COMPARING FUNCTIONAL DATA ANALYSIS AND HYSTERESIS LOOPS WHEN TESTING TREATMENTS FOR REDUCING HEAT STRESS IN DAIRY COWS

S. Maynes

A. M. Parkhurst

J. B. Gaughan

T. L. Mader

See next page for additional authors

Author Information

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S. Maynes[1], A. M. Parkhurst[1], J. B. Gaughan[2], T. L. Mader[3]
[1]Department of Statistics, University of Nebraska – Lincoln
[2]School of Animal Studies, The University of Queensland, Gatton, Australia
[3]Department of Animal Science, University of Nebraska – Lincoln

Abstract

Various techniques are commonly used to reduce heat stress, including sprayers and misters, shading, and changes in feed. Oftentimes studies are performed where researchers do not control the times when animals use shading or other means available to reduce heat stress, making it hard to test differences between treatments. Two methods are used on data from a study where Holstein cows were given free access to weight activated "cow showers." Functional data analysis can be used to model body temperature as a function of time and environmental variables such as the Heat Load Index. Differences between treatment groups can be tested using a Functional Bayesian MCMC model. Alternatively hysteresis loops, such as the ellipse, formed by a plot of air temperature or the Heat Load Index against body temperature over the course of a day can be estimated and their parameters used to test differences between cows with access to showers and cows without. Results from an R package **hysteresis**, which can estimate these loops and their parameters are illustrated. Functional data analysis allows for looser assumptions regarding the body temperature curve and the ability to look for differences between groups at specific time points, while hysteresis loops give the ability to look at heat stress over the course of a day holistically in terms of parameters such as amplitude, lag, internal heat load and central values.

Key words: Thermo-regulatory response, Heat Stress, Energy dissipation, Farm animals.

1. Introduction

Average yearly monetary losses due to heat stress in dairy cattle have been estimated at $897 million in the US alone (St-Pierre et al., 2003). Academic research on this topic is fairly intensive and includes work on the efficacy of using genetics (Howard et al., 2013), diet (Mader et al., 2002), shading (Brown-Brandl et al., 2005), air conditioning, fans, sprinklers, misters (Hillman et al., 2005), or even weight activated cow showers (Legrand et al., 2011, Maynes and Parkhurst, 2012) along with other techniques for reducing heat stress. A Google scholar search for the term "heat stress cattle" gives about 110,000 results. Generally, either body temperature (*Tb*) or panting/breathing rate is used to estimate heat stress (Gaughan et al., 2008). Internal *Tb* offers a more sensitive measure of heat stress, but it is harder to obtain and model due to the need for internal *Tb* loggers and the presence of hysteresis, which is the dependence of *Tb* on both the animal's past and current environment. Often the methods used to model *Tb* are suboptimal in that they fail to fully account for this hysteresis, or for other equally important parts of the model such as the environmental heat load and random variation due to cow. Of the first ten results in Google scholar for the term "heat stress cattle body temperature logger" for papers that look at internal *Tb* (Mitlöhner et al., 2001, Mader et al., 2002, Davis et al., 2003, Brown-Brandl et al., 2005, Hillman et al., 2005, Beatty et al., 2006, Gaughan et al., 2008, Tucker et al., 2008, Dikmen and Hansen, 2009, Schütz et al., 2009), only one paper attempts to model *Tb* as a time dependent process without having to resort to using hour as a categorical variable (Gaughan et al., 2008).

Two possible methods for modeling *Tb* explored in this paper are to use either functional data analysis or a sinusoidal hysteretic model. Functional data analysis (FDA) allows for the use of b-spline or Fourier basis functions that can model body temperature more accurately, but the sinusoidal hysteretic model provides holistic measures of heat stress over the course of a day that may be more informative. A study that gave Holstein cattle unlimited access to weight activated “cow showers” will be used to illustrate the ability of these two techniques to effectively measure the differences between treatment and control cows.

2. Experimental Design

The study (Legrand A. et al., 2011) used 24 Holstein cows. Twelve cows had unlimited access to weight activated “cow showers” that could be used at any time of day, while 12 control cows were not given access to showers. Four cows were tested at a time in four separate pens, two of which were outfitted with cow showers. Six trials of 5 consecutive days were held over the course of the summer. For each trial, 2 cows had access to showers; 2 did not. Each of the 4 pens had a shaded area and an unshaded area; water troughs for all 4 pens were shaded while showers were unshaded.

Table 1. Pen Design for a Single Trial. The water trough was inside the barn while showers and the feed bunk were located outside. (Legrand et al., 2011, Maynes and Parkhurst 2012)

South-most Pen 1	Pen 2	Pen 3	North-most Pen 4
Barn			
Water Trough	Shared Water Trough		Water Trough
Outside Area			
Control	Shower	Control	Shower
Feed Bunk			

Internal *Tb* for each cow was measured every 5 minutes with a temperature logger inserted into the vaginal cavity. Environmental measures such as air temperature (*Ta)*, black globe temperature, humidity, wind speed and wind direction were measured every 5 to 10 minutes. The environmental data was used to compute two separate heat indices; the Temperature-Humidity Index (*THI*) and the Heat Load Index (*HLI*) reported in Igono, M. et al. (1992),and Gaughan, J.B. et al. (2008) respectively.

Table 2. Components and Formulas for Heat Indices

Heat Index	Components	Formula
Temperature-Humidity Index (THI)	T_a, Relative Humidity(RH)	$(1.8 \times T_a + 32) – [(0.55 – 0.0055 \times RH) \times (1.8 \times T_a – 26)]$
Heat Load Index (HLI)	Black Globe Temperature (BGT), Wind Speed(WS), Relative Humidity(RH)	IF BGT >25, $8.62 + (0.38 \times RH)+ (1.55 \times BGT) + \exp(-WS + 2.4) – 0.5 \times WS$ Else, $10.66 + (0.28 \times RH) + (1.3 \times BGT) – WS]$

3. Models

The *Tb* data is sorted by cow(24) and day(5) into 120 separate cow-day curves and standardized for both the FDA and elliptical hysteresis models. The FDA model is then fit using b-splines of order 6 with 14 knots in the R package FDA (Ramsay et al., 2013). A roughness penalty of 0.5 is placed on the second derivative to avoid overfitting. A functional Bayesian MCMC model, as described by Crainiceanu and Goldsmith (2010), is then used to model these b-spline curves in a way that accounts for random effects due to cow and day. Before a functional Bayesian MCMC model can be fit however, the b-spline curves must first be split into their functional principal components, which are conceptually similar to traditional principal components. The eigenfunctions, similar to eigenvectors, are the series of orthogonal curves that explain the largest portion of the variation among Tb curves. Once these are obtained, the functional Bayesian MCMC model fits the curve

$$y_i(t) = \mu(t) + \sum_{k=1}^{K} L_k(t) f_{i,k} + e_i(t) \qquad \textbf{Eq. 1}$$

where $y_i(t)$ is *Tb* for cow*day i at time t, $\mu(t)$ is the mean *Tb* curve, $e_i(t)$ is a normally distributed residual variation term. The $L_k(t)$ are eigenfunctions, where k marks the eigenfunction in question, obtained from separating the *Tb* b-spline curves into their functional principal components, and the $f_{i,k}$ are random coefficients on the eigenfunctions given a T distribution of the form

$$f_{i,k} \sim \mathrm{T}(\bar{f}_{i,k}, \sigma_k, df_k) \qquad \textbf{Eq. 2}$$

where $\bar{f}_{i,k}$ is the expected value of $f_{i,k}$, σ_k is the standard deviation for eigenfunction k, and df_k is the degree of freedom parameter for eigenfunction k. The T distribution is used instead of the normal distribution as certain curves are clear outliers which would not be accounted for by the normal distribution. The $\bar{f}_{i,k}$ are calculated as

$$\bar{f}_{i,k} = \tau_k + \tau_{H,k} * H(1)_i + \sum_{j=1}^{3} \left(H(j)_i * b_{j,k}\right) + c_{i,k} + d_{i,k} \qquad \textbf{Eq. 3}$$

where τ_k is the shower access effect, the $H(j)_i$ are functional principal component scores on the Heat Load Index (*HLI*), $\tau_{H,k}$ is the interaction effect between the first HLI principal component and shower access, the $b_{j,k}$ are coefficients on the *HLI* functional principal components, $c_{i,k}$ is the normally distributed random effect on cow and $d_{i,k}$ is the normally distributed random effect on day. This model is fit with the R package rstan (Stan: A C++ Library for Probability and Sampling, Version 1.3, 2013). The following priors are used

Table 3. Priors for FDA Bayesian MCMC Model (Eq 1- 3).

Parameter	Prior
$e_i(t)$ standard deviation	$\Gamma(1,2)$
$c_{i,k}$ standard deviation	$\Gamma(1,2)$
$d_{i,k}$ standard deviation	$\Gamma(1,2)$
σ_k	$\Gamma(1,2)$
τ_k	N(0,1)
$\tau_{H,k}$	N(0,1)
$b_{j,k}$	N(0,1)
df_k	Unif(1,100)

The sinusoidal hysteretic model is fit using the R package hysteresis, developed by (Maynes et al., 2013). The input variable *Ta* and the output variable *Tb* together form an ellipse from which three parameters calculated by the hysteresis package, cy, ampy, and lag, are calculated. These parameters represent the mean value of *Tb*, the amplitude of *Tb* and the delay between *Ta* and *Tb*. The ellipses are estimated using the 'harmonic2' method and circular block bootstrapping is used to account for residual autocorrelation (Politis and Romano, 1991) and to obtain standard errors for derived parameters such as ampy and lag (Politis and Romano, 1991, Yang and Parkhurst, 2011). A multivariate Bayesian MCMC model is then used to measure the effect of allowing shower access on these parameters, and their responsiveness to the *HLI*. The Bayesian MCMC model is used instead of a traditional multivariate linear model as it is better able to handle the information about measurement error obtained when estimating the ellipse. This model is

$$Y_{ij} = N(\bar{Y}_{i,j}, \Sigma_{i,j})$$

$$\bar{Y}_{i,j} = \mu + \tau_i + hH_j + \tau h_i H_j + c_i + d_j \qquad \textbf{Eq. 5}$$

$$\Sigma_{i,j} = \mathrm{diag}(B * \bar{\sigma}_{i,j} + A)\ \Gamma\ \mathrm{diag}(B * \bar{\sigma}_{i,j} + A) \qquad \textbf{Eq. 6}$$

where Y_{ij} is the vector of cy, ampy, and lag for every cow i and day j, $\bar{Y}_{i,j}$ is the expected value of Y_{ij} and $\Sigma_{i,j}$ is the variance matrix. The vector $\bar{Y}_{i,j}$ is based on the vector of means μ, the treatment effects τ_i, the *HLI* coefficients h, the mean *HLI* over the course of the day H_j, the treatment *HLI* interactions τh_i, and the multivariate normal distributed cow and day random effects c_i and d_j. The variance matrix $\Sigma_{i,j}$ is based on the vector of parameter measurement standard deviations based on bootstrapping $\bar{\sigma}_{i,j}$, its coefficient vector B, and the vector of model based standard deviations A. Priors are given in Table 4.

Table 4. Priors for Elliptical Hysteresis Model, (Eq 4-6)

Parameter	Prior
μ	N(0,100)
τ_i	N(0,10)
h	N(0,10)
τh_i	N(0,10)
c_i	MVN(0,σ_c,ρ_c)
d_j	MVN(0,σ_d,ρ_d)
σ_c	Unif(0,100)
σ_d	Unif(0,100)
ρ_c	A 3x3 matrix of β(2,2)*2-1 correlation coefficients and a diagonal of ones.
ρ_d	A 3x3 matrix of β(2,2)*2-1 correlation coefficients and a diagonal of ones.
B	Inverse Γ(3,3)
A	Unif(0,100)
Γ	A 3x3 matrix of β(2,2)*2-1 correlation coefficients and a diagonal of ones.

This data has been studied before using elliptical hysteresis in Maynes and Parkhurst (2012). The major improvements in this paper are in the use of circular block bootstrapping and the substitution of a multivariate model regressing on ampy, cy, and lag instead of one regressing only on the area of the interior of the ellipse. This multivariate model explains more about how cows experience heat stress over the course of a day.

4. Results

The Tb data was standardized using the mean *Tb* for all 120 cows (38.9^o C) with a standard deviation of 0.40^o C among all observations. Figure 1 shows the b-spline curves fit to *Tb* for all 120 day by cow combinations with mean curves for cows with and without shower access superimposed. Control cows are distinguished by having a higher maximum around 8 p.m. and lower minimum around 8 a.m., and below we will show that both of these differences are statistically significant. A number of curves appear to be outliers, necessitating the use of the T distribution to describe variation between curves. The two red outliers belong to the control group while the two purple outliers belong to the shower group.

FIgure 1. Single Day B-splines for *Tb (n=120),* with Control and Shower means superimposed. *Tb* is standardized to have a mean of 0 and a standard deviation of 1. Non standardized mean is 38.9° C, standard deviation is 0.4° C.

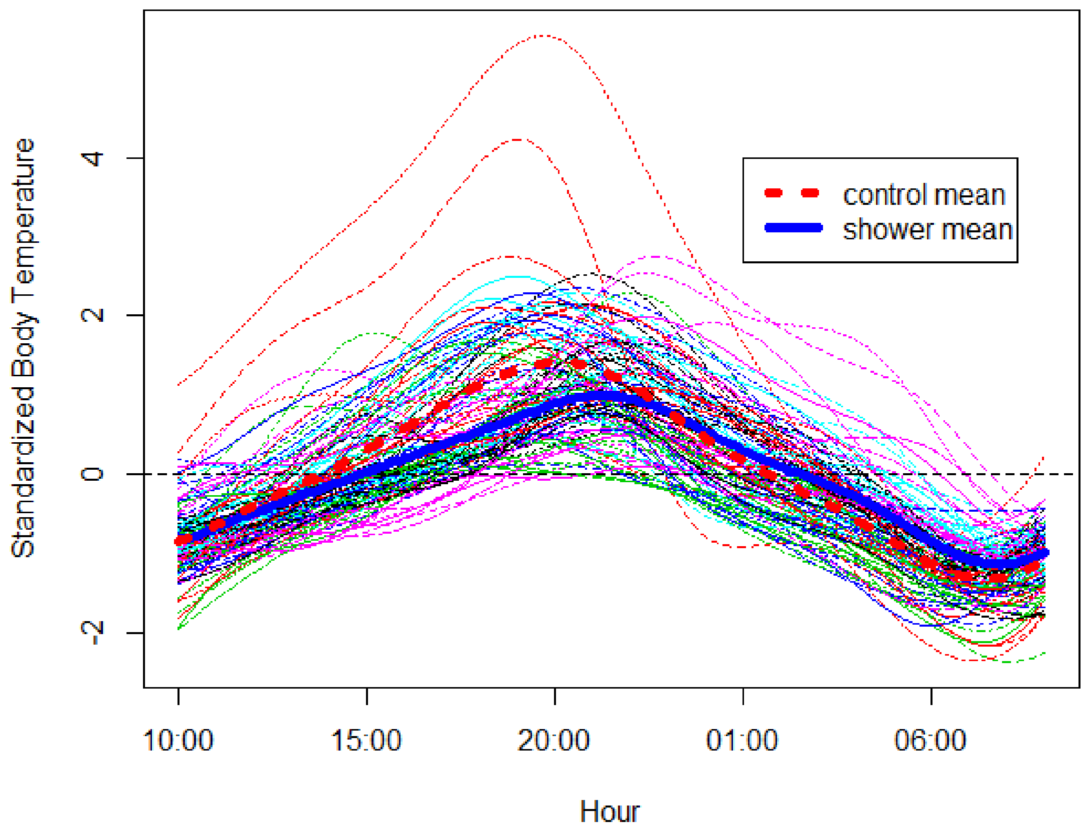
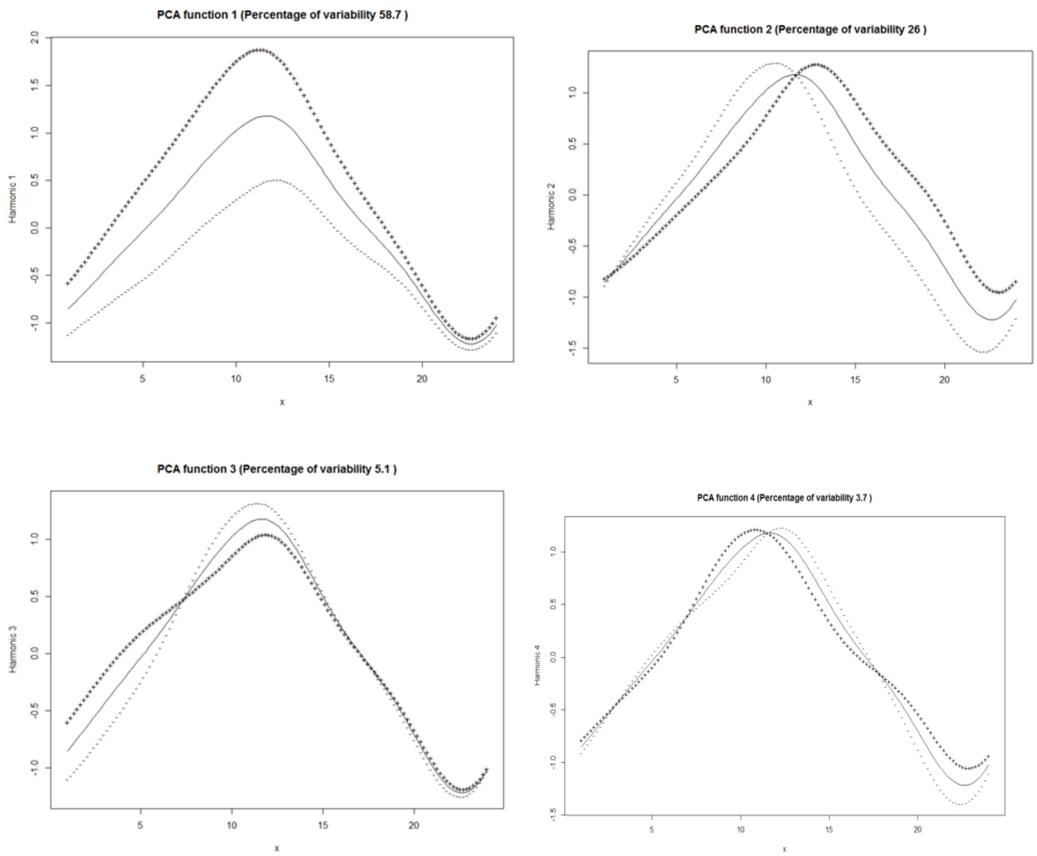


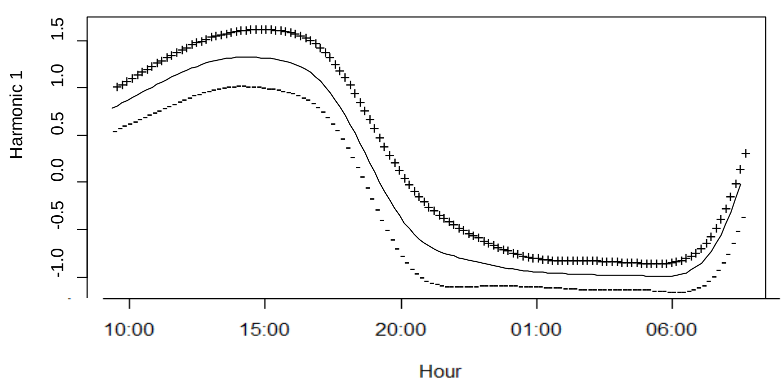
Figure 2 shows the first 4 functional principal components. They explain 94% of the variation in these curves. The first principal component, which explains 59% of the variation in the curves, can be described as the magnitude of *Tb* experienced by a cow, while the second principal component can be characterized as lag since the positive harmonic follows behind the mean curve, and the third as amplitude because the positive harmonic has a higher maximum and lower minimum than the mean curve. It is difficult to produce a description for the quadratic functional principal component.

Figure 2. First 4 Principal Components for *Tb*. Center line is mean *Tb* curve, the darker dotted line is one harmonic above the mean and the lighter dotted line is one harmonic below the mean. Principal components cross the mean line k-1 times, where k is the number of the principal component.



Additionaly the *HLI* curves are also split into their principal components, and the first 3 which explain 92% of the variation in *HLI* are used in the FDA model. The first *HLI* functional principal component, which is shown in figure 3 and used in the *HLI* treatment interaction, (eq. 3) is quite similar to the first *Tb* functional principal component and explains 69% of the variation in the *HLI*.

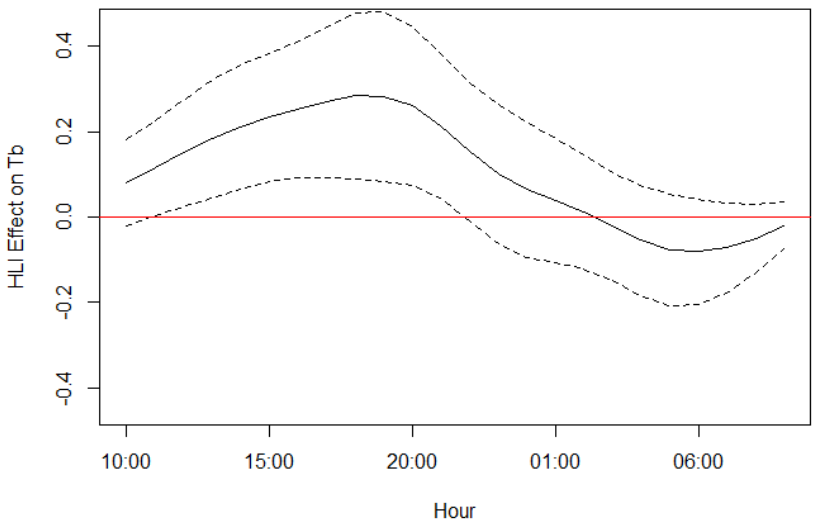
Figure 3. First *HLI* principal component. Center line is mean *Tb* curve, the darker dotted line is one harmonic above the mean and the lighter dotted line is one harmonic below the mean.



The effect of this first *HLI* principal component on *Tb* (see Eq. 1,2,3) is shown in Figure 4 below with a 95% credible interval at each time point. Higher *HLI* leads to greater *Tb* between the hours of 2 p.m. and 10 p.m. (0 time on the plot is 10 a.m.) but does not lead to increased *Tb* elsewhere. The maximum at this time is approximately 0.2 *Tb* standard deviations. As one standard deviation in *Tb* is 0.4^{o} C, this is a 0.08^{o}

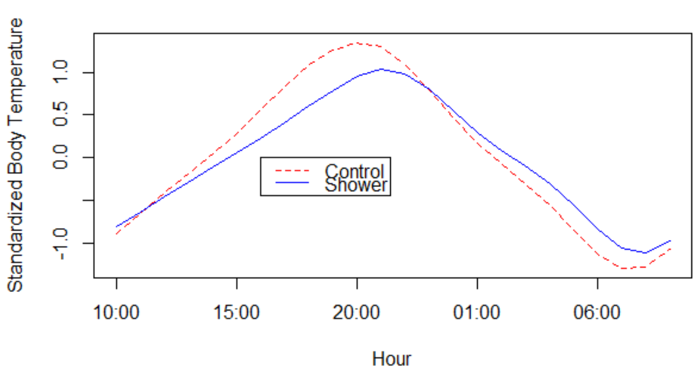
C increase. However, *HLI* varied little from day to day over the course of this experiment, so this small effect size is more a reflection of the size of the *HLI* principal component standard deviation than of the strength of the *HLI/Tb* relationship. Average daily *HLI* had a standard deviation of 5.5 over the course of the experiment, which is small in comparison to the within day mean standard deviation of 19.5.

Figure 4. Effect of 1 Standard Deviation Increase in HLI PC1 on *Tb* Over the Course of a Day with 95% Credible Interval



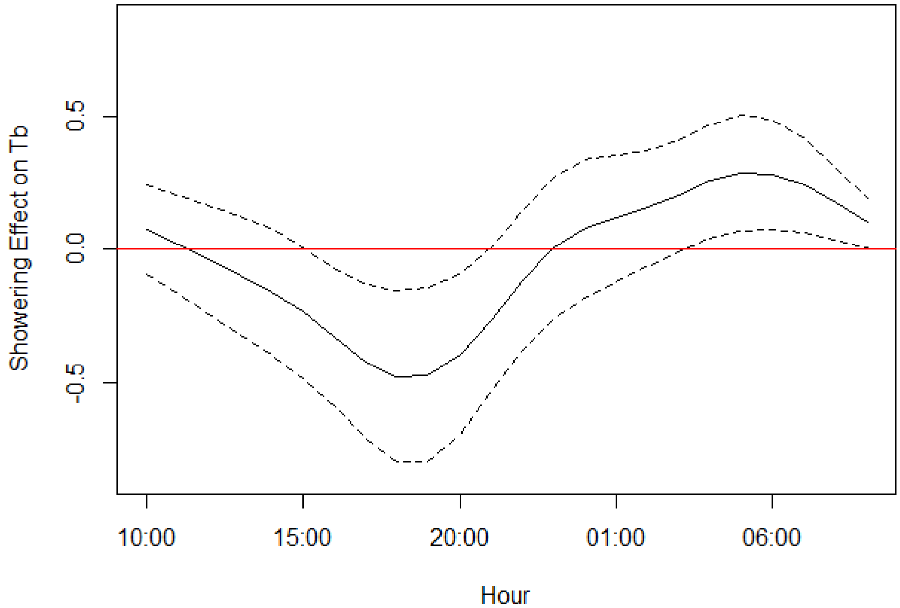
Expected control and shower curves (Eq. 1,2,3) at the mean level of the first *HLI* functional principal component are shown in Figure 5. These look similar to the original mean curves in Figure 1. Cows with access to a shower exhibit lower *Tb* between the hours of 4-9 p.m. and higher *Tb* from 5-10 a.m. The decrease in *Tb* when it is at its highest level is about 0.5 s.d. or 0.2^o C while the increase when it is at its lowest level is about 0.25 s.d. or 0.1^o C. Overall the mean curve for cows with access to a shower appears to be less symmetric than that for control cows, as cows with access to a shower are slower reaching their maximum.

Figure 5. Expected *Tb* Curves for Control and Shower Cows at Mean HLI.



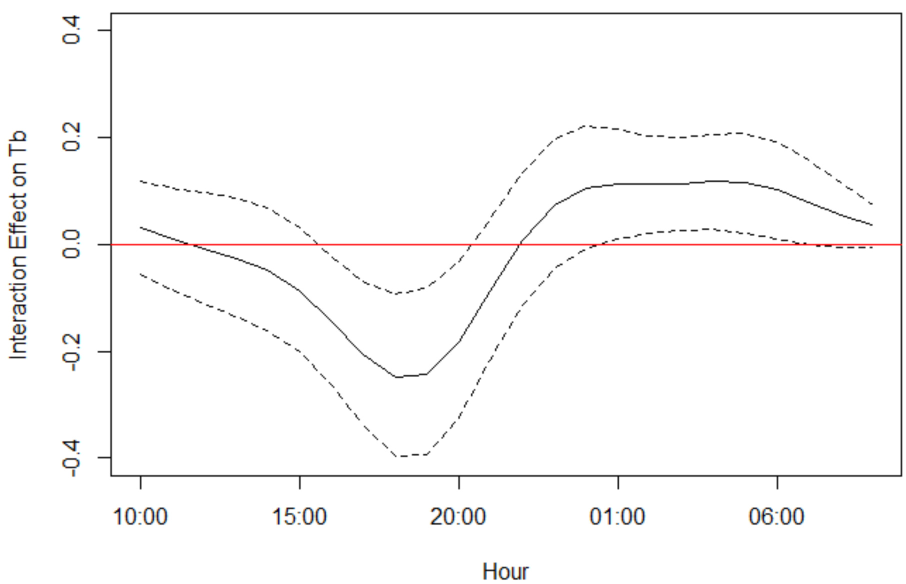
The effect of allowing shower access on *Tb* over the course of the day, which is the difference between the two lines in Figure 5, is shown in Figure 6 along with a 95% credible interval at each point in time. Allowing access to a shower leads to lower *Tb* around 8 p.m. and higher *Tb* around 6 a.m., with probability higher than 97.5% at both times as given by the 95% credible intervals in Figure 6.

Figure 6. Effect of Showering on *Tb* with 95% Credible Intervals.



The shower access *HLI* interaction effect shown in Figure 7 is negative between the hours of 4-9 p.m. and positive between the hours of 2-7 a.m. The maximum decrease in *Tb* is approximately 0.08^{o} C and the maximum increase is approximately 0.04^{o} C.

Figure 7. Interaction of Shower Access and *HLI* on *Tb* with 95% Credible Intervals.



The table below provides mean posterior estimates on random effect parameters. Some important things to note are that while the second and third *Tb* functional principal components appear to be normally distributed with high values for the degrees of freedom parameter, the first and fourth principal components seem to have an almost Cauchy distribution, which shows the importance of using the T distribution to represent the observational variation in these principal components. Additionally most of the variation in the first and second principal components is between cows while most of the variation in the third and fourth principal components is between days. The posterior mean for the standard deviation of the residual is 0.25.

Table 5. Posterior Estimates on *Tb* Functional Principal Components, k=1-4. See Eq. 1,2,3.

Parameter	1st FPC Estimate	2nd FPC Estimate	3rd FPC Estimate	4th FPC Estimate
df_k	3.5	28	48	1.6
σ_k	0.61	0.45	0.30	0.09
$c_{i,k}$ standard deviation	1.1	0.80	0.37	0.26
$d_{i,k}$ standard deviation	0.50	0.50	0.50	0.50

In addition two additional models were tried by replacing *HLI* in the model with either *Ta* or *THI*. The first 3 functional principal components explain 96-97% of the variation for the *Ta* and *THI* indices, so by replacing *HLI* in the model with *THI* or *Ta* a comparison can be made between the effectiveness of these indices in predicting heat stress. Both *THI* and *Ta* are far less effective in predicting *Tb* than is *HLI*. Table 6 below shows a measure of fit, -2*log(probability) for all of these models, which is equivalent to the AIC or BIC without a degrees of freedom adjustment, as all three models have the same number of parameters. The model using *HLI* performs far better than the others, and this difference is not only statistically significant when looking at numerical measures of overall fit but can also be seen in Figure 8 with the credible interval for the effect of the first *HLI* functional principal component smaller in comparison to those for *Ta* and *THI*.

Table 6. Fit for FDA Models with Various Heat Indices.

Heat Index	Measure of Fit = -2*log(Probability)
Air Temperature	-3411
Temperature-Humidity Index (THI)	-3415
Heat Load Index (HLI)	**-5240**

Figure 8. Effect of 1 Standard Deviation Increase in Heat Indices with 95% Credible Intervals. The *HLI* offers far more predictive power.

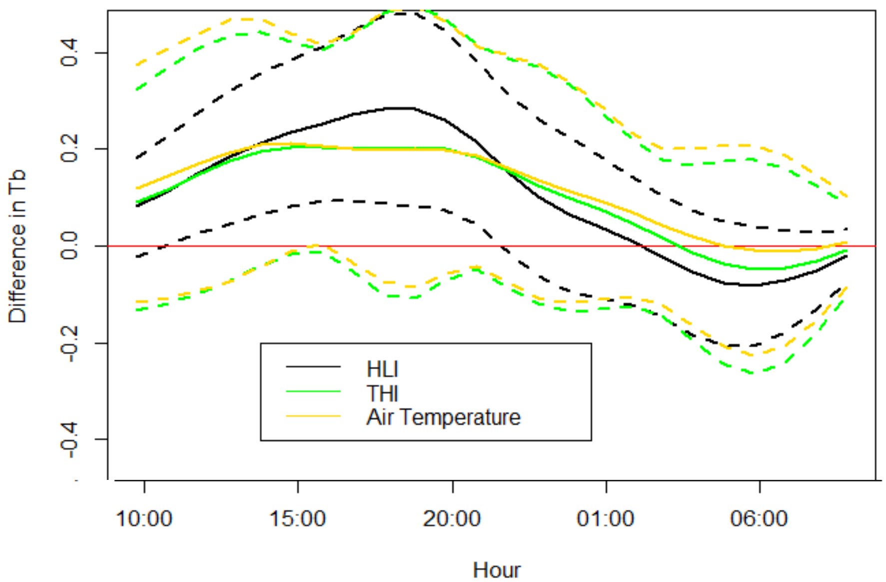
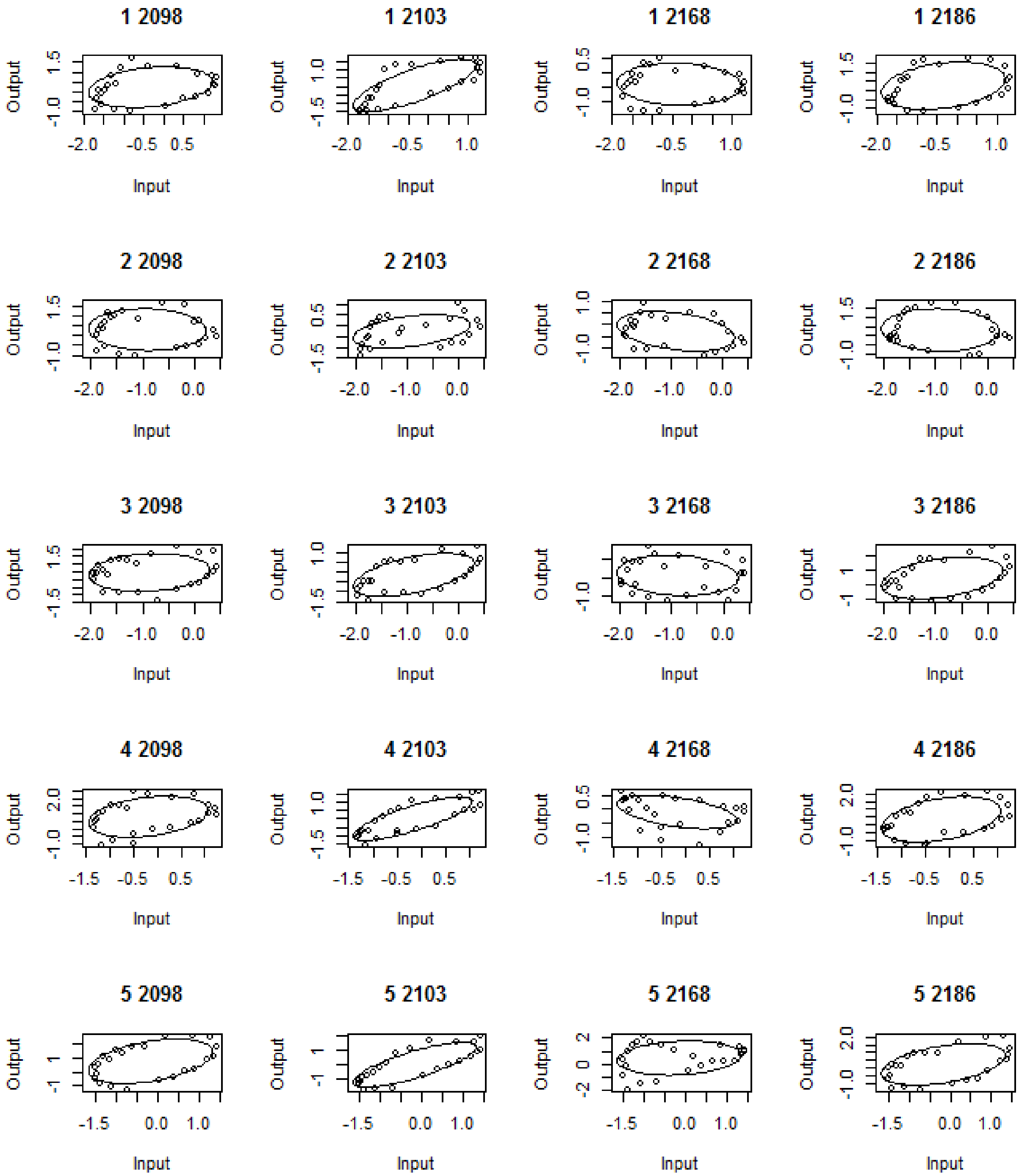


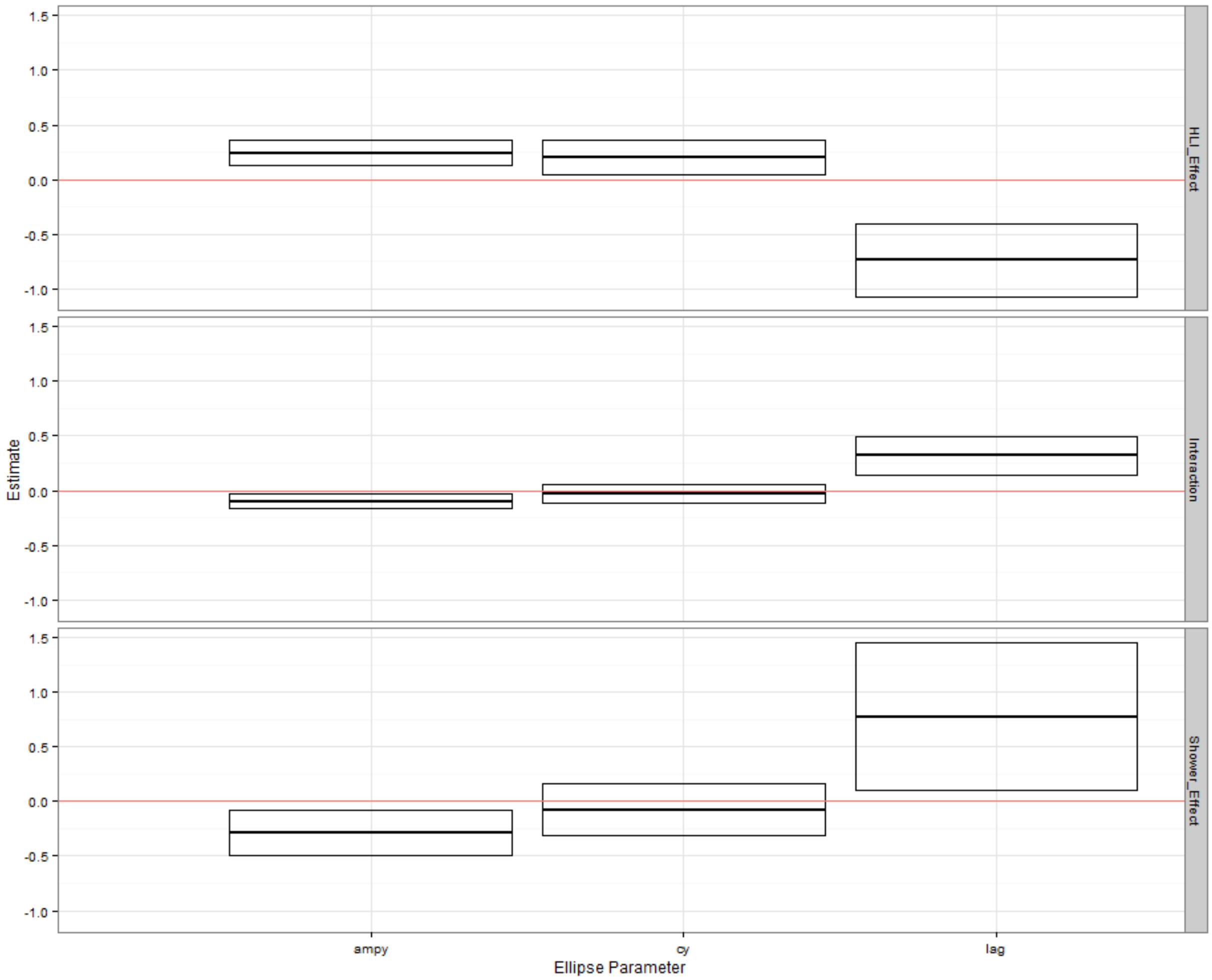
Figure 9 shows the 20 fitted ellipses from the 4th trial. Some days are clearly fit better than others, necessitating the use of bootstrap standard deviations in the Bayesian MCMC model. Despite the fact that the previous analysis showed the superiority of *HLI* in comparison to *Ta* for the prediction of *Tb*, *Ta* is still used as the input for fitting these ellipses as it is more easily described as sinusoidal.

Figure 9. Fitted and Bootstrapped Ellipses for the *Ta/Tb* Relationship from Trial 4. The first number is day while the second is cow.



The model described in equations 4-6 is then fit. Figure 10 below shows posterior means and 90% credible intervals for the effects of the *HLI*, allowing shower access, and the shower *HLI* interaction on the ellipse parameters cy, ampy, and lag.

Figure 10. Posterior Distributions with 90% Credible Intervals for the effects of *HLI*, allowing access to a shower, and the shower *HLI* interaction on Tb mean (cy), Tb amplitude (ampy) and lag.



One standard deviation increase in *HLI* increases the amplitude of the sinusoidal *Tb* curve while also increasing the central value of *Tb*, and decreasing the lag between *Ta an*d *Tb.* Allowing access to a shower at the mean level of *HLI* decreases amplitude while increasing lag, and has no apparent effect on cy. As *HLI* increases the showering effect becomes stronger, as amplitude continues to decrease while the lag continues to increase. Table 7 provides posterior means, standard errors, and the probability that an effect is greater than zero for the model parameters shown in Figure 10.

Table 7. The effects of *HLI*, allowing access to a shower, and the shower *HLI* interaction on cy, ampy and lag. Also includes standard errors and the probability that an effect is greater than zero.

cy

Parameter	Posterior Mean	Standard Error	P(x > 0)
HLI	0.20	0.09	0.98
Interaction	-0.03	0.05	0.28
Shower Access	-0.07	0.14	0.30

ampy

Parameter	Posterior Mean	Standard Error	P(x > 0)
HLI	0.24	0.07	0.9998
Interaction	-0.10	0.04	0.007
Shower Access	-0.29	0.13	0.012

lag

Parameter	Posterior Mean	Standard Error	P(x > 0)
HLI	-0.74	0.21	0.0003
Interaction	0.32	0.11	0.998
Shower Access	0.78	0.42	0.97

Distributions for the standard deviations of random effects on cow, day, and observation can be seen in Figure 11. Most of the variation in ellipse parameters appears to be due to cow, not day or observation. This is not surprising as some of the variation between days was already taken into account with the *HLI* variable.

Figure 11. Posterior Distributions with 90% Credible Errors for the Standard Deviations on the Cow, Day, and Observational normally distributed error terms.

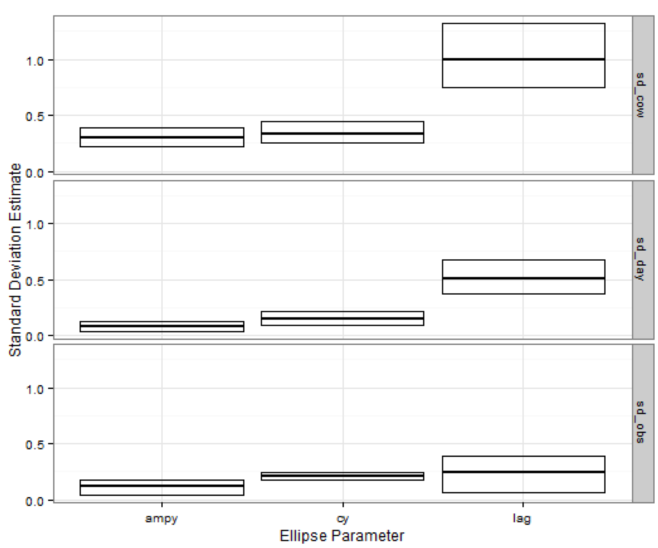
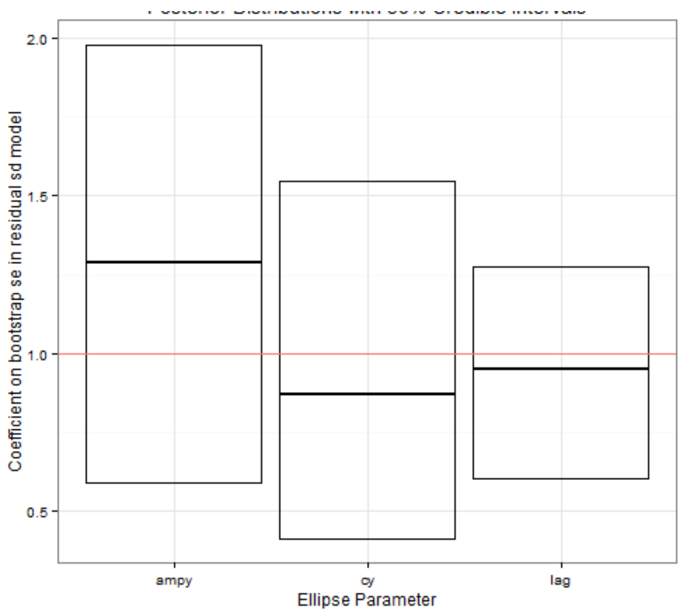


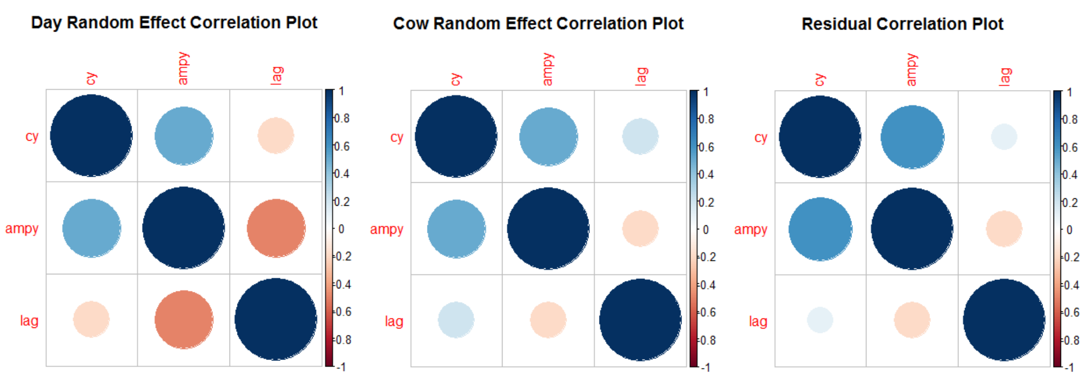
Figure 12 below shows the posterior distributions for the coefficients on bootstrap standard error parameters. These distributions are centered at 1, which provides weak evidence that the bootstrap standard errors correctly account for the ellipse parameter measurement error in the model.

Figure 12. Posterior Distributions with 90% Credible Intervals for the Coefficient on the Bootstrap Standard Error in the Residual Standard Error Model.



The residual correlation plots in Figure 13 suggest that ampy and cy are positively correlated while ampy and lag are negatively correlated. This holds for the random effects on day, the random effects on cow, and the residuals on individual observations.

Figure 13. Residual Correlations Between mean Tb (cy), amplitude (ampy) and lag. The correlations between these parameters are similar for the cow and day random effects along with the observational residuals.



5. Conclusion

Functional data analysis can be used to measure heat stress in animals at specific times of day while hysteresis loop analysis provides estimates of heat stress that summarize changes in body temperature over the course of one day. When applied to the data from Legrand A. et al., 2011, functional data analysis shows that allowing free access to a weight activated cow shower decreases body temperature during the afternoon between 4 and 9 p.m. while increasing body temperature mid-morning between 5 and 8 a.m. at the mean level of the *HLI* in this study. As the *HLI* increases, the difference between cows with and without access to showers also increases in a statistically significant manner in the afternoon and midmorning. In this study, the *HLI* shows a far greater ability to predict changes in body temperature due to the environmental heat load than alternative measures such as *Ta* or the *THI*. The higher body temperature experienced by cows with access to showers, or on days with lower mean *HLI*, in the early morning is similar to results found in other studies such as Lefcourt, 1996. These results indicate it is important to measure heat stress over the course of a full day, and not just during those hours when the heat challenge is strongest, as effects on body temperature many hours later can be significant and counterintuitive.

Hysteresis loops formed by plotting air temperature against body temperature over the course of one day can be used to determine whether the differences between control and shower cows are due to changes in mean body temperature, the amplitude of body temperature or an increase in the time lag between air and body temperatures. There is strong evidence that allowing access to a shower increases the lag between *Ta* and *Tb*, and that it decreases the amplitude or range of the *Tb* curve. However it is difficult to detect a decrease in mean *Tb* due to allowing access to a shower. A higher mean value of the *HLI* over the course of a day leads to an increase in mean body temperature (cy) along with a decreased lag between *Ta* and *Tb* and an increase in the amplitude *Tb*. Both the increase in lag and the decrease in amplitude are greater with shower access at higher levels of *HLI*. Allowing access to a shower appears to be an effective method for reducing heat stress.

Both the FDA model and the elliptical hysteresis model are able to find statistically significant effects of allowing shower use on *Tb* in this study. Whereas FDA can be used to find differences in *Tb* between groups at specific times of day, elliptical hysteresis provides parameter estimates that act as daily summaries. Both methods provide measures that provide important information for studying heat stress.

6. Bibliography

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