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Beyond the Point Charge: Equipotential Surfaces and Electric Fields of Various Charge Configurations

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A laboratory experiment often performed in an introductory electricity and magnetism course involves the mapping of equipotential lines on a conductive sheet between two objects at different potentials. In this article, we describe how we have expanded this experiment so that it can be used to illustrate the electrostatic properties of conductors. Different configurations of electrodes can be used to show that the electric field is zero inside a conductor as well as within a cavity, the electric field is perpendicular to conducting surfaces, and the charge distribution on conducting surfaces can vary.

Students often have difficulty transitioning from configurations comprised solely of point charges to those that include conductors.[1] Rather than applying the ideal conductor model, students often continue to rely on concepts that they just previously studied, such as Coulomb's law, and ignore the presence of a conductor. The students also often equate charge density with equipotential surfaces, which implies that the charge density cannot vary across an equipotential surface. Not only do these difficulties inhibit student success in electrostatics, they can also impact performance later in the course when electrodynamics and circuits are studied.

We have observed similar behavior among our own students on a quiz administered after class instruction, and prior to the laboratory activities described below. When asked to give the direction of the electric field at various locations around a single point charge or a dipole configuration, 82-90% of the students answered correctly. (The one exception was between the two charges of the dipole, where 64% answered correctly and 26% answered that the field was zero.) When presented with situations with conductors, however, the percentage of students answering correctly dropped. The most revealing configuration was a conductor with an interior cavity that contained an off-center point charge. When asked for the direction of the field outside of the conductor, 22% of the students gave the correct response and 66% gave a response consistent with an isolated point charge and no conducting shell.

To facilitate the development of students' electrostatics intuition, we developed several laboratory experiments. These experiments allow students to study electric fields and equipotential surfaces near conductors in a visual manner without the need for mathematics. There are a number of experimental ways to visualize and measure the electric field and potential for static two-dimensional configurations.[2-4] The method presented here relies on a sheet of conductive paper[5] mounted on a wooden board using metal wingbolts to which leads from a power supply can be connected (Fig. 1). Electrodes are drawn on the paper with a conductive silver ink pen, and a potential difference (generally 10 V) is applied to them.

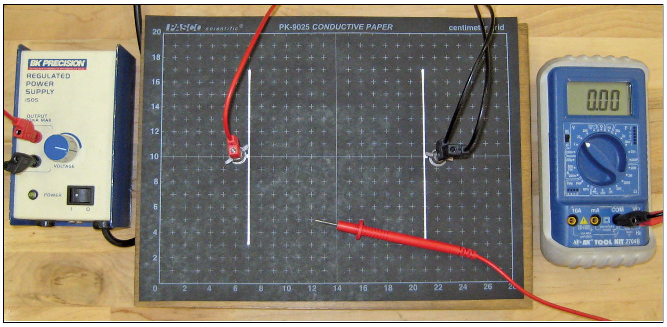

Fig. 1. Photograph showing the apparatus used to plot equipotential surfaces and electric field lines. On the conducting paper are two parallel electrodes connected to a DC power supply via wingbolts. The low-potential side of the multimeter is connected to the low-potential electrode. By moving the high-potential probe across the paper, equipotential surfaces can be identified.

While some instructors have students measure the electric field with fixed distance voltmeter probes,[6] we are presenting the more traditional method that focuses on equipotentials. The students are asked to map out the equipotential lines on the paper corresponding to 1.0 V, 2.0 V, … , 9.0 V using a multimeter probe. Once students have mapped these lines, they proceed to draw the electric field lines based on the principle that these lines must be perpendicular to the equipotentials and that the field lines are directed from high to low potential.

Existing laboratory manuals often include electrostatic experiments with conductive paper, but they typically only suggest configurations such as two small circular electrodes, illustrating the equipotentials and field lines of a dipole, and two parallel lines, illustrating the equipotentials and field lines between the parallel plates of a capacitor.[7,8] It is worth noting that the electric field of the circular electrodes resembles that of a three-dimensional cylinder rather than a sphere.[9] In the following sections, we present new electrode configurations that illustrate the electrostatic behavior inside conductors, at their surfaces, and within cavities.

Point charge and conducting shell

The simple configuration of a point charge outside of a spherical conducting shell can illustrate the principle of the shielding of the electrostatic field by a conductor. The point

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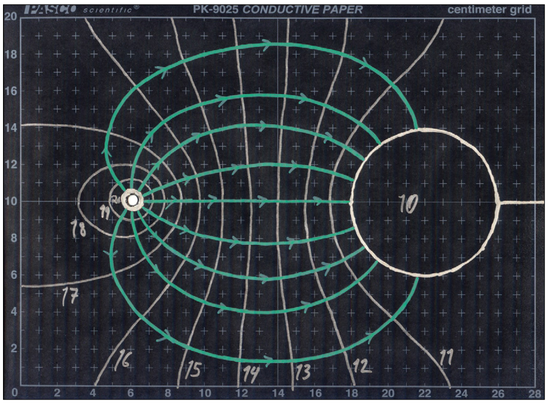

Fig. 2. Point charge and conducting shell configuration. The equipotential surfaces are drawn in silver and labeled in volts. The entire region inside the conducting shell is at the same potential, which indicates that there is no electric field. The electric field lines, which were sketched such that they are perpendicular to the equipotential surfaces, are drawn in green.[12]

charge is represented by a circular dot at a potential of 20 V. The conducting shell is the closed circle, which is kept at a lower potential, say 10 V. Measurements of the potential are made both outside and inside the circle. The results are shown in Fig. 2. Note that near the edges of the paper the equipotentials are non-ideal due to the finite size of the conducting paper.[10]

Outside the shell, the mapped equipotential lines result in electric field lines that emanate from the point charge and end at the lower-potential circle. Within the circle, all points are measured to be 10 V, so the electric field is zero in this region. This demonstrates to the students that the electric field of the point charge does not penetrate the interior of the conducting circle, and that a second point charge within the circle would not be subject to an electric force due to the charge on the outside. Students can also observe that the electric field is perpendicular to the surfaces, which is especially easy to see in the larger conducting shell.

Lightning rod

An important property of a conductor in electrostatic equilibrium is that the surface charge density, and therefore the electric field, is largest at the sharpest parts of the conductor. A prime example of this fact is the lightning rod, which is modeled by the configuration shown in Fig. 3. The horizontal line at the top represents a charged cloud in the atmosphere, and it is kept at 10.0 V. The vertical line represents the lightning rod and is kept at 0 V.

Near the horizontal conductor (cloud), the equipotentials are evenly spaced. Near the tip of the lightning rod, however, they are much more closely spaced. When the students map the electric field lines, they observe that the electric field is strongest at the point on the lightning rod closest to the cloud, and perpendicular to the surface. Compared to the prior configuration, this one more clearly illustrates that the charge density on a conductor can be non-uniform.

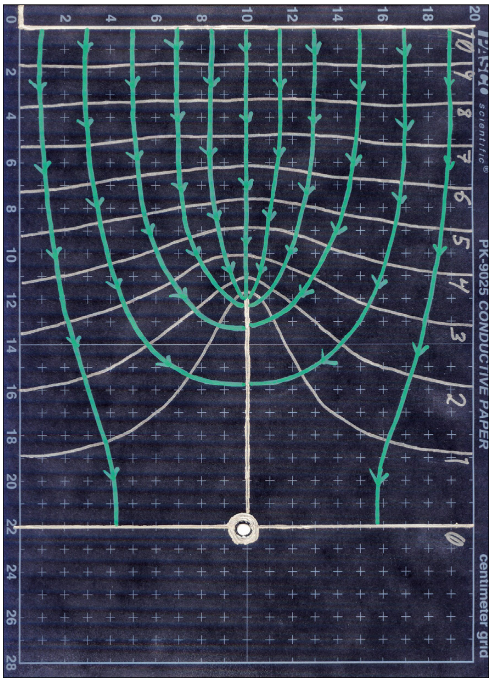

Fig. 3. Lightning rod configuration. The electric field lines, shown in green, are perpendicular to the electrodes and concentrated near the tip of the "lightning rod." This concentration implies that the charge is greatest in this region.

Point charge within a conducting shell

A common example used during the discussion of the electrical properties of conductors and/or Gauss's law involves a point charge (say $+Q$) at the center of a charged conducting shell.[11] Since the electric field is zero within the material of the conducting shell, the students rightly conclude that the amount of charge on the inner shell must be $-Q$ by drawing a circular Gaussian surface in the conducting shell. If the net charge on the shell is given, say $+Q$, then the amount of charge on the outer surface of the shell is the difference between the net charge and the charge on the inner surface, or $+2Q$ in this case. Because of the symmetry of the system, the students all conclude, correctly, that the charge densities on the surfaces of the conducting shell are uniform.

As a follow-up to the discussion of this example, the students are asked to describe the effect of displacing the point charge away from the center. They see that the system has lost its symmetry since the point charge is now closer to one edge of the shell. Most will respond intuitively and incorrectly guess that the charge densities on both the inner *and* outer surfaces of the shell are non-uniform. As discussed earlier, this leads to the very common, incorrect response that the field outside of the shell is the same as that of the off-center point charge.

We have developed an electrode configuration that the students can use to investigate this problem. Figure 4(a) shows a conductive sheet with a point charge (the dot) at the center of a conducting shell. The shell is represented by two concentric conducting circles that are electrically connected by a line drawn on the sheet. This innovation allows students

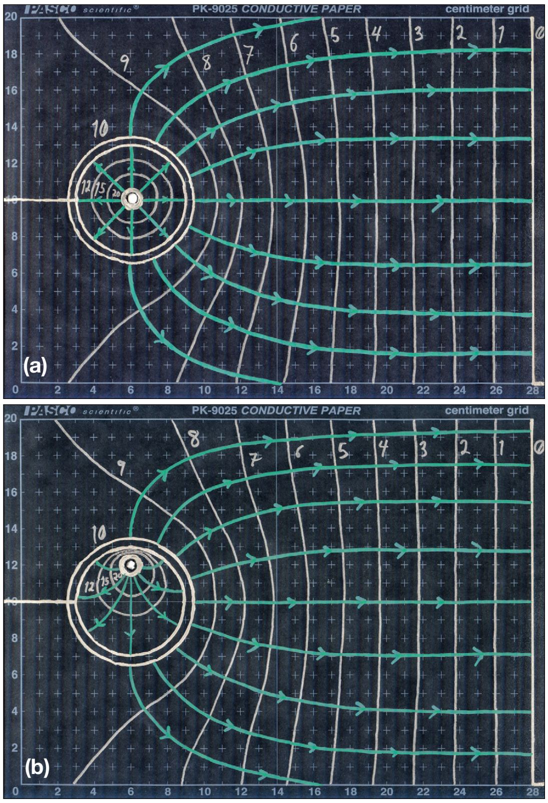

Fig. 4. Configurations for a point charge within a conducting shell: (a) at the center and (b) off-center. The potential inside the shell, between the concentric circular conductors, is constant, implying no electric field. In both configurations, the equipotential and electric field lines outside of the conducting shell are the same despite the difference in charge location. This is unsurprising to students in the symmetric case but initially non-intuitive in the asymmetric case.

to measure the potential inside the shell, between the circles. In the case shown, the point charge is at a potential of 20 V and the shell at 10 V. In order to investigate the electric field outside the shell, we draw a conducting line at the far edge of the sheet that is grounded. As expected, the mapped equipotentials and the electric field lines exhibit cylindrical symmetry both inside and outside the conducting shell. All points between the inner and outer circles are at 10 V, so the electric field is zero in the region within the conducting shell.

In the second part of the experiment, the students are asked to map the equipotentials and the electric field lines on a conductive sheet where the point charge is displaced from the center of the shell. The results are shown in Fig. 4(b). In the region enclosed by the shell, the electric field is no longer symmetric, and is strongest where the distance between the point charge and the inner surface of the shell is smallest. This illustrates that the charge density on the inner surface of the shell is indeed not uniform, but is greatest near the point charge. In the region between the two circles, the electric field remains zero as in Fig. 4(a) since the electric potential is constant. Hence the students can see that although the point charge is moved off-center, there is no subsequent effect on the charge density on the outer surface of the shell, which remains uniform. This point is further illustrated when the equipotential surfaces and the electric field are mapped outside the shell. The field external to the shell is unchanged from that of Fig. 4(a) and remains radially symmetric. It is equivalent to the electric field of a point charge at the center of the configuration even though the point charge enclosed by the shell is off-center.

Discussion

We have observed that students are better able to identify the correct electric field and potential differences in configurations with conductors after the hands-on activities. For example, many students are surprised to learn that the electric field outside the conducting shell is unaffected when the point charge is moved from the center of the shell. This experiment forces students to directly confront common misconceptions about the distribution of charge on a conductor. While not all students correctly identify the electrostatic properties after one lab period, they have now developed questions and are eager for follow-up discussions in class. After the sequence of electrode configurations, students report an increase in their confidence in identifying the electric field and potential.

References

1. R. L. C. Hazelton, M. R. Stetzer, P. R. L. Heron, and P. S. Shaffer, "Investigating student ability to apply basic electrostatics concepts to conductors," *AIP Conf. Proc.* **1513**, 166 (2013).
2. T. B. Jones and D. C. Vacanti, "Conducting-paper—Liquid-crystal fields displays," *Am. J. Phys.* **41**, 1314 (Dec. 1973).
3. W. B. Lynch, "Electric field plotting apparatus," *Phys. Teach.* **19**, 342 (May 1981).
4. R. B. Khaparde and H. C. Pradhan, "An experiment on equipotential curves," *Phys. Educ.* **27**, 27 (2010).
5. From PASCO scientific, 10101 Foothills Blvd., Roseville, CA 95678-9011 or http://www.pasco.com (PASCO PK-9025).
6. D. O. Ludwigsen and G. N. Hassold, "A simple electric field probe in a Gauss's law laboratory," *Phys. Teach.* **44**, 470 (Oct. 2006).
7. D. Loyd, *Physics Laboratory Manual*, 4th ed. (Cengage Learning, 2008).
8. J. D. Wilson and C. A. Hernández-Hall, *Physics Laboratory Experiments*, 8th ed. (Cengage Learning, 2015).
9. M. Lietz, "A potential Gauss's law lab," *Phys. Teach.* **38**, 220 (April 2000).
10. E. Ayars, "Nonphysical results with the electric-field mapping experiment," *Phys. Teach.* **34**, 344 (Sept. 1996).
11. R. A. Serway and J. W. Jewett, *Physics for Scientists and Engineers*, 7th ed. (Thomson Brooks/Cole, 2008), p. 684.
12. When students perform the experiment, they usually draw the equipotentials and field lines using pencil. For ease of viewing, we have traced over the pencil marks using a silver marker for the equipotentials and a green paint pen for the electric field lines in Figs. 2 to 4, which depict the configurations utilized by students in their investigation.

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