Conference on Applied Statistics in Agriculture   1991 - 3rd Annual Conference Proceedings

MESSY EXPERIMENTAL DESIGNS

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Recommended Citation

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Abstract

This paper describes the statistical analysis of an agricultural experiment that was conducted in a very complex, but somewhat reasonable, experimental design. A correct analysis of data collected from the experimental design used requires the estimation of 8 error terms.

1. Introduction

An experiment was conducted to determine the effects of IRRigation, Previous Crop, and GYPsum on the yield of wheat. The plot plan is shown in Figure 1. The irrigation treatments are defined as DRAINED and UNDRAINED, the previous crops are: F-Fallow, P-Perennial Alfalfa, R-Previous three year rotation (sesbania-wheat-sesbania), and W-Wheat straw. Gypsum treatments are denoted by G+-Gypsum applied and G--No gypsum applied.

Note that the west side of the field was drained while the east side was not drained. The sides for the irrigation treatments were selected at random. The east-west rows were grouped into blocks of two rows each, and the gypsum treatments were randomly assigned to the rows within each block. It is not known whether this was done for convenience or whether there was a physical rationale for forming blocks of two rows each for the gypsum treatments. The previous crop treatments were assigned to pairs of rows and the columns of each half of the field. These were assigned in a latin square arrangement with previous crop treatments being rerandomized within each half of the field. The REP notation at the top of each column was provided by the researcher and is not useful for the analysis. One might suppose that the researcher expected that the columns of this plot plan would provide independent replicates of the IRRIGATION treatments. However, true replicates must come from random assignments of the irrigation treatments to the columns of the plot plan. The columns in this design are more accurately called subsamples and do not represent independent replications of the irrigation treatments.

Measurements were taken on each of the smallest sized plots giving a total of 64 measurements.

The questions about the fixed effects which are of interest are those that would be of interest in most 3-way cross-classified treatment structure experiments. The fixed effects are: IRR, PC, GYP, IRR*PC, IRR*GYP, PC*GYP, and IRR*PC*GYP. How should data from this experiment be analyzed? This is the topic of this paper.

One way to answer this question is to consider each of the different sizes of experimental units to which factors have been randomly assigned. This experiment has 8 different sizes of experimental units. These are illustrated in Figures 2-9 by shading a typical experimental unit of each type. The different sizes of experimental units are:

(1) **Half Plot** - The irrigation treatments have been randomly assigned to these. There are two such experimental units. See Figure 2.

(2) **Row Pair** - The row pairs are blocks for the gypsum treatments. The row pairs have been divided into two equal parts and the gypsum treatments have been randomly assigned to the halves in each block. The row pairs are also blocking factors for rows in a latin square design. Each of the previous crop treatments occur in pairs once within each row of each square. There are four such experimental units. See Figure 3.

(3) **Column Block** - The columns in each half plot are blocking factors for columns in a latin square design. Each of the previous crop treatments occur in pairs once within each column of each square. There are eight such experimental units. See Figure 4.

(4) **Half Row Pair within Square** - These are the experimental units for Row Pair*Irrigation effects. There are eight such experimental units. See Figure 5.

(5) **Previous Crop** - These experimental units consist of pairs of the smallest sized plots. The previous crop treatments have been randomly assigned to these in a latin square configuration with a new randomization for each square. There are 32 such experimental units. See Figure 6.

(6) **Gypsum** - These are the experimental units to which the Gypsum treatments have been randomly assigned. These units were blocked into row pairs, and the gypsum treatments were randomly assigned to one-half of each row pair. There are 8 such experimental units. See Figure 7.

(7) **One-fourth row pair** - These are the experimental units for the **Irrigation*gypsum** effects. There are 16 such experimental units. See Figure 8.

(8) **Smallest Plot** - These are the smallest sized experimental units. These are the experimental units to which **Irrigation*   Previous Crop*gypsum** effects have been assigned. See Figure 9.

One way to see how such an experiment can be analyzed is to first consider each of the experimental unit sizes separately. Develop an analysis for each size, and then put all of the separate parts into one complete analysis. Analyses for each of the different sizes of experimental units are described in Sections 1-8, respectively.

1. Half Plot Analysis

There are only two half plots so there is only one contrast for measuring effects amongst the half plots. This single degree of freedom contrast measures the difference between the irrigation treatments. Unfortunately, this part of the experiment is not replicated; hence, there is no measure of experimental error for this experimental unit

size and as a result, there is no legitimate statistical test for comparing the levels of **irrigation** treatments. The columns of the design used represent subsamples and not independent replications. The form of an AoV table for the half plot analysis is given in Table 1.

2. Row Pair Analysis

There are 4 row pairs, so there are 3 orthogonal contrasts for comparing **row pairs**. There is no information about fixed effects in these contrasts. These row pairs act as blocks for the **gypsum** treatments. The form of an AoV table for the row pair analysis is shown in Table 2.

3. Column Block Analysis

There are 8 column blocks, so there are 7 orthogonal contrasts comparing **column** effects. One contrast in the column block effects measures the difference between **irrigation** treatments. This is the same contrast discussed in Section 1. Contrasts which are orthogonal to this one measure subsampling error rather than experimental error for the **irrigation** treatments. These contrasts contain no information about fixed effects, and it is not possible to further partition these contrasts into measures of different effects. The form of an AoV table for the column block analysis is shown in Table 3.

4. Half Row Pair within Square Analysis

There are 8 half row pairs within each square, so there are 7 orthogonal contrasts comparing half row pairs. One such contrast measures the difference between **irrigation** treatments, 3 others are those described in Section 2 which measure differences in complete row pairs. The remaining 3 degrees of freedom come from interaction contrasts between complete **row pairs** and **irrigation**. These interaction contrasts contain no information about fixed effects. One could say that they measure experimental error for these sizes of experimental units, but this error is not appropriate for any comparisons of fixed effects. Thus I prefer to label it as IRR*RP. The form of the AoV table for the half row pair within square analysis is shown in Table 4.

5. Previous Crop Analysis

There are 32 experimental units to which the previous crop treatments have been assigned, so there are 31 orthogonal contrasts measuring differences between these experimental unit sizes. Amongst these contrasts are: seven which measure **column** effects, which can be partitioned as shown in Section 3; three which measure **row pair** effects and three which measure **row pair*irrigation** interaction as described in Section 4; three which measure **previous crop** effects, and three which measure **irrigation*previous** crop interaction. The remaining 12 contrasts contain no information about fixed effects and thus provide measures of experimental error for comparing fixed effect contrasts in these same sizes of experimental units, namely the **previous crop** effect and **irrigation*previous crop** interaction effect. The form of the AoV

table for the analysis of the previous crop experimental units is shown in Table 5.

6. Gypsum Analysis

There are 8 experimental units to which the gypsum treatments have been assigned, so there are 7 orthogonal contrasts comparing these sizes of experimental units. Three of these compare **row pairs** as described in Section 2 and one compares the two **gypsum** treatments. The remaining 3 contrasts come from interaction contrasts of **row pairs** with **gypsum** and contain no information about fixed effects; these measure experimental error for these sizes of experimental units. The form of the AoV table for the analysis of the gypsum experimental units is shown in Table 6.

7. One-fourth Row Pair Analysis

The one-fourth row pair experimental units are shown in Figure 8. There are 16 such experimental units, so there are 15 orthogonal contrasts measuring differences between these sizes of experimental units. Seven of these measure the effects described in Section 6 and one measures the **irrigation** effect described in Section 1. The remaining seven contrasts are formed from the interaction between these two sets of effects. One of these contrast measures **irrigation*gypsum** interaction, and three others measure IRR*RP interaction as described in Section 4. The remaining three contain no information about fixed effects and provide measures of experimental error for these experimental unit sizes. The resulting error term is used to test for the **irrigation*gypsum** interaction effect. The form of the AoV table for the one-fourth row pair analysis is shown in Table 7.

8. Smallest Plot Analysis

There are 64 of the smallest sized plots, so there are 63 orthogonal contrasts for comparisons between these smallest sized experimental units. Amongst the 63 orthogonal contrasts are 31 which measure the effects described in Section 5 and 15 which measure the fixed effects described in Section 7. Seven of these are common to both sections, thus there 63-31-15+7=24 orthogonal contrasts remaining. Three of these measure the **prior crop*gypsum** interaction, three measure the **Irrigation*prior crop*gypsum** interaction, and the remaining 18 measure experimental error for these sizes of experimental units. The form of the AoV table for these smallest sized experimental units is shown in Table 8.

9. A SAS^R Analysis of the Experiment

Now that an appropriate analysis for each of the differing sizes of the experimental units has been determined, the next question is: can the data be analyzed with existing statistical software?

Since the experiment is balanced and since all of the comparisons in Table 8 form orthogonal sets, it should be possible to obtain a computer analysis with a carefully selected model. In addition to modelling the fixed effects (IRR, PC, IRR*PC, GYP, IRR*GYP, PC*GYP, and IRR*PC*GYP),

the model must include terms for the random effects blocking terms (RP, CB(IRR), and IRR*RP). The model must also contain terms which will partition out estimates for each of the experimental error effects (Errors C, D, and E). The residual sum of squares from such a carefully selected model will provide an estimate of the remaining error term, **Error F**.

Note that the analysis must have random effects terms and/or error terms for each experimental unit size. This experiment has 8 different sizes of experimental units, so there should be 8 error and/or random terms in the model. A model for this experiment should have 3 random effects terms and 5 error terms. Unfortunately, for this experiment, **Error A** cannot be estimated, since there were no independent replications of the irrigation treatments. Thus this model will only have 7 such terms.

Actually, random effects terms are also error terms. So far they have not been identified as such, primarily because they are not used as denominators for any F-ratios corresponding to the fixed effects in the analysis. It is important to note that all error and random effects terms must be considered when estimating standard errors for fixed effect means and for comparisons amongst the fixed effects.

The data in this experiment can be analyzed with the GLM procedure of SAS^R by using the following commands (the terms in bold face are random effects terms and/or error terms):

```
PROC GLM;
  CLASSES IRR RP CB PC GYP;
   MODEL Y = IRR RP CB(IRR) IRR*RP PC IRR*PC IRR*RP*PC
            GYP RP*GYP IRR*GYP IRR*RP*GYP PC*GYP IRR*PC*GYP/E E1;
   RANDOM RP CB(IRR) IRR*RP IRR*PC*RP GYP*RP IRR*GYP*RP/TEST;
   LSMEANS IRR|PC|GYP;
```

Errors C, **D**, and **E** are estimated by the effects, **IRR*RP*PC**, **RP*GYP**, and **IRR*RP*GYP**, respectively. **Error F** is estimated by the residual sum of squares.

The TYPE I analysis gives appropriate sums of squares for each of the effects, and this analysis tests the usual fixed effects hypotheses which are tested in balanced three-way experiments. To see this one can examine the results of the E1 option used on the MODEL statement. The E1 option shows the general form of the TYPE I estimable functions, and this general form identifies the functions of the fixed effect parameters being compared to zero by the TYPE I analysis (See Milliken and Johnson (1984), Chapter 10). By using the TEST option on the RANDOM statement above, SAS^R will provide corrected F-statistics for each of the effects in the model using the methods proposed in Milliken and Johnson (1984, Chapter 28).

It might be noted that for many experiments the TYPE III and/or TYPE IV analyses can often be used when the TYPE I analysis cannot. For this experiment, however, the sum of squares for the CB(IRR) term is not computed correctly by the TYPE III and TYPE IV analyses which is why the TYPE I analysis should be used. The TYPE III and IV analyses give the CB(IRR) sum of squares as zero with zero degrees of freedom. The reason for this is that the contrasts which measure this effect are contained

within those that are required for the **Previous Crop Analysis** as
described in Section 5.

The actual data for this experiment are not available. In order to
illustrate an analysis of data from this kind of messy experiment, data
were manufactured. For each of the 64 smallest cell sizes, a data point
was generated from a standard normal distribution. The commands used to
generate the data are:

```
DATA; INPUT IRR GYP CB PC RP @@; Y=RANNOR(12334759); CARDS;
1 1 1 1 1   1 2 1 1 1   1 1 1 1 2 2   1 2 1 2 2   1 1 1 3 3   1 2 1 3 3
1 1 1 4 4   1 2 1 4 4   1 1 2 2 1     1 2 2 2 1   1 1 2 3 2   1 2 2 3 2
1 1 2 4 3   1 2 2 4 3   1 1 2 1 4     1 2 2 1 4   1 1 3 3 1   1 2 3 3 1
1 1 3 4 2   1 2 3 4 2   1 1 3 1 3     1 2 3 1 3   1 1 3 2 4   1 2 3 2 4
1 1 4 4 1   1 2 4 4 1   1 1 4 1 2     1 2 4 1 2   1 1 4 2 3   1 2 4 2 3
1 1 4 3 4   1 2 4 3 4   2 1 5 4 1     2 2 5 4 1   2 1 5 2 2   2 2 5 2 2
2 1 5 1 3   2 2 5 1 3   2 1 5 3 4     2 2 5 3 4   2 1 6 3 1   2 2 6 3 1
2 1 6 1 2   2 2 6 1 2   2 1 6 2 3     2 2 6 2 3   2 1 6 4 4   2 2 6 4 4
2 1 7 1 1   2 2 7 1 1   2 1 7 4 2     2 2 7 4 2   2 1 7 3 3   2 2 7 3 3
2 1 7 2 4   2 2 7 2 4   2 1 8 2 1     2 2 8 2 1   2 1 8 3 2   2 2 8 3 2
2 1 8 4 3   2 2 8 4 3   2 1 8 1 4     2 2 8 1 4
```

The TYPE I AoV table from a GLM analysis of this manufactured data
is given in Table 9. All of the F-Values and their corresponding P-
Values would be wrong in a real data case except those corresponding to
the **PC*GYP** and **IRR*PC*GYP** effects. The results from the RANDOM option
are shown in Table 10, and some of the results from the TEST option are
shown in Table 11.

At this point, we have illustrated the different sizes of
experimental units, have shown how this information can be used to
identify error terms for a statistical analysis, have shown an
appropriate model for the experiment, and illustrated an AoV table. To
effectively complete an analysis of such an experiment, one will also
need to be able to make mean comparisons amongst the fixed effects as
well as estimating fixed effect means and their standard errors. This
can be done by generalizing the results in Milliken and Johnson to a
multiple error term case. The basic ideas are illustrated in Section
10.

10. Estimating Contrasts in Fixed Effects.

This section illustrates the steps required for obtaining correct
standard errors of linear functions of the fixed effects for the
experiment considered in this paper. It should be pointed out that the
LSMEANS option in a GLM analysis of this experiment provides correct
least squares means, but their corresponding standard errors are usually
wrong and p-values for pairwise comparisons are often wrong. A
knowledgeable data analyst may be able to select E= options on the
LSMEANS options to get many of the comparisons carried out correctly,
but it is not usually possible to get the computer to correctly carry
out all comparisons which are likely to be of interest.

Let μ_{ijk} represent the expected response from **irrigation** treatment
i, **prior crop** treatment j, and **gypsum** treatment k. To illustrate the

methods required for correcting the computer analysis consider estimating the following differences in the fixed effects:

1) $\bar{\mu}_{.1.} - \bar{\mu}_{.2.}$: this compares two prior crop main effect means

2) $\bar{\mu}_{11.} - \bar{\mu}_{12.}$: this compares two prior crop means when IRR=1

3) $\bar{\mu}_{..1} - \bar{\mu}_{..2}$: this compares two gypsum main effect means

4) $\bar{\mu}_{.11} - \bar{\mu}_{.12}$: this compares two gypsum means when PC=1

5) $\bar{\mu}_{.11} - \bar{\mu}_{.21}$: this compares two prior crop means when GYP=1

CONTRAST and ESTIMATE statements corresponding to these functions of the fixed effects parameters which can be used in the GLM analysis presented are given in Table 12. Table 13 gives the results of these CONTRAST and ESTIMATE options. The standard errors in Table 13 are usually computed incorrectly and as a result, the t-tests and their corresponding p-values are also incorrect. Likewise the F-tests and their p-values from the contrast options are also incorrectly computed. Nevertheless, the results in Table 13 are useful, because they provide the information required to allow a data analyst to compute corrected standard errors, t-tests, and p-values with a little effort.

To describe how things can be fixed for this experiment, let MS_i, $i=1,2,\ldots,7$, denote the observed mean squares for the random effects and error terms in the TYPE I AoV table. Thus $MS_1=1.9899$, $MS_2=1.7088$, $MS_3=0.5750$, $MS_4=1.7517$, $MS_5=3.0866$, $MS_6=0.2257$, and $MS_7=1.4026$. Next one determines estimates of the individual variance components by solving the equations formed by setting the expected values of the error mean squares to their observed values. To illustrate, the equations which must be solved are:

$$\hat{\sigma}_\epsilon^2 + 4\hat{\sigma}_{IRG}^2 + 8\hat{\sigma}_{RG}^2 + 2\hat{\sigma}_{IRP}^2 + 8\hat{\sigma}_{IR}^2 + 16\hat{\sigma}_{RP}^2 = MS_1,$$

$$\hat{\sigma}_\epsilon^2 + 2\hat{\sigma}_{IRP}^2 + 8\hat{\sigma}_{C(I)}^2 = MS_2,$$

$$\hat{\sigma}_\epsilon^2 + 4\hat{\sigma}_{IRG}^2 + 2\hat{\sigma}_{IRP}^2 + 8\hat{\sigma}_{IR}^2 = MS_3,$$

$$\hat{\sigma}_\epsilon^2 + 2\hat{\sigma}_{IRP}^2 = MS_4,$$

$$\hat{\sigma}_\epsilon^2 + 4\hat{\sigma}_{IRG}^2 + 8\hat{\sigma}_{RG}^2 = MS_5,$$

$$\hat{\sigma}_\epsilon^2 + 4\hat{\sigma}_{IRG}^2 = MS_6, \text{ and}$$

$$\hat{\sigma}_\epsilon^2 = MS_7.$$

Solving one gets

$$\hat{\sigma}_\epsilon^2 = MS_7, \quad \hat{\sigma}_{IRG}^2 = (MS_6 - MS_7)/4, \quad \hat{\sigma}_{RG}^2 = (MS_5 - MS_6)/8, \quad \hat{\sigma}_{IRP}^2 = (MS_4 - MS_7)/2,$$

$$\hat{\sigma}_{IR}^2 = (MS_3 - MS_4 - MS_6 + MS_7)/8, \quad \sigma_{C(I)}^2 = (MS_2 - MS_4)/8, \text{ and}$$

$$\hat{\sigma}^2_{RP} = (MS_1 - MS_3 - MS_5 + MS_6)/16.$$

Next it is noted that the standard error of any linear function of the fixed effect cell means will have the form

$$[k(\sigma^2_\epsilon + c_1\sigma^2_{IRG} + c_2\sigma^2_{RG} + c_3\sigma^2_{IRP} + c_4\sigma^2_{IR} + c_5\sigma^2_{RP} + c_6\sigma^2_{C(I)}]^{1/2}.$$

The values of c_1-c_6 can be obtained from the expected contrast mean squares given in Table 13, and the value of k can be obtained as $k = (PSE)^2/\sigma^2_\epsilon$ where PSE is the printed standard error in Table 13. Then the estimated standard error of a linear function of the fixed effects parameters can be obtained by computing

$$[k(\hat{\sigma}^2_\epsilon + c_1\hat{\sigma}^2_{IRG} + c_2\hat{\sigma}^2_{RG} + c_3\hat{\sigma}^2_{IRP} + c_4\hat{\sigma}^2_{IR} + c_5\hat{\sigma}^2_{RP} + c_6\hat{\sigma}^2_{C(I)}]^{1/2}.$$

The estimates of our contrasts and their corrected standard errors are given in Table 14.

To complete the analysis one has to be able to assign p-values to pseudo t-statistics formed by taking ratios of the estimates and their corresponding corrected standard errors. To do this, degrees of freedom are assigned to the t-tests by using Satterthwaite's Theorem for approximating the distribution of a linear combination of independent chi-square random variables. To illustrate, consider the fourth contrast. From Table 13, the expected mean square of this contrast is $\sigma^2_\epsilon + 4\sigma^2_{IRG} + 4\sigma^2_{RG}$. Its estimate is equal to $.125(MS_5) + .875(MS_6)$. From Satterthwaite's Theorem, the approximate degrees of freedom to assign to this linear function of MS_5 and MS_6 is

$$\hat{\nu} = \frac{[.125MS_5 + .875MS_6]^2}{(.125)^2MS_5^2/\nu_5 + (.875)^2MS_6^2/\nu_6}$$

where ν_5 and ν_6 are the degrees of freedom associated with MS_5 and MS_6, respectively. For the example

$$\hat{\nu} = \frac{[.125(3.0866) + .875(0.2257)]^2}{(.125)^2(3.0866)^2/3 + (.875)^2(0.2257)^2/3} = \frac{.5833^2}{.0626} = 5.4,$$

$t = -0.0563/0.2700 = -0.2085$, and the resulting p-value is 0.5785. In a similar manner the degrees of freedom and approximate p-values can be obtained for the other contrasts. The results are given in Table 14.

11. REFERENCES

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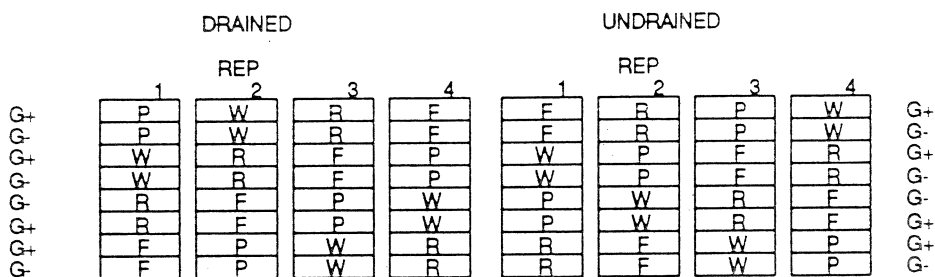
DRAINED UNDRAINED

REP REP

	1	2	3	4	1	2	3	4	
G+	P	W	R	F	F	R	P	W	G+
G-	P	W	R	F	F	R	P	W	G-
G+	W	R	F	P	W	P	F	R	G+
G-	W	R	F	P	W	P	F	R	G-
G-	R	F	P	W	P	W	R	F	G-
G+	R	F	P	W	P	W	R	F	G+
G+	F	P	W	R	R	F	W	P	G+
G-	F	P	W	R	R	F	W	P	G-

Figure 1. Plot plan for a 3-factor cross-classified experiment.

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REP REP

	1	2	3	4	1	2	3	4	
G+	P	W	R	F	F	R	P	W	G+
G-	P	W	R	F	F	R	P	W	G-
G+	W	R	F	P	W	P	F	R	G+
G-	W	R	F	P	W	P	F	R	G-
G-	R	F	P	W	P	W	R	F	G-
G+	R	F	P	W	P	W	R	F	G+
G+	F	P	W	R	R	F	W	P	G+
G-	F	P	W	R	R	F	W	P	G-
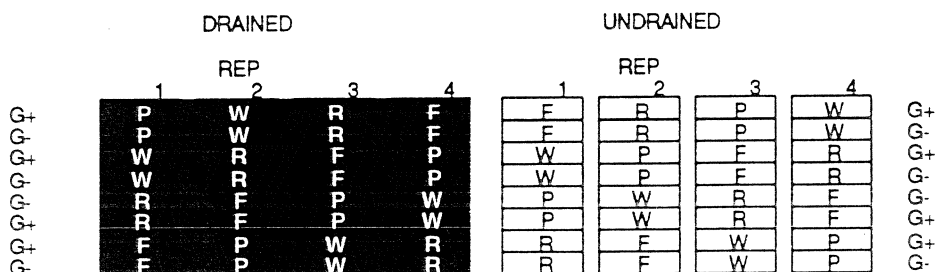
Figure 2. Size 1 experimental unit - IRR treatment.

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REP REP

	1	2	3	4	1	2	3	4	
G+	P	W	R	F	F	R	P	W	G+
G-	P	W	R	F	F	R	P	W	G-
G+	W	R	F	P	W	P	F	R	G+
G-	W	R	F	P	W	P	F	R	G-
G-	R	F	P	W	P	W	R	F	G-
G+	R	F	P	W	P	W	R	F	G+
G+	F	P	W	R	R	F	W	P	G+
G-	F	P	W	R	R	F	W	P	G-
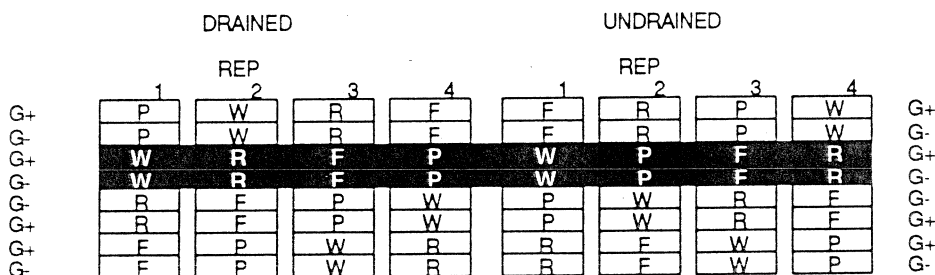
Figure 3. Size 2 experimental unit - RP (row pair block).

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REP REP

	1	2	3	4	1	2	3	4	
G+	P	W	R	F	F	R	P	W	G+
G-	P	W	R	F	F	R	P	W	G-
G+	W	R	F	P	W	P	F	R	G+
G-	W	R	F	P	W	P	F	R	G-
G-	R	F	P	W	P	W	R	F	G-
G+	R	F	P	W	P	W	R	F	G+
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G-	F	P	W	R	R	F	W	P	G-
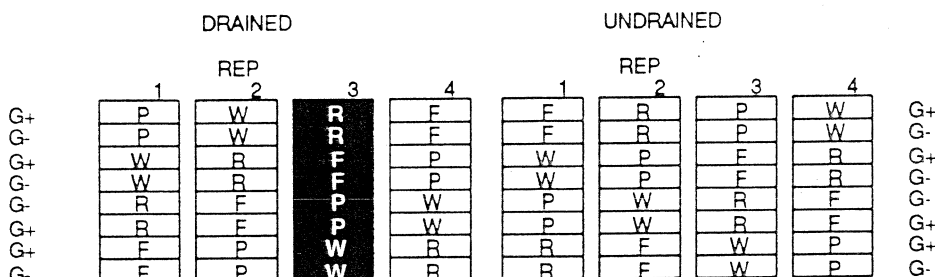
Figure 4. Size 3 experimental unit - CB (column block).

DRAINED UNDRAINED

REP REP

	1	2	3	4	1	2	3	4	
G+	P	W	R	F	F	R	P	W	G+
G-	P	W	R	F	F	R	P	W	G-
G+	W	R	F	P	W	P	F	R	G+
G-	W	R	F	P	W	P	F	R	G-
G-	R	F	P	W	P	W	R	F	G-
G+	R	F	P	W	P	W	R	F	G+
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G-	F	P	W	R	R	F	W	P	G-
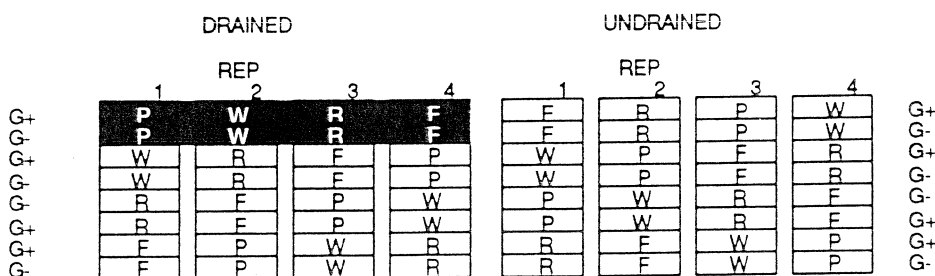
Figure 5. Size 4 experimental unit - RP*IRR.

DRAINED   UNDRAINED

REP   REP

	1	2	3	4		1	2	3	4	
G+	P	W	R	F		F	R	P	W	G+
G-	P	W	R	F		F	R	P	W	G-
G+	W	R	F	P		W	P	F	R	G+
G-	W	R	F	P		W	P	F	R	G-
G-	R	F	P	W		P	W	R	F	G-
G+	R	F	P	W		P	W	R	F	G+
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G-	F	P	W	R		R	F	W	P	G-
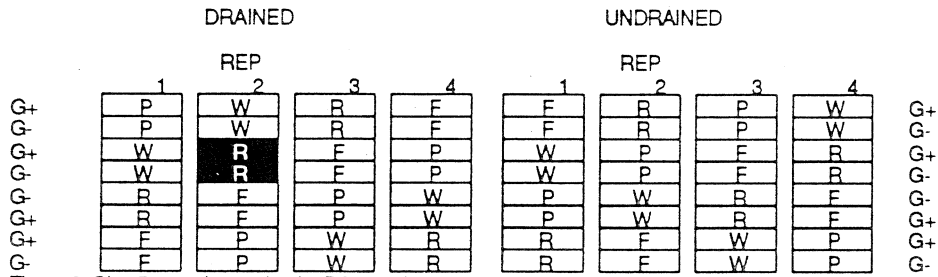
Figure 6. Size 5 experimental unit - PC (previous crop).

DRAINED   UNDRAINED

REP   REP

	1	2	3	4		1	2	3	4	
G+	P	W	R	F		F	R	P	W	G+
G-	P	W	R	F		F	R	P	W	G-
G+	W	R	F	P		W	P	F	R	G+
G-	W	R	F	P		W	P	F	R	G-
G-	R	F	P	W		P	W	R	F	G-
G+	R	F	P	W		P	W	R	F	G+
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G-	F	P	W	R		R	F	W	P	G-
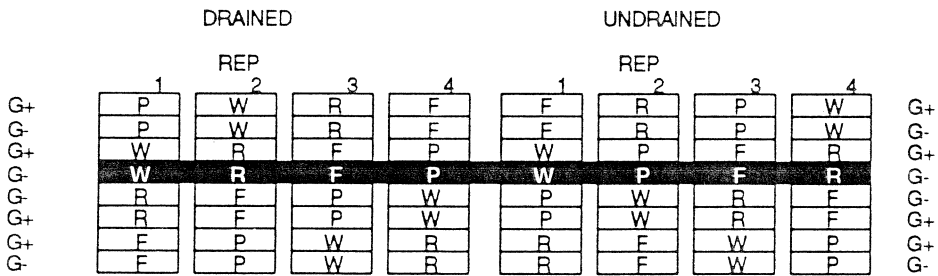
Figure 7. Size 6 experimental unit - GYP (gypsom treatment).

DRAINED   UNDRAINED

REP   REP

	1	2	3	4		1	2	3	4	
G+	P	W	R	F		F	R	P	W	G+
G-	P	W	R	F		F	R	P	W	G-
G+	W	R	F	P		W	P	F	R	G+
G-	W	R	F	P		W	P	F	R	G-
G-	R	F	P	W		P	W	R	F	G-
G+	R	F	P	W		P	W	R	F	G+
G+	F	P	W	R		R	F	W	P	G+
G-	F	P	W	R		R	F	W	P	G-
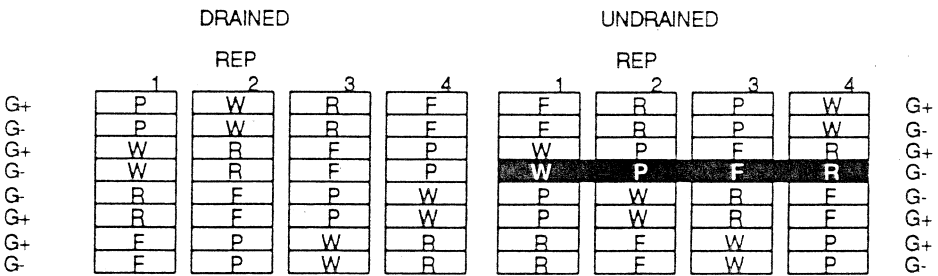
Figure 8. Size 7 experimental unit - IRR*GYP.

DRAINED   UNDRAINED

REP   REP

	1	2	3	4		1	2	3	4	
G+	P	W	R	F		F	R	P	W	G+
G-	P	W	R	F		F	R	P	W	G-
G+	W	R	F	P		W	P	F	R	G+
G-	W	R	F	P		W	P	F	R	G-
G-	R	F	P	W		P	W	R	F	G-
G+	R	F	P	W		P	W	R	F	G+
G+	F	P	W	R		R	F	W	P	G+
G-	F	P	W	R		R	F	W	P	G-
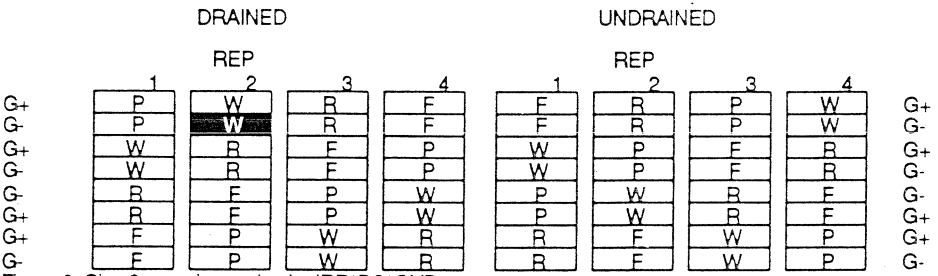
Figure 9. Size 8 experimental unit - IRR*PC*GYP.

Table 1. AoV for the Half Plot Analysis.

Source	df
IRR	1
Error A	0

Table 2. AoV for the Row Pair Analysis.

Source	df
RP	3

Table 3. AoV for the Column Block Analysis.

Source	df
IRR	1
Error A	0
CB(IRR)	6

Table 4. AoV for the Half Row Pair within Square Analysis.

Source	df
IRR	1
Error A	0
RP	3
IRR*RP	3

Table 5. AoV for the Previous Crop analysis.

Source	df
IRR	1
Error A	0
CB(IRR)	6
RP	3
IRR*RP	3
PC	3
IRR*PC	3
Error C	12

Table 6. AoV for the Gypsum analysis.

Source	df
RP	3
GYP	1
Error D	3

Table 7. AoV for the One-fourth Row Pair Analysis.

Source	df
IRR	1
Error A	0
RP	3
IRR*RP	3
GYP	1
Error D	3
IRR*GYP	1
Error E	3

Table 8. AoV for the Smallest Sized Experimental Units.

Source	df
IRR	1
Error A	0
CB(IRR)	6
RP	3
IRR*RP	3
PC	3
IRR*PC	3
Error C	12
GYP	1
Error D	3
IRR*GYP	1
Error E	3
PC*GYP	3
IRR*PC*GYP	3
Error F	18

Table 9. TYPE I AoV from GLM.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	45	68.46703683	1.52148971	1.08	0.4417
Error	18	25.24615400	1.40256411		
Corrected Total	63	93.71319083			

R-Square	C.V.	Root MSE	Y Mean
0.730602	1631.140	1.184299	0.072606

Source	DF	Type I SS	Mean Square	F Value	Pr > F
IRR	1	0.69916919	0.69916919	0.50	0.4892
RP	3	5.96976122	1.98992041	1.42	0.2700
CB(IRR)	6	10.25263401	1.70877234	1.22	0.3422
IRR*RP	3	1.72509224	0.57503075	0.41	0.7478
PC	3	3.72119063	1.24039688	0.88	0.4679
IRR*PC	3	1.36406611	0.45468870	0.32	0.8078
IRR*RP*PC	12	21.02049815	1.75170818	1.25	0.3253
GYP	1	0.51506547	0.51506547	0.37	0.5521
RP*GYP	3	9.25975536	3.08658512	2.20	0.1232
IRR*GYP	1	0.24236915	0.24236915	0.17	0.6825
IRR*RP*GYP	3	0.67722329	0.22574110	0.16	0.9212
PC*GYP	3	4.05700782	1.35233594	0.96	0.4311
IRR*PC*GYP	3	8.96320418	2.98773473	2.13	0.1320

Table 10. Expected Mean Squares of the Effects in the Type I Analysis.

Source	Type I Expected Mean Square
IRR	Var(Error) + 4 Var(IRR*RP*GYP) + 2 Var(IRR*RP*PC) + 8 Var(IRR*RP) + 8 Var(CB(IRR)) + Q(IRR,IRR*PC,IRR*GYP,IRR*PC*GYP)
RP	Var(Error) + 4 Var(IRR*RP*GYP) + 8 Var(RP*GYP) + 2 Var(IRR*RP*PC) + 8 Var(IRR*RP) + 16 Var(RP)
CB(IRR)	Var(Error) + 2 Var(IRR*RP*PC) + 8 Var(CB(IRR))
IRR*RP	Var(Error) + 4 Var(IRR*RP*GYP) + 2 Var(IRR*RP*PC) + 8 Var(IRR*RP)
PC	Var(Error) + 2 Var(IRR*RP*PC) + Q(PC,IRR*PC,PC*GYP,IRR*PC*GYP)
IRR*PC	Var(Error) + 2 Var(IRR*RP*PC) + Q(IRR*PC,IRR*PC*GYP)
IRR*RP*PC	Var(Error) + 2 Var(IRR*RP*PC)
GYP	Var(Error) + 4 Var(IRR*RP*GYP) + 8 Var(RP*GYP) + Q(GYP,IRR*GYP,PC*GYP,IRR*PC*GYP)
RP*GYP	Var(Error) + 4 Var(IRR*RP*GYP) + 8 Var(RP*GYP)
IRR*GYP	Var(Error) + 4 Var(IRR*RP*GYP) + Q(IRR*GYP,IRR*PC*GYP)
IRR*RP*GYP	Var(Error) + 4 Var(IRR*RP*GYP)
PC*GYP	Var(Error) + Q(PC*GYP,IRR*PC*GYP)
IRR*PC*GYP	Var(Error) + Q(IRR*PC*GYP)

Table 11. Some of the Corrected F-tests from the TEST option.

General Linear Models Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Dependent Variable: Y

Source: IRR *
Error: MS(CB(IRR)) + MS(IRR*RP) - MS(IRR*RP*PC)

		Denominator	Denominator		
DF	Type I MS	DF	MS	F Value	Pr > F
1	0.6991691922	0.33	0.5320949032	1.314	0.6416

* - This test assumes one or more other fixed effects are zero.
NOTE: This above test is wrong since it does not use Error A.

Source: PC *
Error: MS(IRR*RP*PC)

		Denominator	Denominator		
DF	Type I MS	DF	MS	F Value	Pr > F
3	1.2403968764	12	1.7517081794	0.708	0.5655

* - This test assumes one or more other fixed effects are zero.

Source: IRR*PC *
Error: MS(IRR*RP*PC)

		Denominator	Denominator		
DF	Type I MS	DF	MS	F Value	Pr > F
3	0.4546887018	12	1.7517081794	0.260	0.8531

* - This test assumes one or more other fixed effects are zero.

Source: GYP *
Error: MS(RP*GYP)

		Denominator	Denominator		
DF	Type I MS	DF	MS	F Value	Pr > F
1	0.5150654698	3	3.0865851197	0.167	0.7103

* - This test assumes one or more other fixed effects are zero.

Source: IRR*GYP *
Error: MS(IRR*RP*GYP)

		Denominator	Denominator		
DF	Type I MS	DF	MS	F Value	Pr > F
1	0.242369155	3	0.2257410966	1.074	0.3763

* - This test assumes one or more other fixed effects are zero.

Table 12. Contrast and Estimate options for GLM.

```
CONTRAST 'PC1-PC2' PC 1 -1 0 0;

CONTRAST 'PC(IRR 1)' PC 1 -1 0 0 IRR*PC 1 -1 0 0   0 0 0 0;

CONTRAST 'GYP1-GYP2' GYP 1 -1;

CONTRAST 'GYP(IRR 1)' GYP 4 -4 RP*GYP 1 -1 1 -1 1 -1 1 -1
 IRR*GYP 4 -4 0 0 IRR*RP*GYP 1 -1 1 -1 1 -1 1 -1 0 0 0 0 0 0 0 0
 PC*GYP 1 -1 1 -1 1 -1 1 -1
 IRR*PC*GYP 1 -1 1 -1 1 -1 1 -1 0 0 0 0 0 0 0 0;

CONTRAST 'GYP(PC1)' GYP 8 -8 RP*GYP 2 -2 2 -2 2 -2 2 -2
 IRR*GYP 4 -4 4 -4
 IRR*RP*GYP 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1
 PC*GYP 8 -8 0 0 0 0 0 0
 IRR*PC*GYP 4 -4 0 0 0 0 0 0 4 -4 0 0 0 0 0 0;

CONTRAST 'PC(GYP1)' PC 8 -8 0 0 IRR*PC 4 -4 0 0 4 -4 0 0
 IRR*RP*PC 1 -1 0 0 1 -1 0 0 1 -1 0 0 1 -1 0 0 1 -1 0 0 1 -1 0 0
    1 -1 0 0 1 -1 0 0 PC*GYP 8 0 -8 0 0 0 0 0
 IRR*PC*GYP 4 0 -4 0 0 0 0 0 4 0 -4 0 0 0 0 0;

ESTIMATE 'PC1-PC2' PC 1 -1 0 0;

ESTIMATE 'PC(IRR 1)' PC 1 -1 0 0 IRR*PC 1 -1 0 0   0 0 0 0;

ESTIMATE 'GYP1-GYP2' GYP 1 -1;

ESTIMATE 'GYP(IRR 1)' GYP 4 -4 RP*GYP 1 -1 1 -1 1 -1 1 -1
 IRR*GYP 4 -4 0 0 IRR*RP*GYP 1 -1 1 -1 1 -1 1 -1 0 0 0 0 0 0 0 0
 PC*GYP 1 -1 1 -1 1 -1 1 -1
 IRR*PC*GYP 1 -1 1 -1 1 -1 1 -1 0 0 0 0 0 0 0 0/DIVISOR=4;

ESTIMATE 'GYP(PC1)' GYP 8 -8 RP*GYP 2 -2 2 -2 2 -2 2 -2
 IRR*GYP 4 -4 4 -4
 IRR*RP*GYP 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1
 PC*GYP 8 -8 0 0 0 0 0 0
 IRR*PC*GYP 4 -4 0 0 0 0 0 0 4 -4 0 0 0 0 0 0/DIVISOR=8;

ESTIMATE 'PC(GYP1)' PC 8 -8 0 0 IRR*PC 4 -4 0 0 4 -4 0 0
 IRR*RP*PC 1 -1 0 0 1 -1 0 0 1 -1 0 0 1 -1 0 0 1 -1 0 0 1 -1 0 0
    1 -1 0 0 1 -1 0 0 PC*GYP 8 0 -8 0 0 0 0 0
 IRR*PC*GYP 4 0 -4 0 0 0 0 0 4 0 -4 0 0 0 0 0/DIVISOR=8;
```

Table 13. Results from Contrast and Estimate Options.

Contrast	Contrast Expected Mean Square
PC1-PC2	Var(Error) + 2 Var(IRR*RP*PC) + Q(PC,IRR*PC,PC*GYP,IRR*PC*GYP)
PC(IRR 1)	Var(Error) + 2 Var(IRR*RP*PC) + Q(PC,IRR*PC,PC*GYP,IRR*PC*GYP)
GYP1-GYP2	Var(Error) + 4 Var(IRR*RP*GYP) + 8 Var(RP*GYP) + Q(GYP,IRR*GYP,PC*GYP,IRR*PC*GYP)
GYP(IRR 1)	Var(Error) + 4 Var(IRR*RP*GYP) + 4 Var(RP*GYP) + Q(GYP,IRR*GYP,PC*GYP,IRR*PC*GYP)
GYP(PC1)	Var(Error) + Var(IRR*RP*GYP) + 2 Var(RP*GYP) + Q(GYP,IRR*GYP,PC*GYP,IRR*PC*GYP)
PC(GYP1)	Var(Error) + Var(IRR*RP*PC) + Q(PC,IRR*PC,PC*GYP,IRR*PC*GYP)

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
PC1-PC2	0.34316390	0.82	0.4232	0.41871293
PC(IRR 1)	0.75042384	1.27	0.2212	0.59214950
GYP1-GYP2	-0.17942015	-0.61	0.5521	0.29607475
GYP(IRR 1)	-0.05634265	-0.13	0.8945	0.41871293
GYP(PC1)	-0.28286897	-0.48	0.6386	0.59214950
PC(GYP1)	0.50929944	0.86	0.4011	0.59214950

Table 14. Estimates and their Standard Errors.

Parameter Function	Estimate	k	Corrected Standard Error of Estimate	pseudo t	Estimated Degrees of Freedom
PC1-PC2	0.3432	.125	0.4679	0.734[*]	12[*]
PC(IRR 1)	0.7504	.25	0.6618	1.134	12
GYP1-GYP2	-0.1794	.0625	0.4392	-0.408	3
GYP(IRR 1)	-0.0563	.125	0.2700	-0.209	5.4
GYP(PC1)	-0.2829	.25	0.6752	-0.419	12.8
PC(GYP1)	0.5093	.25	0.5264	0.811	19.7

[*]The first 3 of these do not need to be approximated.
These contrasts have expectations which are the same as
one of the error terms in the AoV table.