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Conference on Applied Statistics in Agriculture — 2005 - 17th Annual Conference Proceedings

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Recommended Citation

Shamsuddin, Md and Adnan, Mian A S (2005). "THE USE OF PSEUDO FACTORS IN 4^n, 8^n, 2^mX4^n, 2^nx8^n FACTORIAL DESIGNS," *Conference on Applied Statistics in Agriculture*. https://doi.org/10.4148/2475-7772.1145

THE USE OF PSEUDO FACTORS IN 4^n, 8^n, 2^mX4^n,2^nx8^n FACTORIAL DESIGNS

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ABSTRACT

The confounded factorial designs of the experiments 4^n, 2^mX4^n, 8^n, etc, and their different fractions can be constructed and analyzed using a suitable method of pseudo factors known as Rotation-Conversion Method. An example is shown.

INTRODUCTION

Pseudo factors are generally used in social science, when any alternative approach is required. The approach of pseudo factors is also used in constructing and analyzing indirectly factorial designs whose levels are in prime power. When the number of level is prime power, the direct approach for construction is very difficult. Because when the level of any factor is the power of the prime like $4 = 2^2$ (or like $8 = 2^3$, $9 =3^2$, etc), the finite field formed by the 2^2 classes residues, called Galois Field of order 2^2 and denoted by $GF(2^2)$ is used. The field exerts the roots or elements $u_0=0$, $u_1=1$, $u_2 = x$, $u_3 = x + 1$ of the minimum function or modulus polynomial $Q(x) = x^2 + x + 1$. It is not easy to manipulate these numbers u_0, u_1, u_2, u_3 of $GF(2^2)$ like the numbers 0 and 1 of GF (2).

Many authors have used the pseudo factor technique for constructing the symmetrical factorial designs of levels of prime powers. Yates (1937), Raghvarao (1971), John (1970, 71), Kempthorne (1952) used pseudo factors showing the relation of a real factor at 4 levels and two pseudo factors of two levels. Shamsuddin (1987, 90, 94), Shamsuddin and Aggarwal (1987) used the pseudo factors in symmetrical row-column designs. The technique of pseudo factors has also been used by Nath (1993) and Hossain (1995) respectively in 4^n,4^{n-k} and 8^n designs. Farid (1994) has constructed and analyzed the asymmetrical factorial design like 2^mx 4^n factorials by use of pseudo factors and Haque (1996) used the same in their regular and irregular fractions. We like to discuss in details the different steps of construction and analysis patterns used in these factorial designs and their fractions by use of pseudo factors through the conversion rule.

INDIRECT CONSTRUCTION.

First the conversion method of pseudo factors into the real factors through their level correspondence is discussed so that it can be used to construct the confounding structure of both symmetrical and asymmetrical factorial designs composed of prime power of levels partly or fully. The levels of a real factor of a 4^n design belong to the elements u_0, u_1, u_2, u_3 of Galios Field (2^2) and those of pseudo factors of 2^n designs belong to elements 0,1 of GF(2). So, in order to correspond between their levels, we are to replace the level 0 of 4^n design by 00 of 2^{2n} design, the level 1 by 10, the level x by 01 and 1+x by 11. In the same way we can correspond the levels between 8^n and 2^{3n} designs. As $GF(2^3)$ having the minimum function $f(x) = x^3 + x^2 + 1$ possesses the elements $u_0=0$, $u_1=1$, $u_2= x$, $u_3= x^2$, $u_4=x^3 =1 + x^2$, $u_5=x^4 =x^3 +x =1+x+x^2$, $u_6= x^5=x^3 +x^2 + x =1 +x$, $u_7= x^6 = x +x^2$, we replace the level 0 of 8^n design by 000 of 2^{3n} design, the level 1 by 100, the level x by 010, the level x^2 by 001 and similarly the level 1+x is replaced by 110, the level $1 + x^2$ is replaced by 101, $x+x^2$ by 011 and $1 +x +x^2$ by 111. (Hossain 1995) Accordingly, the level correspondence of real factors with pseudo factors for different factorials of prime power of levels is shown below:

Level Correspondence for 4^n Factorials:

Level of a real factor	Level combination of 2 pseudo factors
u_0	00
u_1	10
u_2	01
u_3	11

Level Correspondence for 8^n Factorials:

Level of a real factor	Level combination of 3 pseudo factors
u_0	000
u_1	100
u_2	010
u_3	001
u_4	101
u_5	111
u_6	110
u_7	011

Conversion Procedure:

In the conversion procedure it is shown how the real factors and their interactions are being replaced by their pseudo factors components. This conversion procedure named as Rotation-Conversion Procedure [Shamsuddin (1994)] finalized by trial and error method consists of two table: Rotation Table and Conversion Table.

Rotation-Conversion Procedure for 4^n factorials:

Let a real factor A at u_1 level be replaced by the pseudo factors A_1,A_2, A_1A_2 and the real factor B at u_1 level by B_1,B_2, B_1B_2 then, in a rotation table, B at the power level u_2 written as B^{u_2} is replaced respectively by the pseudo factors B_2, B_1B_2, B_1 ,and B at the power level u_3 denoted as B^{u_3} is replaced by B_1B_2, B_1,B_2 (as the different power levels of B are changing rotationally the positions of the pseudo factors B_1,B_2, B_1B_2) for multiplying with other pseudo factors A_1,A_2, A_1A_2 of A in order to get, in the conversion table, the interactions AB, A B^{u_2} , A B^{u_3} of A and B. These are shown below in details in rotation and conversion tables for 4^n factorials:

Table 1:Rotation table for 4^n factorials

	Real Factors at different power levels							
	A	B	B^{u_2}	B^{u_3}	C	C^{u_2}	C^{u_3}	… …
PSEUDO FACTOR	A_1	B_1	B_2	$B_1\ B_2$	C_1	C_2	C_1C_2	
	A_2	B_2	$B_1\ B_2$	B_1	C_2	C_1C_2	C_1	
	$A_1\ A_2$	$B_1\ B_2$	B_1	B_2	C_1C_2	C_1	C_2	

Table 2: Conversion table for 4^n factorials

	REAL FACTOR INTERACTIONS					
	AB	AB^{u_2}	AB^{u_3}	…	…	…
PSEUDO	A_1B_1	A_1B_2	$A_1B_1B_2$	…	…	…
FACTOR	A_2B_2	$A_2B_1B_2$	A_2B_1	…	…	…
INTERACTION	$A_1A_2B_1B_2$	$A_1A_2B_1$	$A_1A_2B_2$	…	…	…

Similar Conversion Procedure for 8^n is as follows:

Table 3: Rotation Table of 8^n Factorials

	A	B^{u_1}	B^{u_2}	B^{u_3}	B^{u_4}	B^{u_5}	B^{u_6}	B^{u_7}	…
P	A_1	B_1	B_3	B_2B_3	$B_1B_2B_3$	B_1B_2	B_1B_3	B_2	…
S	A_3	B_3	B_2B_3	$B_1B_2B_3$	B_1B_2	B_1B_3	B_2	B_1	…
E	A_2A_3	B_2B_3	$B_1B_2B_3$	B_1B_2	B_1B_3	B_2	B_1	B_3	…
U	$A_1A_2A_3$	$B_1B_2B_3$	B_1B_2	B_1B_3	B_2	B_1	B_3	B_2B_3	…
D	A_1A_2	B_1B_2	B_1B_3	B_2	B_1	B_3	B_2B_3	$B_1B_2B_3$	…
O	A_1A_3	B_1B_3	B_2	B_1	B_3	B_2B_3	$B_1B_2B_3$	B_1B_2	…
	A_2	B_2	B_1	B_3	B_2B_3	$B_1B_2B_3$	B_1B_2	B_1B_3	…

Table 4: Conversion table of 8^n Factorials

	REAL FACTORIAL EFFECTS						
	AB^{u_1}	AB^{u_2}	AB^{u_3}	AB^{u_4}	AB^{u_5}	AB^{u_6}	AB^{u_7}
P	A_1B_1	A_1B_3	$A_1B_2B_3$	$A_1B_1B_2B_3$	$A_1B_1B_2$	$A_1B_1B_3$	A_1B_2
S	A_3B_3	$A_3B_2B_3$	$A_3B_1B_2B_3$	$A_3B_1B_2$	$A_3B_1B_3$	A_3B_2	A_3B_1
E	$A_2A_3B_2B_3$	$A_2A_3B_1B_2B_3$	$A_2A_3B_1B_2$	$A_2A_3B_1B_3$	$A_2A_3B_2$	$A_2A_3B_1$	$A_2A_3B_3$
U	$A_1A_2A_3B_1B_2B_3$	$A_1A_2A_3B_1B_2$	$A_1A_2A_3B_1B_3$	$A_1A_2A_3B_2$	$A_1A_2A_3B_1$	$A_1A_2A_3B_3$	$A_1A_2A_3B_2B_3$
D	$A_1A_2B_1B_2$	$A_1A_2B_1B_3$	$A_1A_2B_2$	$A_1A_2B_1$	$A_1A_2B_3$	$A_1A_2B_2B_3$	$A_1A_2B_1B_2B_3$
O	$A_1A_3B_1B_3$	$A_1A_3B_2$	$A_1A_3B_1$	$A_1A_3B_3$	$A_1A_3B_2B_3$	$A_1B_1B_2B_3$	$A_1A_3B_1B_2$
	A_2B_2	A_2B_1	A_2B_3	$A_2B_2B_3$	$A_2B_1B_2B_3$	$A_2B_1B_2$	$A_2B_1B_3$

In an example it is found that the 4^2 design of 4 pseudo factors A_1,A_2,B_1,B_2 confounds A_1B_2, $A_2B_1B_2$, and$A_1A_2B_1$ with its 4 blocks [Table 5]. After conversion, this 4^2 design of 2 real factor A and B confounds one effect AB^{u_2} with its 4 blocks [Table 6] .

Table 5:The 4^2 design of 4 pseudo factors in 4 blocks

0000	1010	1111	0101
1101	0111	0010	1000
1011	0001	0100	1110
0110	1100	1001	0011

Table 6:The 4^2 design of 2 real factors in 4 blocks

u_0u_0	u_1u_1	u_3u_3	u_2u_2
u_3u_2	u_2u_3	u_0u_1	u_1u_0
u_1u_3	u_0u_2	u_2u_0	u_3u_1
u_2u_1	u_3u_0	u_1u_2	u_0u_3

Similarly any 8^n design of n real factor in 8^r (r < n) blocks confounding $(8^r-1)/(r-1)$ effects can made indirectly through 2^{3n} design of 3n pseudo factors in 8^r blocks confounding $(2^{3r}-1)/(r-1)$ pseudo factorial effects (shown in tables 7 & 8).

Table 7: The 2^6 design of 6 pseudo factors A_1, A_2,A_3,B_1,B_2,B_3 confounding Pseudo factorial effect A_1B_2, A_3B_1, $A_2A_3B_3$ with 8 blocks

000000	000001	000100	001000	100000	000110	110000	101000
100010	100011	100110	101010	000010	100100	010010	001010
001101	001100	001001	000101	101101	001011	111101	100101
011100	011101	011000	010100	111100	011010	101100	110100
101111	101110	101011	100111	001111	101001	011111	000111
111110	111111	111010	110110	011110	111000	001110	010110
010001	010000	010101	011001	110001	010111	100001	111001
110011	110010	110111	111011	010011	110101	000011	011011

Table 8: The 8^2 factorial design of 2 factors A and B confounding factorial effect AB^{u_7} with 8 blocks

u_0u_0	u_0u_3	u_0u_1	u_3u_0	u_1u_0	u_0u_6	u_6u_0	u_4u_0
u_1u_2	u_1u_7	u_1u_6	u_4u_2	u_0u_2	u_1u_1	u_2u_2	u_3u_2
u_3u_4	u_3u_1	u_3u_3	u_0u_4	u_4u_4	u_3u_7	u_5u_4	u_1u_4
u_7u_1	u_7u_4	u_7u_0	u_2u_1	u_5u_1	u_7u_2	u_4u_1	u_6u_1
u_4u_5	u_4u_6	u_4u_7	u_1u_5	u_3u_5	u_4u_3	u_7u_5	u_0u_5
u_5u_6	u_5u_5	u_5u_2	u_6u_6	u_7u_6	u_5u_0	u_3u_6	u_2u_6
u_2u_3	u_2u_0	u_2u_4	u_7u_3	u_6u_3	u_2u_5	u_1u_3	u_5u_3
u_6u_7	u_6u_2	u_6u_5	u_5u_7	u_2u_7	u_6u_4	u_0u_7	u_7u_7

The 2^mx4^n Factorial Designs:

This type of design of m two leveled factors and n four leveled factors can be indirectly constructed by 2^{m+2n} factorials of two leveled m real factors and 2n pseudo factors. Here, as an example, we like to discuss 2^2x4^2 factorial design. It can be indirectly constructed by 2^6 factorial of two real factors and four pseudo factors, each at two levels.

Table 9a: The row-column 2^6 factorial design confounding A_1B_2, $A_2B_1B_2$ and $A_1A_2B_1$ with 4 blocks

000000	001000	000100	000001
000110	001110	000010	000111
001011	000011	001111	001010
001101	000101	001001	001100
110000	111000	110100	110001
110110	111110	110010	110111
111011	110011	111111	111010
111101	110101	111001	111100
100000	101000	100100	100001
100110	101110	100010	100111
101011	100011	101111	101010
101101	100101	101001	101100
010000	011000	010100	010001
010110	011110	010010	010111
011011	010011	011111	011010
011101	010101	011001	011100

Table 9b: The row-column 2^2x4^2 factorial design confounding AB^{u_2} with 4 blocks

$00u_0u_0$	$00u_1u_0$	$00u_2u_0$	$00u_0u_2$
$00u_2u_1$	$00u_3u_1$	$00u_0u_1$	$00u_2u_1$
$00u_1u_3$	$00u_0u_3$	$00u_3u_3$	$00u_1u_1$
$00u_3u_2$	$00u_2u_2$	$00u_1u_2$	$00u_3u_0$
$11u_0u_0$	$11u_1u_0$	$11u_2u_0$	$11u_0u_2$
$11u_2u_1$	$11u_3u_1$	$11u_0u_1$	$11u_2u_1$
$11u_1u_3$	$11u_0u_3$	$11u_3u_3$	$11u_1u_1$
$11u_3u_2$	$11u_2u_2$	$11u_1u_2$	$11u_3u_0$
$10u_0u_0$	$10u_1u_0$	$10u_2u_0$	$10u_0u_2$
$10u_2u_1$	$10u_3u_1$	$10u_0u_1$	$10u_2u_1$
$10u_1u_3$	$10u_0u_3$	$10u_3u_3$	$10u_1u_1$
$10u_3u_2$	$10u_2u_2$	$10u_1u_2$	$10u_3u_0$
$01u_0u_0$	$01u_1u_0$	$01u_2u_0$	$01u_0u_2$
$01u_2u_1$	$01u_3u_1$	$01u_0u_1$	$01u_2u_1$
$01u_1u_3$	$01u_0u_3$	$01u_3u_3$	$01u_1u_1$
$01u_3u_2$	$01u_2u_2$	$01u_1u_2$	$01u_3u_0$

Fractional Factorial Designs:

In constructing 4^{n-k} factorial designs or 8^{n-k} designs indirectly we can use the pseudo factorial like 2^{2n-2k} or 2^{3n-3k} designs respectively through the relations shown in the conversion table. Thus, as for example, the 4^{3-1} design containing one block defined by $1 = AB^{u_2}C^{u_3}$ can be made using 2^{6-2} pseudo factorial design defined by $1 = A_1B_2C_1C_2 = A_2B_1B_2C_1 = A_1A_2B_1C_2$……(i). And the 8^{3-1} factorial design containing one block defined by $1 = AB^{u_2}C^{u_7}$ can be done by using the 2^{9-3} pseudo factorial design defined by defining contrast $1 = A_1B_3C_2 = A_3B_2B_3C_1 = A_2A_3B_1B_2B_3C_3 = A_1A_2A_3B_1B_2C_1C_2 = A_1A_2B_1B_2C_1C_2C_3 = A_1A_3B_2C_1C_2 = A_2B_1C_1C_3$.…..(ii). The same rule is followed for a single block or the odd numbers of blocks of a pseudo factorial design to construct the regular or irregular fraction of a 2^mx4^n factorial design.

The constructions of different types of full as well as fractions of 2^mx8^n, 4^mx8^n ,$2^mx4^nx8^n$ designs will be similarly made.

ESTIMATION and ANALYSIS

We can estimate all factorial effects in all two-leveled full factorials of pseudo factors by direct method or Yates method and in the fractions of pseudo factorials by direct method. Then these estimated values are converted to real factorial effects by conversion procedure shown in the conversion tables. In the fractional factorial the same procedure of conversion is followed as shown in the conversion table required for 4^n or 8^n factorial. After estimating the pseudo factorial effects (main effect or interaction effect) group-wise, followed by the conversion table, from its alias structure we get the estimates of real main effects and interaction effects by conversion for the 4^n or 8^n factorial design. The sum of square of a pseudo factorial effect in these two leveled full or fractional factorials is estimated as usual procedure by [square of total factorial effect / total d.f.]

In analysis the real factor ss or the real factorial effect ss is obtained as the total of the ss's of all pseudo factors or pseudo factorial effects used for conversion of the real factor or real factorial effect. Thus, as for example in calculating the sum of squares of 4^2 factorials, we see ss(A) is ss(A_1)+ss(A_2) +ss(A_1A_2) and ss(AB^{u_2}) is the total of ss(A_1B_2), ss($A_2B_1B_2$,) and ss($A_1A_2B_1$) as shown in the conversion table. And similarly in the 8^2 factorial , as for examples using its conversion table, ss(AB^{u_2}) =ss(A_1B_3) + ss($A_3B_2B_3$,) + ss($A_2A_3B_1B_2B_3$) + ss($A_1A_2A_3B_1B_2$) + ss($A_1A_2B_1B_3$) + ss($A_1A_3B_2$) + ss(A_2B_1).

As an example we show here the ANOVA table of 4^2 factorial design of 2 real factors A and B confounding AB^{u_3} with 4 blocks using the 2^4 design of 4 pseudo factors A_1, A_2, B_1, B_2 through the conversion rule of 4^n factorial.

Table 10. The Analysis of Variance Table of 4^2 Design

2^4 factorial design			4^2 factorial design		
S.V.	D.F.	SS	S.V.	D.F.	SS
A_1	1	S(A_1)			
A_2	1	S(A_2)	A	3	S(A)
A_1A_2	1	S(A_1A_2)			
B_1	1	S(B_1)			
B_2	1	S(B_2)	B	3	S(B)
B_1B_2	1	S(B_1B_2)			
A_1B_1	1	S(A_1B_1)			
A_2B_2	1	S(A_2B_2)	AB	3	S(AB)
$A_1A_2B_1B_2$	1	S($A_1A_2B_1B_2$)			
$A_1B_1B_2$	1	S($A_1B_1B_2$)			
A_2B_1	1	S(A_2B_1)	$AB^u{}_3$	3	S($AB^u{}_3$)
$A_1A_2B_2$	1	S($A_1A_2B_2$)			
Block	3	S_b	Block	3	S_b
Error	by subtraction		Error	by subtraction	

Summary

In constructing and analyzing indirectly the 4^n, 2^m x 4^n and 8^n types of designs the pattern of use of pseudo factors are shown here in detail.

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