Matryoshka Inspired Statistical Surrogate for Turbulent Mixing

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‘Matryoshka’ inspired statistical surrogate for turbulent mixing

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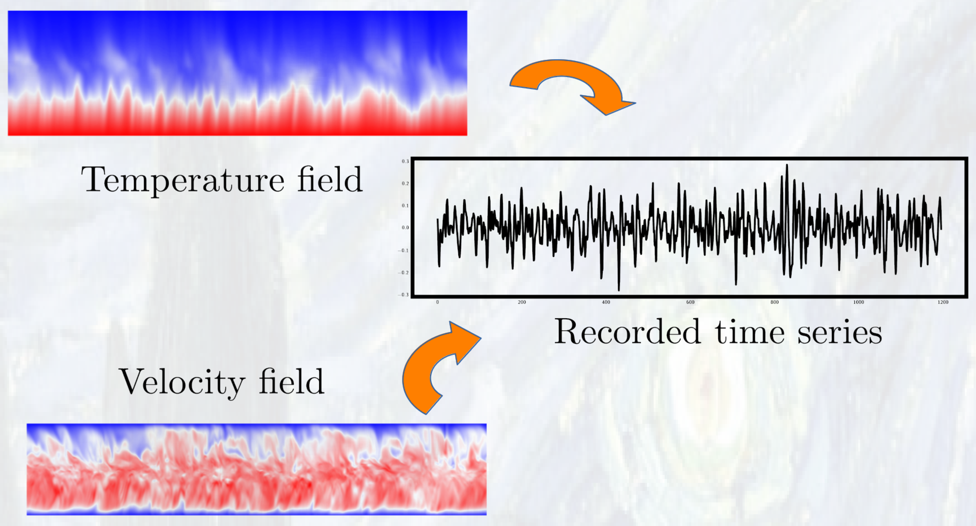
Scalar transport in turbulent flows

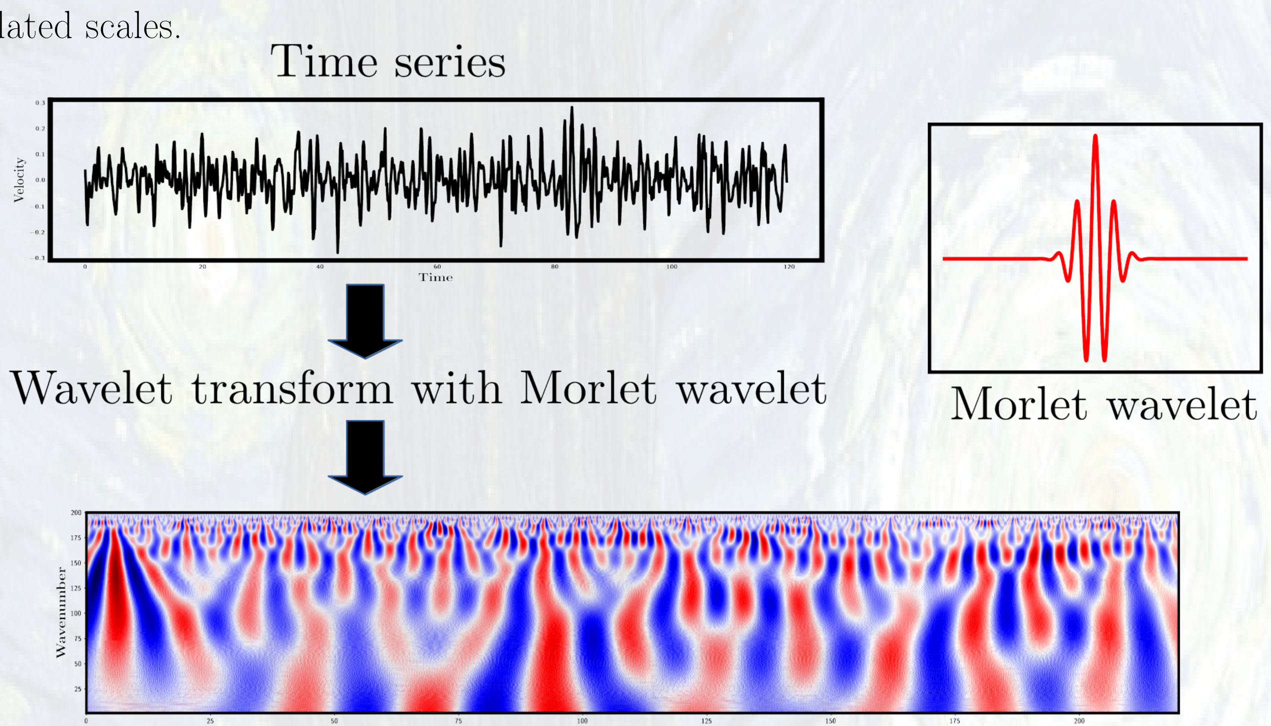
- Turbulent mixing has serious implications on reactor safety, particularly due to complex fluid dynamics in reactor plena and containments.
- Difficult to capture scalar turbulence properly without Direct Numerical Simulations (DNS) of Navier-Stokes equations or Experiments.
- Statistical learning–Lagrangian description of turbulent mixing–*passive scalar getting advected by fluctuating velocity.*



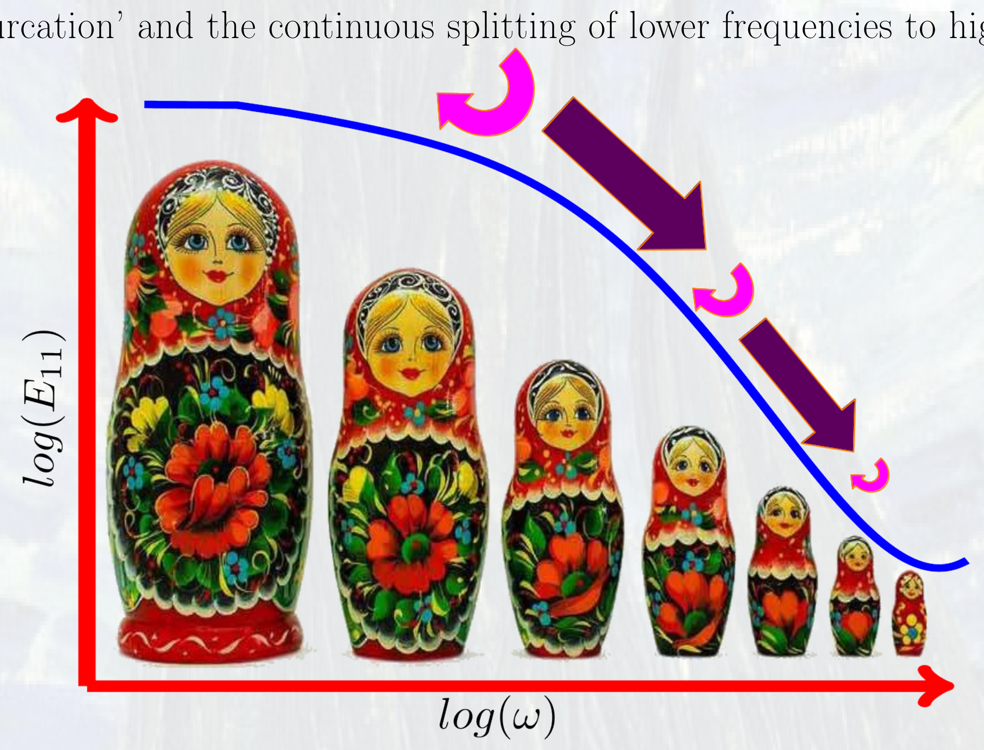
DNS of liquid metal flow in a channel with hot wall at the bottom (Paul Scherrer Institute, SUI)

Energy cascading

A system, for example turbulent flow between two plates have multiple ‘hidden’ and differentially correlated scales.



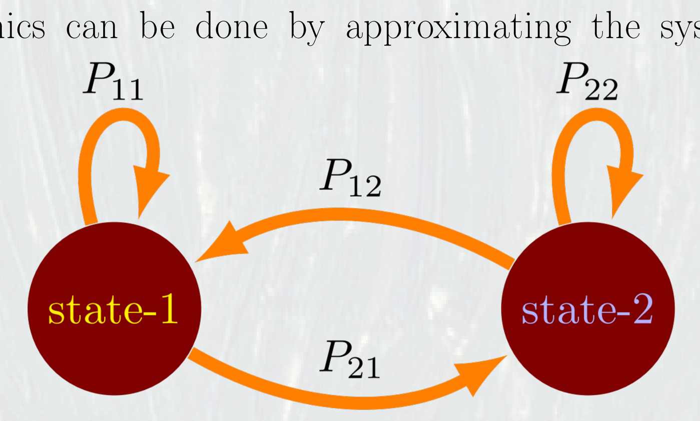
This fact was surmised by Lewis Fry Richardson in an elegantly written poem: *“Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity”.* The interaction of these scales are highly localized in time. This can be appreciated by performing a ‘wavelet’ transform on the turbulent signal as presented in the figure above. This beautifully depicts the presence of ‘pitchfork bifurcation’ and the continuous splitting of lower frequencies to higher frequencies.



An analogy with the Russian ‘Matroyshka dolls’ i.e. ‘doll in a doll’ can be drawn which portrays the ‘Richardson’s’ cascade by a multilayer structure. Here, in this picture the scales are arranged in a multi-layered ‘Matryoshka’. They interact with each other in a highly localized manner in time. This non-trivial interaction together with the fine scale structures makes the computational process expensive. Hence, coarse grained statistical models are the indispensable tools.

Statistical analysis

Coarse graining of dynamics can be done by approximating the system via a ‘Markov’ process.



Markov process

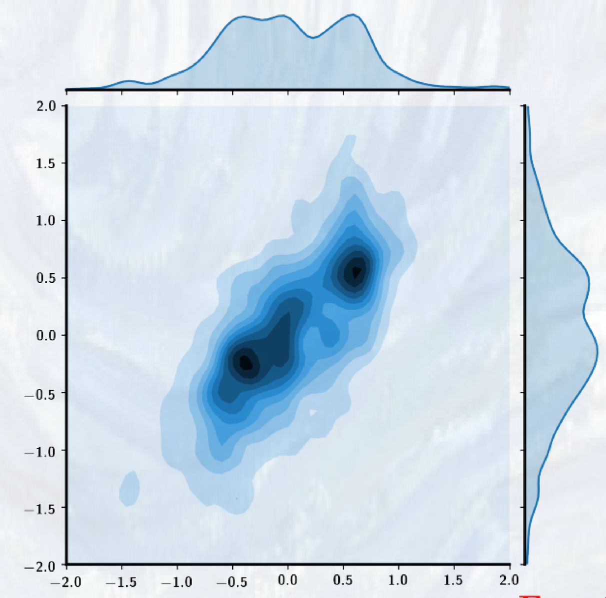
- If the jump from one time scale to another is bigger than a certain scale then the process is ‘Markovian’ or ‘Memoryless’.
- By removing the length/time scale corresponding to memory it is possible to approximate the process via a ‘Markov’ model.
- Renner et al. (2001) showed that this length scale corresponds to the Taylor microscale.
- For jumps greater than ‘Taylor’ microscale the condition of ‘Markovianity’ will remain satisfied.

A simple Markov model

‘Markovian’ dynamics can be described by an Ito semi martingale process.

$$\underbrace{dx = \mu dt + \sigma dW}_{Ito-SDE} \qquad \underbrace{\frac{\partial P}{\partial t} = \mu\frac{\partial P}{\partial x} + D\frac{\partial^2 P}{\partial x^2}}_{Fokker-Planck-equation}$$

An Ito semi-martingale comes equipped with a Fokker-Planck equation for the probability current which can be interpreted in a variety of ways. Following the analysis of Friedrich & Peinke (1997) it is possible to compute the parameters of Ito equation. These parameters accounts for the drifting and diffusive behavior of the probability current.



$$\underbrace{M_n(x,t,\Delta t) = \lim_{\Delta t\to 0}\frac{1}{n!\Delta t}\int_{-\infty}^{\infty}(x'-x)^n P(x',t+\Delta t|x,t)dx}_{Moment-equation}$$

$$\underbrace{M_1 = \mu, M_2 = D}_{First-two-moments}$$

“First two moments computed directly from the DNS data”

$$M_1 = \mu = ax^3 + bx^2 + cx + d$$

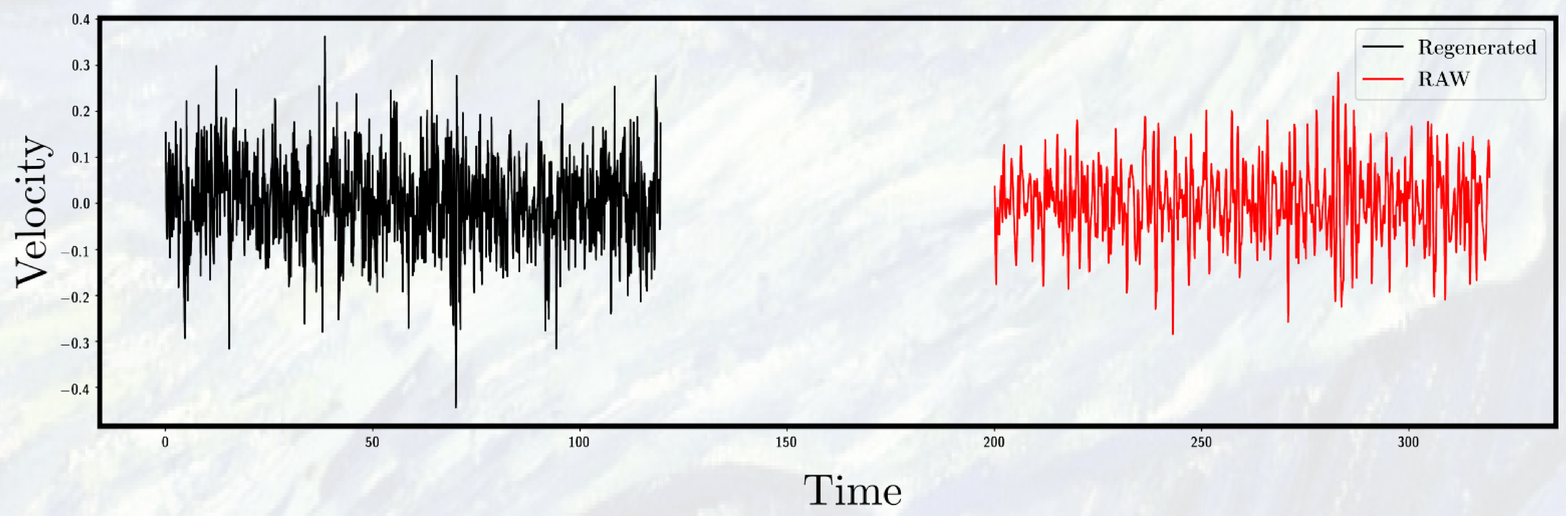
$$M_2 = D = ex^2 + fx + g$$

Functional forms of first two moments–for data driven surrogate

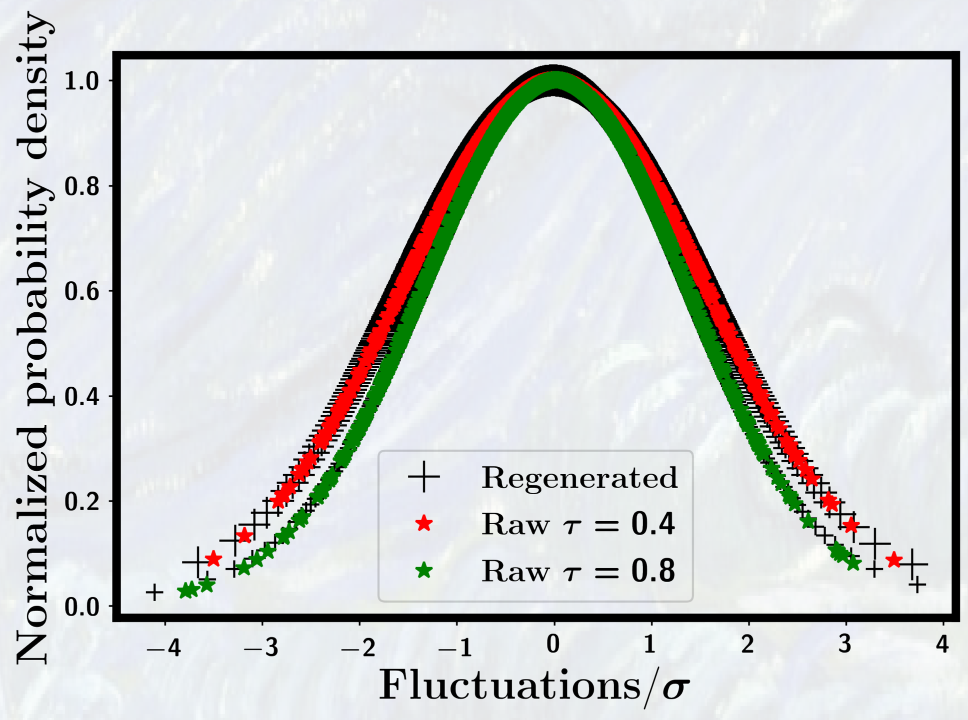
Joint probability-a vertical cut in it reveals $P(x', t+\Delta t|x,t)$-the propagator of Markovian states

Statistical simulation results

The disentangling of the deterministic and stochastic trends quantify an understandable picture of the recorded time series. Carefully separating the time scales bigger than the Taylor microscale it is possible to estimate the process via a simple ‘Markov’ model.



In red is the DNS data shifted in time from regenarated data for clarity



Computed pdf from raw and regenerated data

The model can be treated as a data driven surrogate of the original system. Comparison of ‘pdfs’ show that lower time scales correspond to fatter tails thus closer to capturing the smaller scale fluctuations. Evaluating $x(t+\tau) - x(t)$ from the trained time series will reveal higher and higher scales or a ‘Matryoshka’ like setup. The trained model can be coupled to the passive scalar tracking Lagrangian equation for turbulent mixing studies. To fully resolve the scales of turbulence the model should be supplemented by an appropriate *‘memory kernel’.*

Acknowledgment

We would like to thank Dr. Bojan Ničeno and Wentao Guo of Paul Scherrer Institute, Switzerland for providing us the DNS data of liquid metal channel flow.

References

Friedrich, R. & Peinke, J. (1997), ‘Description of a turbulent cascade by a fokker-planck equation’, *Physical Review Letters* **78**(5), 863.

Renner, C., Peinke, J. & Friedrich, R. (2001), ‘Experimental indications for markov properties of small-scale turbulence’, *Journal of Fluid Mechanics* **433**, 383–409.

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Background: Starry Night by Vincent Van Gogh.
Multi-scale picture of luminance: statistically accurate depiction of turbulence in distant stars.
More can be learned from: `https://arxiv.org/pdf/physics/0606246.pdf`