performance of the  
7 proposed KSVQR model, the another medium size dataset of Singapore is selected.  
8 Without considering real-time price, the power load datasets from December 1, 2014  
9 to December 26, 2014 are selected as training datasets. The power load from  
10 December 27, 2014 to January 30, 2014 are chosen as testing data set. Under the  
11 condition of considering real-time price, the power load and real-time price datasets  
12 from December 1, 2014 to December 26, 2014 are chosen as training data set, the  
13 power load from December 27, 2014 to December 30, 2014 is chosen as testing  
14 dataset. This case predicts power load for 4 days, and there are 48 points for a day in  
15 both conditions. All samples are normalized in advance

16 The empirical results, comparison and discussion among these cases are  
17 demonstrated to verify our research. We choose three different kernel-based SVQR  
18 models in our comparative studies. The two cases select 20 quantiles with the interval  
19 of 0.05, and the quantile is from 0.01 to 0.96. All the parameters for the three different  
20 kernel functions of two cases are shown in Table 2, where  $C$  is penalty parameter;  
21  $c$ ,  $d$  and  $\delta^2$  are the parameters of kernel functions. The selection of penalty  
22 parameters and the parameters of the kernel functions have a considerable impact on  
23 prediction ability of the KSVQR method [40]. The forecasting errors and time of  
24 mode (the highest probability point) in the probability density curve from the datasets  
25 in 2014 are summarized in Table 3. Two forecasting error measurements, MAPE and  
26 MAE, are employed to verify the forecasting accuracy of the proposed model. For the  
27 case of Singapore in November, 2014, Gaussian kernel SVQR obtains the optimal  
28 result with considering real-time price, the MAPE and MAE values are 0.81% and  
29 47.20, respectively. For the case of Singapore in December, 2014, Gaussian kernel  
30 SVQR also obtains the optimal result with considering real-time price, the MAPE and  
31 MAE values are 1.91% and 98.71, respectively. Furthermore, it is easy to find that the  
32 calculation time of three different KSVQR methods is similar, and it spends much  
33 time to obtain accurate results when the forecasting period is extended to 4 days. The  
34 results are reasonable because the principle of SVQR probability density forecasting

1 method is to construct probability density function by means of the kernel density  
2 function and point prediction results under different quantiles. The complexity will  
3 continually increase with the expansion of forecasting period and sample scale.

4 In order to further explain the superiority of the proposed method, the PICP and  
5 PINAW are considered as the evaluation metrics and the values of the two measures  
6 are shown in Table 4. We can discover that the constructed PIs cover the real values in  
7 a great percentage. For the case of Singapore in November, 2014, the PICP of the  
8 three different kernels SVQR is 100%, regardless of whether real-time price is  
9 considered or not, which means that all the targets are covered by PIs. It shows that  
10 the perfect results are obtained by KSVQR model when the forecasted period is only  
11 one day. On the other hand, the width of PIs for Singapore load is narrow. For the case  
12 of Singapore in December, 2014, it has the high PICP values with 99.48% for  
13 Gaussian kernel function with considering real-time price. The other two KSVQR  
14 models also have high coverage. This also means that the constructed PIs cover the  
15 target values with a high probability. However, PINAW value of the Gaussian-based  
16 SVQR models is wider than the other two models.

17 In order to better illustrate the advantages of KSVQR method, this paper compares  
18 the forecasting errors of KSVQR, SVR and BP models in Table 5. It shows the  
19 prediction errors of three different methods under the condition of considering  
20 real-time price and the condition of not considering real-time price. The penalty  
21 parameter of SVR is 8000 and the insensitive loss function is 0.001 for 2014.11  
22 datasets of Singapore. For 2014.12 datasets of Singapore, the penalty parameter of  
23 SVR is 1000 and the insensitive loss function is 0.1. The iteration number of BP  
24 neural network is 1000, and the neural network structure for the two cases are 11-3-1  
25 and 7-1-1, respectively. KSVQR method is superior to the other methods based on the  
26 results in both cases of considering or not considering the real-time power price. The  
27 proposed KSVQR method shows strong generalization capability by comparing the  
28 forecasting results of different methods. Also, Forecasting results of KSVQR method  
29 with considering real-time price are better than that of KSVQR method without  
30 considering real-time power price. Therefore, real-time price factors should be  
31 considered as an important factor for STLF. No matter what kind of method is  
32 adopted.

33 Fig.4 demonstrates that the prediction results of Singapore on November 20, 2014

1 and prediction intervals based on real-time price and Gaussian kernel SVQR, which  
 2 show that the actual value always falls in the prediction interval and the mode of  
 3 predicted results is close to the true value curve. It comes to the conclusion that the  
 4 proposed method can accurately depict power load fluctuations. Fig.5 shows the  
 5 prediction results and prediction intervals for the case of Singapore from December 27,  
 6 2014 to December 30, 2014 based on real-time price and Gaussian kernel SVQR. It  
 7 can be seen from the diagram that the actual value almost falls in the prediction  
 8 interval. This also illustrates the Gaussian kernel SVQR method can better describe  
 9 power load fluctuations. Fig.6 and Fig.7 give the diagram of probability density curve  
 10 based on real-time price and Gaussian kernel SVQR on November 20, 2014 and  
 11 December 30, 2014, respectively. It gives completely probability distribution of future  
 12 power load and the real value also appears in the density function with high  
 13 probability, which can explain the advantages of probability density forecasting  
 14 method in quantifying the uncertainty and improving prediction accuracy. It can be  
 15 seen from Fig.6 that the rest of the actual values are mostly appear in the probability  
 16 density curve with the highest probability, in addition to the actual value on 12:00 that  
 17 appears in the tail of probability density curve. Similarly, it can be seen from Fig.7  
 18 that the rest of the actual values arise in the middle of the probability density curve,  
 19 except for the actual value on 23:30. However, the density curve drawn in Fig 7 is less  
 20 smooth than the results of Fig. 6.

21 **Table 2**

22 The parameters used in the KSVQR model.

Kernel function type	Parameters value		
For 2014.11 datasets of Singapore			
Linear	C=2		
Polynomial	C=1	c=1	d=1
Gaussian	C=8000	$\delta^2=10000$	
For 2014.12 datasets of Singapore			
Linear	C=1		

Polynomial	C=1000	c=1	d=1
Gaussian	C=1000	$\delta^2=4500$	

1 **Table 3**

2 Forecasting errors and time of mode from the datasets in 2014

	Without considering real-time power price			Considering real-time power price		
	MAPE(%)	MAE(MW)	Time(s)	MAPE(%)	MAE(MW)	Time(s)
For the case of Singapore in 2014.11						
Linear SVQR	1.29	75.94	0.97	1.16	67.94	1.42
Polynomial SVQR	1.13	66.73	1.03	0.87	50.63	1.44
Gaussian SVQR	1.10	64.43	1.13	0.81	47.20	1.52
For the case of Singapore in 2014.12						
Linear SVQR	2.28	119.80	141.58	2.06	106.61	150.43
Polynomial SVQR	2.29	120.77	173.95	2.03	105.44	238.91
Gaussian SVQR	2.20	115.51	167.29	1.91	98.71	169.64

3

4 **Table 4**

5 PI evaluation indices from the datasets in 2014

	Without considering real-time power	Considering real-time power
--	-------------------------------------	-----------------------------

	power price		price	
	PICP(%)	PINAW(%)	PICP(%)	PINAW(%)
For the case of Singapore in 2014.11				
Linear SVQR	100	16.48	100	11.62
Polynomial SVQR	100	15.95	100	11.83
Gaussian SVQR	100	15.76	100	11.75
For the case of Singapore in 2014.12				
Linear SVQR	91.15	23.93	93.23	22.20
Polynomial SVQR	82.81	23.68	93.23	21.11
Gaussian SVQR	96.35	30.65	99.48	29.60

1

2 **Table 5**

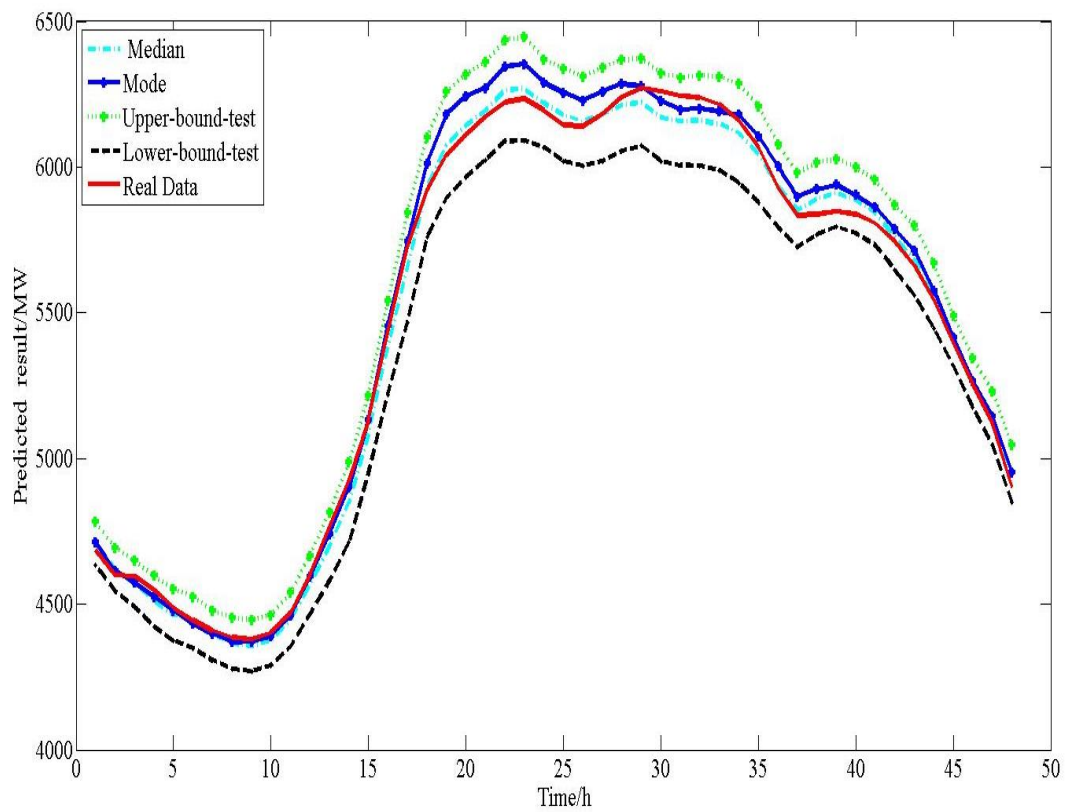
3 Forecasting errors of SVR , SVQR and BP from the datasets in 2014

algorithm	Without considering real-time power price		Considering real-time power price	
	MAPE(%)	MAE(MW)	MAPE(%)	MAE(MW)
For the case of Singapore in 2014.11				
SVR	3.41	200.00	2.97	154.36
SVQR(Gaussian kenel)	1.10	64.43	0.81	47.20
BP	3.65	215.96	3.30	174.80

For the case of  
Singapore in 2014.12

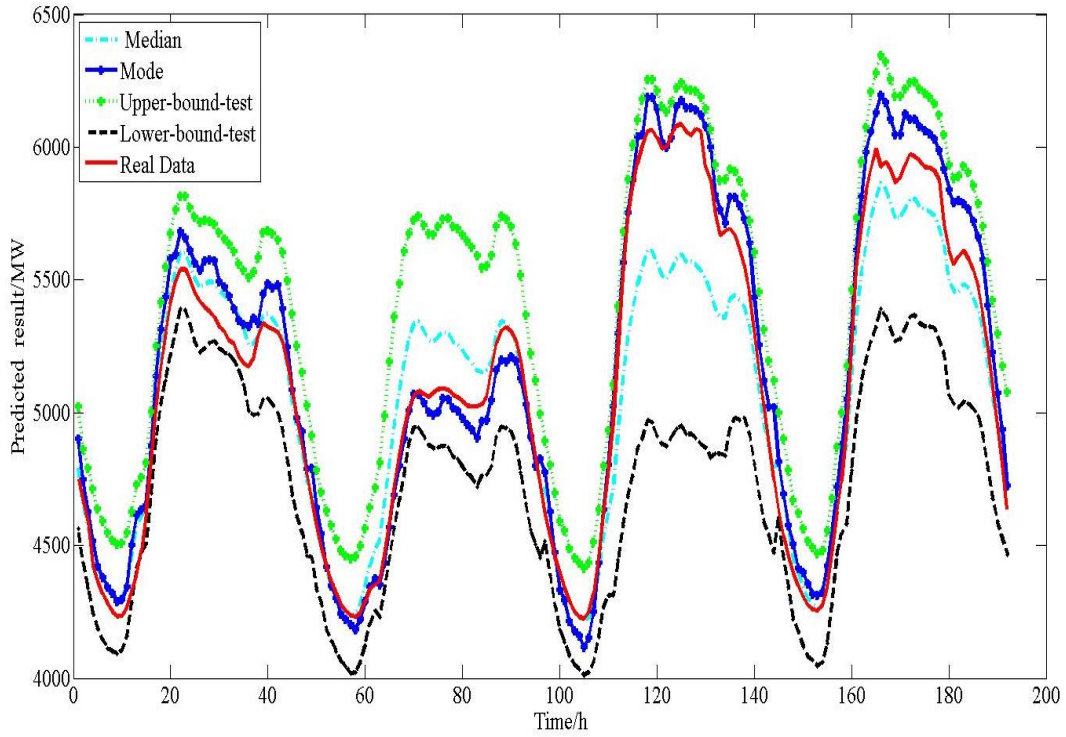
SVR	4.22	203.16	3.23	155.79
SVQR(Gaussian kenel)	2.20	115.51	1.91	98.71
BP	4.33	223.68	3.32	167.43

1



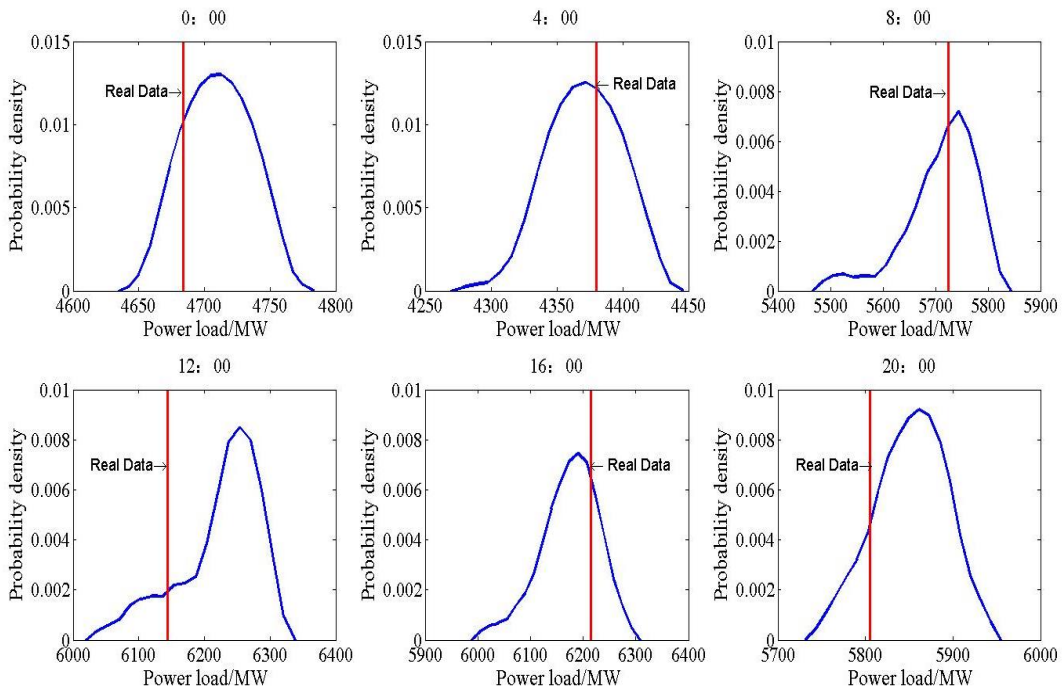
2

3 Fig. 4 Prediction results and prediction intervals based on Gaussian kernel SVQR for  
4 the case of Singapore on November 20, 2014.



1

2 Fig. 5 Prediction results and prediction intervals based on Gaussian kernel SVQR for  
 3 the case of Singapore from December 27, 2014 to December 30, 2014.

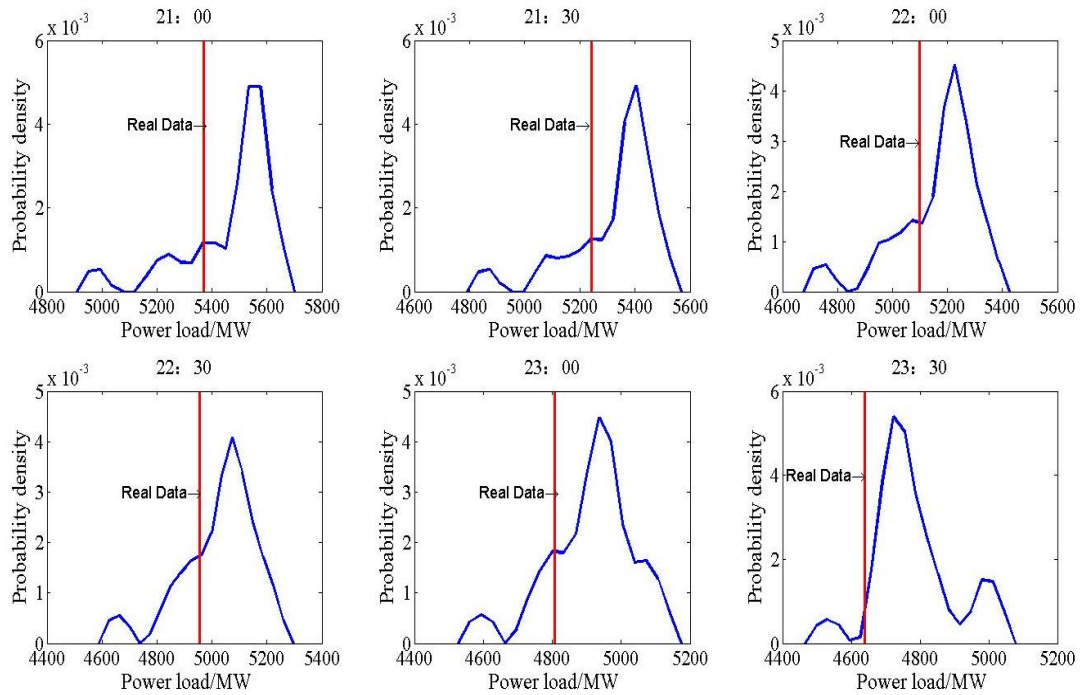


4

5 Fig. 6 Diagram of probability density curve based on real-time price and Gaussian  
 6 kernel SVQR on November 20, 2014.

7





1

2 Fig. 7 Diagram of probability density curve based on real-time price and Gaussian  
 3 kernel SVQR on December 30, 2014 at the last 6 half-hours.

#### 4 4.3. Empirical results and analysis of Singapore in July, 2016

5 In order to further evaluate the generalization capability of the proposed method, an  
 6 real-time dataset of Singapore in the summer of 2016 is adopted to predict the future  
 7 load of 10 days. The case selects the practical power load and real-time price datasets  
 8 of Singapore from July 1, 2016 to July 30, 2016 as a smaller training sample and  
 9 longer test sample example. As compared with the case in December, 2014, the period  
 10 of training sample is decreased to 20 days, and the period of test sample is increased  
 11 to 10 days. Under the conditions of considering and not considering real-time price,  
 12 this dataset is divided into training datasets and testing datasets. Namely, the datasets  
 13 from July 1, 2016 to July 20, 2016 are selected as training datasets, and the datasets  
 14 from July 21, 2016 to July 30, 2016 are chosen as testing datasets. Our task is to  
 15 predict power load for 10 days, and there are 48 points for a day in both conditions. It  
 16 comes to the conclusion that the total datasets are same as the other cases, but the  
 17 sample size of training datasets is smaller. Because we have shown that the Gaussian  
 18 kernel function can lead to the optimal results in our previous two case studies, we  
 19 also choose Gaussian kernel SVQR for our analysis here. Under the both conditions  
 20 of considering and not considering real-time price, all parameter values of the  
 21 KSVQR model are kept the same. The penalty parameter of the KSVQR model is

1  $l(C=1)$ , the width of Gauss kernel is  $20(\delta^2 = 20)$ . The experimental results are  
 2 summarized in Table 6. It can be concluded that the KSVQR model also has stable  
 3 forecasting results. At the same time, with the decrease of training data and the  
 4 increase of forecasting period, it can be seen that real-time price has a certain  
 5 influence on the results of power load forecasting. However, the effect is not obvious.  
 6 On the other hand, it has similar PICP and PINAW values though the result is a bit  
 7 better when the real-time power price is considered. Furthermore, the PINAW value  
 8 of this case is wider than the other two cases.

9 In order to clearly show the advantages of KSVQR method, Fig. 8 provides  
 10 prediction results and prediction intervals for the case of Singapore from July 21,  
 11 2016 to July 30, 2016 based on Gaussian kernel SVQR, which shows the actual value  
 12 mostly falls in the prediction interval. Furthermore, the mode of predicted results is  
 13 closer to the true value curve, which illustrates the KSVQR method accurately depicts  
 14 power load fluctuations. Fig. 9 shows the probability density curve based on real-time  
 15 price and Gaussian kernel SVQR on July 30, 2016. It is clear to see that the curve is  
 16 less smooth than those in Figs. 6 and 7, though the real value also appears in the  
 17 probability density curve.

18

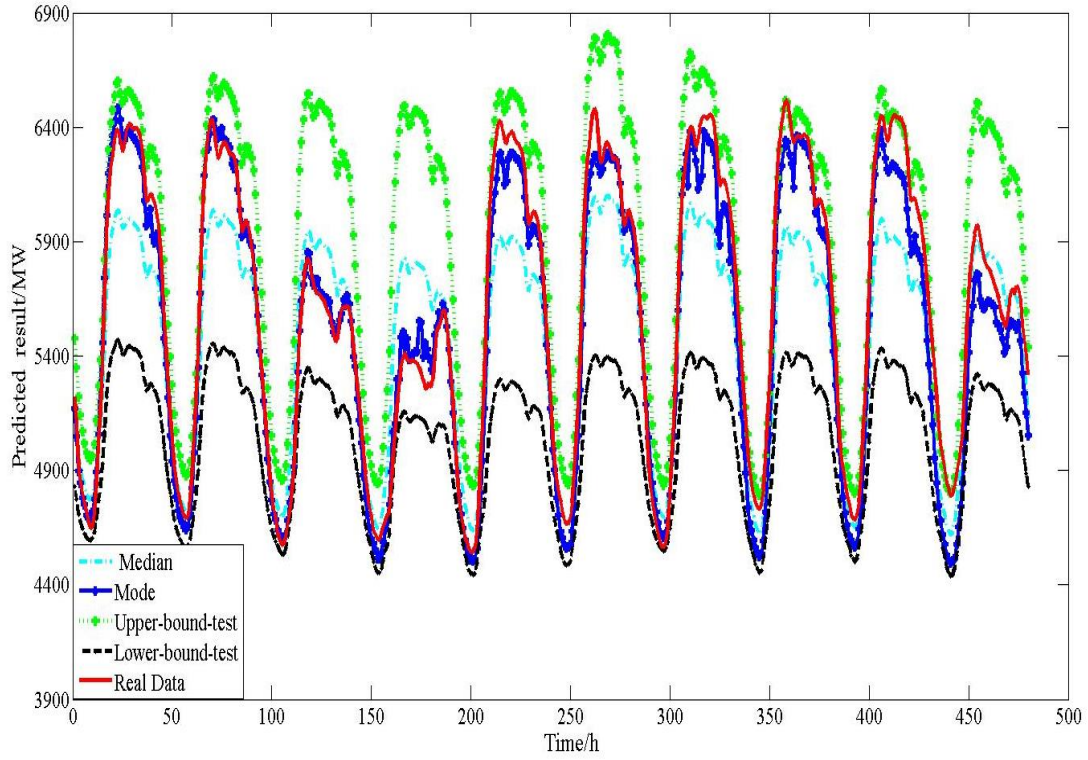
19

20 **Table 6**

21 Experimental results of KSVQR (Gaussian kernel) from the datasets in 2016

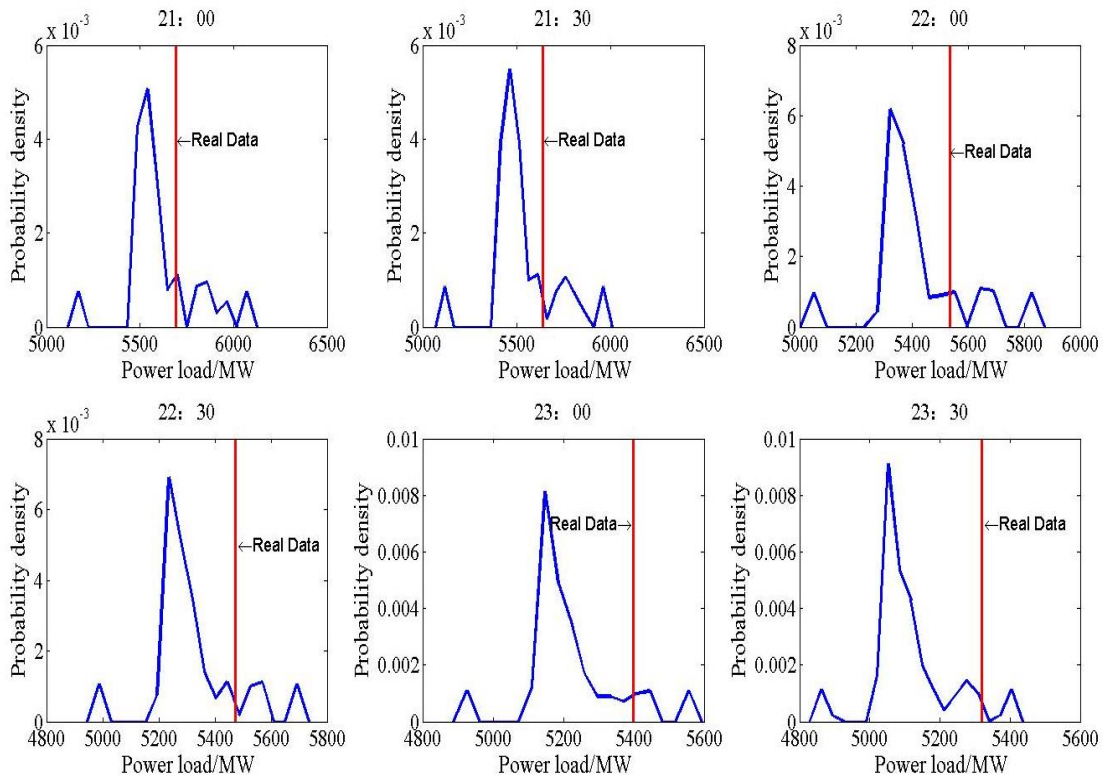
		For the case of Singapore in 2016.7			
		MAPE(%)	MAE(MW)	PICP(%)	PINAW(%)
Without	considering	1.84	102.40	94.79	37.04
real-time power price					
Considering	real-time	1.80	99.83	95.21	36.98
power price					

22



1

2 Fig. 8 Prediction results and prediction intervals based on Gaussian kernel SVQR for  
 3 the case of Singapore from July 21, 2016 to July 30, 2016.



4

5 Fig. 9 Diagram of probability density curve based on real-time price and Gaussian  
 6 kernel SVQR on July 30, 2016 at the last 6 half-hours.

#### 1 **4.4. Observations for the experiments**

2 Base on the above results in the experiment, several observations can be made: (1)  
3 The KSVQR probability density forecasting method not only shows better  
4 performance than traditional point prediction methods, but also provides much useful  
5 information, which is the probability distribution of forecasting results. (2) Gaussian  
6 kernel SVQR is more effective than Polynomial SVQR and linear SVQR. It can  
7 obtain nearly 100% PICP when the real-time price is considered and the forecasting  
8 period is less than 4 days. (3) The real-time price can improve PICP, PINAW, and the  
9 forecasting accuracy of all models adopted. However, the improvement may be  
10 reduced when the training data are decreased and the forecasting period is increased.  
11 (4) The smoothness of obtained probability density curves will reduce when the test  
12 data are increased

### 13 **5 Conclusion and future work**

14 STLF is a fundamental and vital task for operation and controlling of smart grids.  
15 The random factors and the penetration of the renewable energies greatly increase the  
16 uncertainty of power systems and calculation errors. In order to achieve better  
17 prediction results, the paper proposes a short-term power load probability density  
18 forecasting method using KSVQR and Copula theory. Since kernel function can  
19 effectively approximate any function, the selection of appropriate kernel function is  
20 important to the learning target of forecasting model. Therefore, this paper adopts  
21 KSVQR method to compares three different kinds of kernel functions and selects the  
22 optimal kernel function for the learning target. Copula theory is introduced to provide  
23 the correlation analysis between real-time price and power load. As to the probability  
24 density forecasting method, the forecasting results under the different quantiles are  
25 input into the kernel density estimation function [5]. Hence, we design a multi-stage  
26 scheme to construct probability density functions for measuring the uncertainty of  
27 electricity load. The advantage of our probability density prediction method is the  
28 effectiveness on quantifying the uncertainty, which contributes to the improvement of  
29 the forecasting accuracy of the power load.

30 Based on the analysis of three datasets in Singapore, the three KSVQR methods  
31 have excellent forecasting performance, and Gaussian kernel is shown to be the

1 optimal kernel function. Couple function can measure the nonlinear relationship  
2 between power load and real-time price. The fluctuation of power load and real-time  
3 price affects each other. The prediction accuracy can be improved, the high PICP and  
4 narrow PINAW are acquired when the real-time is considered. However, the quality of  
5 PICP and PINAW is reduced when forecasting sample scale is expanded. The  
6 smoothness of probability density function may also decrease with the increase of  
7 forecasting period.

8 In the future, we will carry out the following research: (1) More information on  
9 probability density function will be considered to improve the forecasting potential of  
10 SVQR. (2) It is difficult to obtain high PICP and narrow PINAW because a higher  
11 PICP may result in a wider PINAW. To improve the reliability of probability density  
12 forecasting methods, the appropriate objective function need to be constructed to  
13 balance the result between PICP and PINAW. (3) The quality of probability density  
14 curve will reduce when the forecasting period and sample scale are expanded. Better  
15 bandwidth estimation methods will be investigated (4) The parameters of SVQR are  
16 very important for probability density forecasting. Several intelligence optimization  
17 algorithms may be applied to tune the parameter of SVQR.

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