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Distributing Indivisible Goods Fairly: Evidence from a Questionnaire Study*

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Abstract. We report the results of a questionnaire study on the fair distribution of indivisible goods. We collected data from three different subject pools, first- and second-year students majoring in economics, law students, and advanced economics students with some background knowledge of fairness theories. The purpose of this study is to assess the empirical relevance of various fairness criteria such as inequality aversion, the utilitarian principle of maximizing the sum of individual payoffs, the Rawlsian “maximin” principle of maximizing the payoff of the worst-off individual, and the criterion of envy-freeness (in the sense of Foley, 1967).

1. Introduction

The question of which allocations are perceived by individuals as fair, or just, has received considerable attention in recent years. The interest in this question has been further sparked by the extensive experimental literature in economics demonstrating that individual behavior is not only guided by the pursuit of material self-interest, but to a significant extent also by fairness considerations (see, among many others, Fehr and Schmidt 1999, Bolton and Ockenfels 2000, Charness and Rabin 2002, Camerer 2003, and Engelmann and Strobel 2004). But what do people consider to be fair? There are several dimensions in which one can expect different answers to that question. First, individual perceptions of fairness are known to be context-dependent. For instance, fairness plays a fundamentally different role in strategic settings versus market settings, in allocation problems with indivisible goods versus divisible goods, in situations with “strangers” versus “partners,” etc. Secondly, fairness perceptions vary across individuals, depending, among other things, on gender (see, e.g., Andreoni and Vesterlund 2001), educational background (see, e.g., Marwell and Ames 1981), or cultural background (see, e.g., Henrich et al. 2001). In this paper, we address the issue of fairness in a non-strategic context with indivisible goods. We report the results of a questionnaire study conducted in three classes with different students at the University of Bonn in 2001. The students were lower-level economics students (first and second year), law students, and more advanced economics students attending a course on “Theories of Distributive Justice” held by one of us. The students were given time at the end of the three classes to do the questionnaire – all students completed the questionnaire latest by the end of class time (they were free to leave once they were done). Participants were instructed to not communicate with each other whilst working on the questionnaire. The number of participants was 58 in the advanced economics course, 158 in the lower-level economics course (“Mathematics for Economists”), and 51 in the law course. In total 267 individuals participated and filled out the questionnaire.

The questionnaire contained 10 hypothetical allocation problems with two or three individuals.¹ For each allocation problem, the subjects were asked to determine the distribution they personally considered to be the fairest. We stressed that there is no right or wrong answer to this problem and emphasized the normative content of the question. With the exception of two allocation problems, we explicitly asked the subjects to take on the role of an impartial arbiter; in the last two allocation problems, subjects were asked to take on the role of one particular concerned individual (see Appendix 1 for the exact wording of the questionnaire).

In each allocation problem, there were between three and six indivisible goods that could be given to any of the individuals, or not distributed at all. In three allocation problems there was an additional divisible amount of money that could be distributed. Each good could be given to at most one individual, whereas money could be split in any way between individuals or thrown away. An important feature of our allocation problems is that the same good generally has different values for different individuals. This feature is crucial in order to test the criterion of envy-freeness in the sense of Foley

¹ The law students were given only 8 of the 10 allocation problems (see Section 2 below).

(1967), according to which an allocation is envy-free if no individual would like to swap his or her bundle for the bundle of anybody else.

The main purpose of the questionnaire study reported here is to determine the empirical relevance of competing fairness criteria such as envy-freeness, inequality aversion, the utilitarian principle, and the Rawlsian maximin solution.² At times, some of these criteria can be in conflict with Pareto optimality, and we were also interested in finding out how this particular conflict would be resolved. While often criticized for its lack of providing sufficient incentives for subjects, the questionnaire approach has some tradition in the fairness literature, see Yaari and Bar-Hillel (1984), Gaertner et al. (2001, 2002) for related studies, and Gaertner (2006, Chapter 9) and Konow (2003) for overviews on empirical fairness studies more generally. In our context, the absence of such incentives is mitigated by the fact that individuals were put in the position of impartial arbiters who do not directly benefit from the individuals' payoffs anyway.

Our motivation for introducing different subject pools was to investigate whether economics students respond differently to fairness issues than law students, and more importantly, whether an exposure to theoretical economic research on justice has an impact on what subjects consider to be fair.³

The present study complements our previous experimental work in this area, specifically Herreiner and Puppe (2007), where the criterion of envy-freeness has been experimentally investigated for the first time, and Herreiner and Puppe (2006), where we study the possible trade-off between fairness and Pareto optimality in laboratory experiments. In the latter paper, we explain individual behavior by means of a simple procedure according to which bargainers first determine an “egalitarian reference distribution” and then choose a Pareto improvement, provided that this does not result in too much inequality. Our results concerning the relevance of envy-freeness in the context of laboratory experiments are mixed (see Herreiner and Puppe 2007 for details). That envy-freeness has empirical relevance in certain contexts is demonstrated in Herreiner (2007), also using the questionnaire method.

The remainder of this paper is organized as follows. In the next section, we present the allocation problems and list the allocations chosen most frequently by the subjects. In Section 3, we use our data to test whether the economics students make different choices than the law students, and whether an exposure to fairness theories alters attitudes towards fairness criteria. Although not in all problems, in general, law students seem to be more prone to pick Pareto optimal and potentially less equal allocations than economics students.

We emphasize that the present questionnaire approach is *explorative* in the sense that we do not test specific hypotheses on the behavior of individuals. Rather, we hope that the data and the results will be used and analyzed further to develop a sense of what kind of fairness criteria matter in what kind of allocation problems.

² For overviews of the theoretical literature on fair allocations, see e.g. Brams and Taylor (1996), Thomson (2005) and Young (1994).

³ The upper-level students attending the course on distributive justice had a previous exposure to the basic fairness concepts considered here in the context of an exchange economy with divisible goods. The aspects arising from the indivisibilities were new to them. The other students had no previous exposure to fairness concepts in economics and theories of distributive justice.

2. The Allocation Problems and Most Frequent Choices

We now present and discuss the allocation problems presented to the subjects. In order to facilitate the assessment of our results, we provide a summary of the most frequent choices directly after the description of each problem. We present and comment on only what we consider to be the most interesting and striking results – all allocation choices reported in the tables below are significantly different from random choices.⁴ The complete data set is provided in Appendix 2 below, and we invite the interested reader to have a closer look there. Unless stated otherwise, all numbers and percentages refer to the choices aggregated over all three subject pools, i.e. lower- and upper-level economists and law students.⁵

Problem 1

The first problem involved three hypothetical goods, labeled A, B and C, and two individuals, labeled I and II. The following table gives the value of the goods for each individual. Good A is worth 49 units to individual I and 47 units to individual II. Similarly, good B is worth 46 to individual I and 48 to individual II. Finally, good C is worth 5 to both individuals.⁶ Each good can be given to at most one individual. Subjects had the opportunity to “throw away” goods, i.e. not to allocate goods to either individual.

PROBLEM 1

	A	B	C
I	49	46	5
II	47	48	5

Since each good can be given to either individual or not be distributed, there are thus in total $3^3 = 27$ different allocations. Clearly, efficiency requires that all goods be distributed.

The distribution problem reflects a simple but basic conflict between payoff equality and efficiency, or alternatively, between envy-freeness and efficiency. The “most equal” allocations in which all goods are distributed are the allocation (A,BC), i.e. good A goes to individual I, and goods B and C go to individual II, with a resulting payoff vector (49,53), and the allocation (BC,A) with a resulting payoff vector (51,47). Both allocations display an interpersonal payoff difference of four units. Of these two allocations, only the first is Pareto optimal, since the allocation (AC,B) with payoff vector (54,48) Pareto dominates the allocation (BC,A). All allocations in which all three

⁴ Based on χ^2 -tests considering all allocations and subsets of the most frequently chosen allocations; all respective p-values are so close to zero to be significant at all relevant significance levels. The only exception to this is Problem 2 when considering only the two most frequently chosen allocations.

⁵ The figures below always give the percentages in relation to *all* observations including invalid and no answers.

⁶ Note that the values in the rows add up to 100. This is true for all allocation problems considered here. Our reason for designing the problems in this way was to create a kind of ex-ante symmetry in the sense that the bundle consisting of all goods should have the same value for all individuals.

goods are distributed involve *envy* in the sense that at least one individual would be better off with the bundle of the other individual.⁷ For instance, individual I is envious of individual II under the allocation (A,BC), since with the bundle BC individual I would receive a payoff of 51 (instead of the payoff 49 resulting from good A). The only way to avoid envy in this sense is to not distribute at least one good. A natural way to create envy-freeness is to dispose of good C and to give the two other goods to the individuals who value them most, yielding the allocation (A,B) with payoff vector (49,48). Note that the resulting payoff difference of one unit is smaller than the smallest payoff difference in all Pareto optimal allocations.

The following table indicates the most frequently chosen allocations and shows their relevant properties (EF stands for envy-free, U for the utilitarian solution, MM for the Rawlsian maximin solution, and IA for the allocation that minimizes the (maximal) difference between individual payoffs).

Allocation	Payoff vector	Choices in %	Pareto optimal	Fairness criteria
(A,B)	(49,48)	70.4	No	EF, IA
(A,BC)	(49,53)	23.2	Yes	U, MM

Results for Problem 1

Thus, more than 70% of the subjects proposed to dispose of good C and thus opted for a Pareto inferior allocation. Whether this behavior was guided by inequality aversion or envy considerations cannot be decided using our data; see Problem 4 for a related discussion. Our results in Herreiner and Puppe (2007) suggest that interpersonal equality is empirically more relevant than the more complex intrapersonal criterion of envy-freeness. Efficiency considerations are certainly not completely ignored, as can be seen from the fact that the overwhelming majority of the subjects that distributed two or three goods allocated goods A and B according to their maximal value (giving good C to the person with the lower payoff in the latter case).⁸

Problem 2

The second problem involved four goods to be distributed among three individuals.

⁷ Formally, an individual is *envious* at a certain allocation if he or she would be better off with the bundle allocated to some other individual than with his or her own bundle. An allocation is *envy-free* if no individual is envious.

⁸ All allocations not shown in the table were chosen with a frequency of less than 2%.

PROBLEM 2

	A	B	C	D
I	5	47	45	3
II	45	5	48	2
III	23	25	32	20

A distinctive feature of this division problem is that there is the possibility to create a perfectly egalitarian distribution via the allocation (C,A,BD) in which individual I receives good C, individual II receives good A, and individual III receives goods B and D. The resulting payoff vector is (45,45,45). As in the first problem, there is a conflict between inequality and efficiency, but now efficiency is compatible with envy-freeness. First, note that the egalitarian allocation (C,A,BD) is neither Pareto efficient, nor envy-free. The allocation (B,A,CD) with a resulting payoff vector (47,45,52) Pareto dominates (C,A,BD); moreover, under the egalitarian allocation individual I envies individual III and individual II envies individual I. The allocation (B,A,CD) is itself Pareto efficient but not envy-free, since both individuals I and II envy individual III. There is a Pareto efficient and envy-free allocation in this example, namely (B,C,AD) with the resulting payoff vector (47,48,43). Note that this allocation is not Pareto superior to the egalitarian distribution and does therefore not respect the CPIES criterion (“Conditional Pareto Improvement from Equal Split”) suggested in Herreiner and Puppe (2006).⁹

In this example, there is another noteworthy contender in terms of fairness, namely the “utilitarian” allocation (B,AC,D) which maximizes the sum of the payoffs. The resulting payoff vector is (47,93,20). By definition, the utilitarian allocation is Pareto efficient. As the present example shows, it may result in significant inequality across individuals. The utilitarian distribution obviously emerges by giving each good to the individual who values it most.

The following table gives the most frequent choices. All allocations not shown in the table were chosen with a frequency of less than 5%.

Allocation	Payoff vector	Choices in %	Pareto optimal	Fairness criteria
(C,A,BD)	(45,45,45)	26.2	No	IA, MM
(B,C, AD)	(47,48,43)	26.2	Yes	EF
(B,A,CD)	(47,45,52)	12.7	Yes	MM
(B,AC,D)	(47,93,20)	9.0	Yes	U
(B,A,C)	(47,45,32)	7.9	No	-

Results for Problem 2

⁹ The term “conditional” refers to the fact that only those Pareto improvements which do not involve too much inequality may qualify as “fair.”

As can be seen from the table, the egalitarian and the Pareto optimal envy-free allocation tie for the most frequently chosen allocation. The third most chosen allocation is a Pareto improvement from the equal distribution and therefore respects the fairness criterion proposed in Herreiner and Puppe (2006).

Problem 3

The third problem involved five goods to be distributed among three individuals. Problems 3 and 5 below were the most complex problem that we presented to our subjects.

PROBLEM 3

	A	B	C	D	E
I	40	2	3	25	30
II	14	26	8	26	26
III	10	26	26	12	26

The distinctive feature of this problem is that there are two payoff-equivalent Pareto efficient allocations, of which only one is envy-free. These two allocations are (A,BD,CE) and (A,DE,BC). The resulting payoff vector is (40,52,52) in both cases. The first allocation is envy-free, but under the second allocation individual I envies individual II.¹⁰ While envy-freeness can always be achieved by disposing goods (for instance, trivially, by disposing all goods), the above allocation (A,BD,CE) is the only envy-free *and* Pareto optimal distribution in this example. The two allocations above are also the two allocations that maximize the payoff of the worst-off individual. In other words, they both represent the “Rawlsian” solution in this example. There are two utilitarian allocations, namely (AE,BD,C) with payoff vector (70,52,26) and (AE,D,BC) with payoff vector (70,26,52).

Problem 3 was not presented to the law students. The total number of observations was therefore $58 + 158 = 216$. In this problem, we observed many different allocations. The following table only shows the two most frequent choices. These are the two payoff-equivalent Rawlsian solutions, of which only one is envy-free. The envy-free variant is chosen more than twice as frequently.¹¹ All other allocations were chosen with a frequency of less than 10%.¹²

¹⁰ Since envy-freeness is the only fairness criterion that can distinguish the two allocations, examples like this one are particularly suitable as “pure” tests of envy-freeness, i.e. of the relevance of envy-freeness independently of any other fairness considerations. Herreiner (2007) provides detailed results and tests based on an independent and richer data set.

¹¹ This is in line with the findings of Herreiner (2007), where it is shown that the envy-free allocation is chosen significantly more often in a number of similar allocation problems.

¹² Other choices were (A,BC,DE) at 9.7%, (A,D,C) at 7.9%, and (AE,D,C) at 5.1%. The two utilitarian allocations were chosen with a total probability of 7.0%.

Allocation	Payoff vector	Choices in %	Pareto Optimal	Fairness criteria
(A,BD,CE)	(40,52,52)	27.8	Yes	EF, MM
(A,DE,BC)	(40,52,52)	12.5	Yes	MM

Results for Problem 3

Problem 4

The fourth problem is a three-person variant of the conflict between equality and efficiency, respectively envy-freeness and efficiency, already addressed in Problem 1 above.

PROBLEM 4

	A	B	C	D
I	30	31	32	7
II	33	29	31	7
III	31	32	30	7

The allocation (C,A,B) is envy-free with resulting payoff vector (32,33,32). Clearly, since good D is not distributed, this allocation is not Pareto optimal. There is also a perfectly egalitarian solution, (B,C,A) with payoff vector (31,31,31), which is, however, Pareto dominated by the envy-free allocation (C,A,B). There are three utilitarian solutions, (CD,A,B), (C,AD,B), and (C,A,BD), which differ only by who receives good D. Note that all three of these, as well as the above envy-free allocation, involve a minimal individual payoff of 32.

In terms of the conflict between Pareto optimality and equality/ envy-freeness, the results of Problem 4 are very similar to Problem 1: An overwhelming majority of subjects is willing to sacrifice “some” efficiency for fairness in the sense of equality, or of envy-freeness. In Problem 4 subjects are willing to sacrifice efficiency significantly more frequently ($P=0.00004$ based on a χ^2 -test) by throwing away one good, C and D respectively. Of the relevant choices, in Problem 4 the additional good is thrown out in 89% of the cases, whereas it is thrown out “only” in 77% of the choices in Problem 1. In Problem 4 the additional good has greater value and can therefore create more inequality if assigned to one of the individuals. Given the choices shown in the table below, one could argue that sacrificing efficiency in Problem 4 supports envy-freeness more than inequality aversion. However, (C,A,B) also Pareto dominates the perfectly egalitarian

allocation (B,C,A) at a minimal increase of inequality. All allocations not shown in the table were chosen with a frequency of less than 5%.¹³

Allocation	Payoff vector	Choices in %	Pareto optimal	Fairness Criteria
(C,A,B)	(32,33,32)	64.4	No	EF, MM
(B,C,A)	(31,31,31)	16.5	No	IA

Results for Problem 4

Problem 5

The fifth problem was the most complex and involved six goods, as follows.

PROBLEM 5

	A	B	C	D	E	F
I	5	20	32	3	25	15
II	26	7	23	20	2	22
III	24	17	6	21	30	2

Again, there is an equality-efficiency conflict. The allocation (BE,CF,AD) results in the perfectly egalitarian payoff distribution (45,45,45), but even though all goods are distributed this allocation is neither Pareto optimal nor envy-free. Indeed, at the egalitarian allocation individual I envies individual II, individual II envies individual III who, in turn, envies individual I. There is thus a feasible exchange of bundles that would make everyone better off, yielding the allocation (CF,AD,BE) with payoffs (47,46,47); hence, the first allocation is not Pareto optimal. Besides the perfectly egalitarian distribution, the other focal allocation is (BC,AF,DE) yielding the payoff vector (52,48,51). This allocation is the utilitarian solution, and is thus Pareto optimal. It also represents the Rawlsian solution, is envy-free, and Pareto dominates the other two allocations. There are also other Pareto optimal and envy-free allocations in this example.

The following table gives the four allocations that were chosen with a frequency of more than 5%. A total of 58.3% of the subjects opted for a Pareto improvement relative to the perfectly equal distribution (the first and the third allocation in the table), confirming the analysis of Herreiner and Puppe (2006). Clearly, the payoff inequality at allocation (BC,AF,DE) is sufficiently small so that participants chose that allocation much more frequently than the more equal Pareto improvement (CF,AD,BE). Note that in the fourth allocation in the table, every individual only receives his or her most

¹³ The three utilitarian allocations together were chosen by slightly more than 10% of the subjects. There were no significant differences in the probability of each of these being chosen ($P=0.3679$ based on a χ^2 -test).

valuable single good. This problem was the second problem not presented to the law students.

Allocation	Payoff vector	Choices in %	Pareto Optimal	Fairness Criteria
(BC,AF,DE)	(52,48,51)	50.0	Yes	EF, U, MM
(BE,CF,AD)	(45,45,45)	9.3	No	IA
(CF,AD,BE)	(47,46,47)	8.3	No	EF
(C,A,E)	(32,26,30)	6.9	No	EF

Results for Problem 5

Problem 6

The sixth problem is similar to Problem 2, and related to Problem 5 above.

PROBLEM 6

	A	B	C	D
I	48	4	3	45
II	25	20	40	15
III	2	1	45	52

As in Problems 2 and 5 above, there is a perfectly egalitarian allocation, (D,AB,C) with payoff vector (45,45,45), which is not envy-free and Pareto dominated by the Rawlsian solution (A,BC,D) which is Pareto optimal with payoff vector (48,60,52).¹⁴ As an important difference to Problem 2, here the Rawlsian solution does not coincide with the utilitarian solution (A,B,CD) with payoff vector (48,20,97). Moreover, in contrast to Problem 5, both the Rawlsian and the utilitarian solution involve significant individual payoff differences.

Although in this problem the Rawlsian solution is also envy-free, the Pareto dominated egalitarian allocation is chosen significantly more often, as shown in the following table. Despite its considerable payoff differences the utilitarian allocation is still chosen with probability 7.9%. All allocations not shown in the table were chosen with a frequency of less than 3%.

¹⁴ In Problem 2, the egalitarian allocation (C,A,BD) with payoff vector (45,45,45) is Pareto dominated by (B,A,CD) with payoff vector (47,45,52). While both involve a minimal payoff of 45, the latter represents the lexicographic refinement of the “Rawlsian” solution. What we call the “Rawlsian” solution is sometimes referred to more precisely as the “leximin” solution.

Allocation	Payoff vector	Choices in %	Pareto Optimal	Fairness Criteria
(D,AB,C)	(45,45,45)	32.6	No	IA
(A,BC,D)	(48,60,52)	28.1	Yes	EF, MM
(A,C,D)	(48,40,52)	18.4	No	EF
(A,B,CD)	(48,20,97)	7.9	Yes	U

Results for Problem 6

Problem 7

In addition to the indivisible goods, problem 7 (and two other problems below) also involved a divisible amount of money that could be distributed among the individuals. In this example, the amount was given by $M=5$ units. Participants were told that money is comparable to the value of the goods, so that a good with a certain value should be considered equivalent to the corresponding amount of money.

PROBLEM 7

	A	B	C	MONEY AMOUNT
I	45	30	25	M=5
II	35	40	25	
III	50	5	45	

Our reason to introduce the money amount was to test whether subjects would use the money to compensate for envy, or alternatively, for inequality. The two focal allocations are (A,B,C) with the money given to individual II and the same distribution (A,B,C) but with money going to individual III. The first allocation yields the perfectly equal payoff vector (45,40+5,45); it is also Pareto optimal, but individual III envies individual I. The second allocation results in the payoff vector (45,40,45+5) and is envy-free and also Pareto optimal.

These two allocations were chosen the most frequently. More than 80% of the subjects chose the goods allocation (A,B,C). Of these 223 subjects, a majority of 65.9% allocated the 5 units of money to individual II, thus creating the egalitarian payoff vector (45,40+5,45). Another 15.2% of these 223 subjects allocated the 5 units of money to individual III, thus compensating envy.

Problem 8

The eighth problem is similar in structure to Problem 2 and the same as Problem 6 with goods A and D swapped, however here there are in addition 7 units of money to be distributed.

PROBLEM 8

	A	B	C	D	MONEY AMOUNT M=7
I	45	4	3	48	
II	15	20	40	25	
III	52	1	45	2	

There are two focal allocations of the goods. The allocation (A,BD,C) results in the egalitarian payoff vector (45,45,45). Given this, the crucial question is again how the money is used. Note that the 7 monetary units are just enough to compensate individual III for his/her envy (since individual III would receive a payoff of 52 from good A). On the other hand, Pareto optimality and envy-freeness can be achieved by the goods allocation (D,BC,A) together with any distribution (x_1, x_2, x_3) of the money such that $x_1 + x_2 + x_3 = 7$ and $x_1 \geq x_3 - 3$.

The following table indicates the five most frequently chosen goods allocations, irrespective of the money allocation. Since the properties of the entire allocation (of the goods and the money) also depend on how the money was distributed, we cannot list the respective fairness criteria in the table.¹⁵

Goods Allocation	Payoff vector (based on goods only)	Choices in %	Pareto Optimal
(D,BC,A)	(48,60,52)	32.2	Yes
(A,BD,C)	(45,45,45)	22.5	No
(D,C,A)	(48,40,52)	16.9	No
(D,B,AC)	(48,20,97)	9.4	Yes
(BD,C,A)	(52,40,52)	7.5	depends

Results for Problem 8

Thus, 32.2% of the subjects chose the goods allocation (D,BC,A) which is Pareto optimal (if no money is thrown away) and envy-free, while only 22.5% chose the “egalitarian” goods allocation (A,BD,C). Another 16.9% chose the goods allocation (D,C,A) which gives each individual the single good he or she values most. The utilitarian goods allocation (D,B,AC) was chosen by 9.4%, and the allocation (BD,C,A) which equalizes the payoffs of individuals I and III by 7.5%. All other goods allocations were chosen with frequency of less than 5%.

¹⁵ Whether an allocation is Pareto optimal of course also depends on the money distribution. A “Yes” in the right column only presupposes that no money is thrown away. A “No” means that for no money distribution the relevant allocation is Pareto optimal. Whether the fourth allocation in the table is Pareto optimal depends non-trivially on the money distribution.

In 87% of the choices of the goods allocation (D,BC,A) all money was distributed, whereas this is true of only 27% of the choices of the goods allocation (A,BD,C) (see the data summary in the appendix). In the latter case, more than half of the participants either gave two units of money to everybody¹⁶ or nothing. Money thus seems to matter much less if the goods allocation alone yields equal payoffs, whereas it matters if it can be used to make payoffs more equal. Among the (D,BC,A) allocation choices almost all address the payoff inequalities through money: more than half gave the largest amount to the first individual, the second most to the third individual, and the least to the second individual. Another 28% distributed the money such that the second individual with the highest payoff from the goods received the least. For 97% of the (D,BC,A) choices money was allocated also in a manner that guaranteed envy-freeness.

A comparison of Problems 6 and 8 shows that here the Pareto optimal and envy-free allocation is chosen significantly more frequently whereas in the identical Problem 6 the egalitarian allocation is chosen more frequently (P-value of 0.0204 based on a χ^2 -test). The fact that money can be used to address some of the “inequality shortcomings” of the Pareto optimal allocation seems to be sufficient to reverse the choice frequencies. The utilitarian allocation, however, is chosen with comparable frequencies in both problems. The inequalities inherent in that goods allocation are too large to be mitigated by the relatively small amount of money available in Problem 8.

Problem 9

The final two problems differ from the rest since they abandon the neutral framing. Instead of suggesting an allocation for two or three *other* individuals, subjects were now asked to imagine themselves in the role of one of the concerned individuals (see Appendix 1 for the exact formulation). The ninth problem was given as follows.

PROBLEM 9

	A	B	C	D
YOUR VALUATION	23	40	20	17
2. PLAYER	2	43	1	54
3. PLAYER	49	4	4	43

Except for the non-neutral framing, this problem is structurally the same as Problem 6 above.¹⁷ There is a perfectly egalitarian allocation, (AC,B,D) with payoff vector (43,43,43), at which both individuals II and III (but not the person who proposes the distribution) are envious. The Rawlsian solution (BC,D,A) with payoff vector (60,54,49) is envy-free and also Pareto optimal.

The following table gives the three most frequent choices (all other allocations were chosen with frequency less than 5%).

¹⁶Making the perfectly equal payoff vector (47,47,47) the most frequently chosen single allocation of the goods *and* the money.

¹⁷The problems are the same for all relevant fairness criteria. Specifically, up to a payoff difference of two units, Problem 6 is obtained from Problem 9 by swapping goods B and C, and permuting individual preferences.

Allocation	Payoff vector	Choices in %	Pareto optimal	Fairness criteria
(BC,D,A)	(60,54,49)	34.1	Yes	EF, MM
(AC,B,D)	(43,43,43)	30.0	No	IA
(B,D,A)	(40,54,49)	17.6	No	EF

Results for Problem 9

The choices here are not significantly different from those of the similar Problem 6 (P-value of 0.2068 based on a χ^2 -test) contrary to Problem 8, although here the Pareto optimal allocation is chosen more frequently than the egalitarian allocation (contrary to Problem 6). The individual making the choice is the one that benefits most from the Pareto optimal allocation. However, as our results show this framing aspect is not as important as supplementing the allocation problem with a money amount to address the inequality in order to make the Pareto optimal allocation attractive.

Problem 10

The tenth and final problem combines the non-neutral framing of the preceding problem with the possibility to distribute a divisible amount of money. The problem is structurally similar to Problem 7 with individual I and III's preferences swapped.

PROBLEM 10

	A	B	C	MONEY AMOUNT
YOUR VALUATION	53	3	44	M=9
2. PLAYER	35	36	29	
3. PLAYER	44	30	25	

The two focal goods allocations are (C,B,A) and (A,B,C). The allocation (C,B,A) yields the egalitarian payoff vector (44,44,44) by giving 8 monetary units to individual II. The proposer (individual I), however, is envious under this allocation. On the other hand, individual I's envy at the allocation (C,B,A) could be compensated for by giving him or her the 9 monetary units (thereby creating envy on the part of individual II).

The allocation (A,B,C) gives both the proposer and individual II their most preferred good, but leaves individual III with envy even if he or she receives all the money.

In this problem, a clear majority of 198 subjects (74.2%) chose the goods allocation (C,B,A). The majority of them (61.6%) gave eight or nine monetary units to individual II, thus creating an egalitarian (or almost egalitarian) payoff distribution; 23 of the 198 subjects (11.6%) gave all the money to the envious individual I (the "proposer"). The goods allocation (A,B,C) was chosen by 31 subjects (11.6%); 20 of them gave more

money to individual III than to the other individuals, and another 9 of these subjects gave all the money to individual III. Thus, 29 of the 31 subjects who chose the goods allocation (A,B,C) at least partly compensated individual III for his or her envy.

Comparing the structurally similar Problems 7 and 10, in both cases over 80% of the subjects who chose the allocation that allowed to either address payoff inequality or envy (the goods allocation (A,B,C) in Problem 7, the goods allocation (C,B,A) here) opted to address primarily the inequality. Despite putting the deciding individual into the potentially envious position, envy was not compensated more frequently. As in Problem 9, putting an individual into a specific position in the questionnaire does not seem to influence choices significantly, in particular not to that individual's advantage. The share of individuals caring about equality as opposed to envy-freeness seems to be remarkably stable.

3. Relevance of Experience and Knowledge

We now investigate whether the different subject groups made consistently different choices. Comparing all three groups based on the most frequently chosen allocations yields the following results (the column “# alloc” gives the number of these allocations,¹⁸ and the column “# obs” their total frequency).

	# alloc	# obs	χ^2	P-value
Problem 1	2	250	0.53	0.7655
Problem 2	2	140	5.09	0.0785
Problem 3	2	87	0.86	0.3534
Problem 4	2	216	1.50	0.4723
Problem 5	3	146	3.15	0.2073
Problem 6	3	211	7.51	0.1114
Problem 7	2	181	0.16	0.9250
Problem 8	7	173	23.02	0.0276
Problem 9	3	218	6.22	0.1831
Problem 10	7	214	27.27	0.0071

Table 1: Differences in choice behaviour across all three subjects groups

Table 1 shows P-values from one-tailed χ^2 -test for three independent samples. Significant differences emerge only in Problems 2, 8, and 10 (at 10%, 5%, and 1% significance levels, respectively, and indicated in bold in the table). For Problem 2, the two most and equally frequently chosen allocations (C,A,BD) and (B,C,AD) that address payoff equality and Pareto optimality paired with envy freeness, respectively, are chosen in a

¹⁸ In Problem 7, the test was based on the allocations (A+0,B+5,C+0) and (A+0,B+0,C+5). In Problem 8 the following allocations were considered (A+x,BD+x,C+x), (BD+y,C+x,A+y), (D+y,BC+z,A+x), (D+x,BC+y,A+x), (D+x,BC+z,A+y), (D+x,C+z,A+y), and (D+0,C+7,A+0), where $x>y>z$. In Problem 10, the relevant allocations are (C+y,B+x,A+y), (C+x,B+y,A+y), (C+y,B+x,A+z), (C+x,B+x,A+x), (A+y,B+y,C+x), (C+x,B+y,A+z), (A+z,B+y,C+x). In the data summary in the appendix, the three groups are referred to as “Econ” (advanced-level economics students), “Math” (lower-level economics students since the questionnaire was conducted in their Math class) and “Law” (law students).

significantly different way by the three groups (10% significance level): Law students chose the Pareto optimal and envy-free allocation more frequently than the economics students, both at the lower and more advanced level.

The same emerges for Problems 8 and 10.¹⁹ These results suggest that there is a systematic difference between the choices by economics and law students, which we investigate directly next. The two groups of economics students did generally not exhibit significant differences in their choices for all problems.²⁰

The following table shows results that compare law students' choices with both economist groups combined (lower-level and advanced level students).

	# alloc	# obs	χ^2	P-value
Problem 1	2	250	0.17	0.6836
Problem 2	2	140	4.46	0.0346
Problem 4	2	216	0.01	0.9189
Problem 6	3	211	6.10	0.0474
Problem 7	2	181	0.04	0.8505
Problem 8	7	173	12.67	0.0485
Problem 9	3	218	2.62	0.2695
Problem 10	7	214	12.38	0.0540

Table 2: Differences in choice behaviour of law students vs. economics students

In four problems, choice behaviour was significantly different between law students and economics students at least at a 10% level (again in highlighted bold in the table). The earlier results are confirmed: Law students generally chose more frequently a Pareto optimal allocation whereas economics students chose relatively more frequently an allocation that equalizes payoffs. In Problem 2 the Pareto optimal and envy free allocation (B,C,AD) was chosen more frequently than the egalitarian allocation (C,A,BD) by law students – 68% of the two most chosen allocations as compared to 46% by economists. In Problem 6, the Pareto optimal and envy free allocation (A,BC,D) is chosen more frequently than the other two allocations, the egalitarian allocation (A,BC,D) and the envy free allocation (A,C,D), by law students (50% vs. 32% among those three allocations). In Problem 8, the goods allocation (D,BC,A) with the largest money amount to the first and/or third individual was chosen more frequently by law students than economics students (63% vs. 35%); economics students chose more frequently (BD,C,A) and (D,C,A), almost exclusively with the money given to the person with the lowest payoff. In problem 10, law students chose relatively more frequently the goods allocation (A,B,C) giving money to the individual(s) with the lowest payoff(s)

¹⁹ The significant results for Problems 8 and 10 reported in this section hold irrespective of what allocations exactly are considered, whether the ones considered here based on comparable payoff patterns, whether only the most frequently chosen specific allocations, or whether only the goods allocations.

²⁰ P-values for χ^2 -tests are generally above 0.20 and mostly substantially higher. Contrary to all other tests reported here, for Problems 8 and 10 the P-value depends on exactly which allocations are included and grouped together when comparing the two economist subject groups (in each case one combination makes the differences between the two economist groups just significant at 10%).

(28% vs. 11% among the 7 considered allocations), whereas economics students focused more on allocation (C,B,A) using the money to compensate the worst-off individual or distributing money equally.

Distinguishing the participants instead on the basis of prior exposure to envy-freeness, i.e. comparing advanced-level economics students to lower-level economics students and law students, based on a two-sample one-tailed χ^2 -test gives the following results:²¹

	# alloc	# obs	χ^2	P-value
Problem 1	2	250	0.23	0.6306
Problem 2	2	140	1.87	0.1719
Problem 4	2	216	1.45	0.2291
Problem 6	3	211	0.29	0.8632
Problem 7	2	181	0.08	0.7794
Problem 8	7	173	5.93	0.4313
Problem 9	3	218	3.56	0.1610
Problem 10	7	214	14.84	0.0216

Table 3: Differences in choices of experienced vs. non-experienced subjects

The only significant difference in choice behaviour emerges for Problem 10. Here, the experienced subjects (those that had heard of envy-freeness before participating in this questionnaire) chose the allocation (C,B,A) giving most of the money (21% vs. 12%) or more money to the envious than to the third person (19% vs. 6% of the 7 considered allocations) more frequently than the other participants. That pattern, however, does not emerge in the related Problem 7. Hence, introduction to the concept of envy-freeness has to be combined with personal exposure to envy, to lead an individual to choose an envy free allocation more frequently.

4. Conclusion

The analysis above focuses on the outcomes suggested by participants and therefore on the distributive fairness aspects considered. It discusses the tension between efficiency, equity considerations and other fairness criteria such as envy-freeness. It also shows that there are no significant differences between allocations chosen by individuals with different backgrounds; in particular, economics students are not more likely to choose efficient or envy-free outcomes than law students. There is one obvious difference between economists enrolled in a class on fairness and justice (“experienced subjects”) and all other participants (“non-experienced subjects”) – experienced subjects use standard economics terminology to explain their choices (although not always correctly), whereas other participants use terms like efficiency or envy only very sparingly.

²¹ Problem 3 and 5 were not given to the law students, the corresponding test is therefore the same as in Table 1 above, where all three groups are compared and where neither case showed any significant difference in choice behaviour.

Our analysis of the written explanations of participant choices yields two further interesting results. For one, individuals substantiate their choices mostly with procedural aspects.²² The most appealing procedures are based on assigning the goods to who values them most. Two versions of such a procedure exist; they do generally lead to different outcomes. The first possibility is to give each good to the individual that values it the most (with an appropriate tie-breaking rule). Alternatively, one could give each person the good that he or she values most (again, with an appropriate tie-breaking rule and some provision for allocating remaining goods). The first procedure yields an efficient outcome which also maximizes the utility sum; the second does not guarantee efficiency. Our participants relied on the second alternative in almost all cases where procedural arguments were made - efficiency seems to be less of a concern than satisfying individual preferences. In the cases where procedural aspects were not the main focus of participants' explanations, equity concerns were the predominant aspect considered.

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²² Extracting the empirically relevant procedural aspects of fair division problems is a central aim of the experiments using video or tape recordings, see Klemisch-Ahlert (1996) and Hennig-Schmidt (1999).

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Questionnaire

This questionnaire is part of a research project by Prof. Dr. Puppe and Dr. Herreiner. It will be evaluated anonymously. We ask for your collaboration by filling out the questionnaire.

Below, we introduce several problems where goods need to be allocated among different individuals. Imagine that the individuals involved in these cases approach you and ask you to determine a fair allocation of the available goods. The goods to be allocated are indivisible, that is, you have to give the good as a whole to one person or you can decide not to allocate it at all, i.e. you throw it away. In some problems you only have to allocate goods, whereas in other problems there is also money to be distributed. The money can be divided among some or all of the individuals, or you can decide to not distribute it at all. The allocation problems are independent of each other, i.e. the individuals in the different problems are *unrelated*. The allocation you decide upon cannot be altered by individuals involved in the problems, neither by swapping goods, nor in any other way.

In all problems your task is to determine the allocation that *you consider to be the fairest* – there exists no “correct” or “incorrect” answer. If possible, provide a short explanation as to why you consider the specific allocation to be the fairest for each problem.

Please work on your questionnaire on your own – do not discuss the problems with your neighbor (you are welcome to do so *after* you handed the questionnaire in).

The notation used in the questionnaire is as follows

Individuals: I, II, III

Goods: A, B, C, D, E, F

Money: M

The tables show the valuations of the goods. Each row indicates the values a particular person assigns to the different goods. Each column shows the values a good has for different individuals; usually, individuals assign different values to the same good.

In problems where you are asked to allocate money in addition to the goods (allocation problems 7, 8 and 10) money amounts are comparable to good values, i.e. a specific amount of money is worth exactly as much as a good of the same value.

Please consider the problems carefully: Take your time and analyze the tables and possible allocations thoroughly.

Any questions?

Allocation Problem 1

	A	B	C
I	49	46	5
II	47	48	5

Proposed Allocation:

Individual Goods	I	II

Explanation:

Allocation Problem 2

	A	B	C	D
I	5	47	45	3
II	45	5	48	2
III	23	25	32	20

Proposed Allocation:

Individual Goods	I	II	III

Explanation:

Allocation Problem 3

	A	B	C	D	E
I	40	2	3	25	30
II	14	26	8	26	26
III	10	26	26	12	26

Proposed Allocation:

Individual Goods	I	II	III

Explanation:

Allocation Problem 4

	A	B	C	D
I	30	31	32	7
II	33	29	31	7
III	31	32	30	7

Proposed Allocation:

Individual Goods	I	II	III

Explanation:

Allocation Problem 5

	A	B	C	D	E	F
I	5	20	32	3	25	15
II	26	7	23	20	2	22
III	24	17	6	21	30	2

Proposed Allocation:

Individual Goods	I	II	III

Explanation:

Allocation Problem 6

	A	B	C	D
I	48	4	3	45
II	25	20	40	15
III	2	1	45	52

Proposed Allocation:

Individual Goods	I	II	III

Explanation:

Now imagine that you are one of the 3 individuals that have to distribute the goods among themselves. Pick an allocation you consider to be fair and that you think is acceptable to the other participants (assume that your proposal can only be realized if all participants agree).

Allocation Problem 9

	A	B	C	D
Your Valuation	23	40	20	17
Player 2	2	43	1	54
Player 3	49	4	4	43

Proposed Allocation:

Individual Goods	You	Player 2	Player 3

Explanation:

Allocation Problem 10

	A	B	C	Money
Your Valuation	53	3	44	
Player 2	35	36	29	M=9
Player 3	44	30	25	

Proposed Allocation:

Individual Goods Money	You	Player 2	Player 3

Explanation:

Allocation Problem 7

	A	B	C	Money
I	45	30	25	
II	35	40	25	M=5
III	50	5	45	

Proposed Allocation:

Individual Goods Money	I	II	III

Explanation:

Allocation Problem 8

	A	B	C	D	Money
I	45	4	3	48	
II	15	20	40	25	M=7
III	52	1	45	2	

Proposed Allocation:

Individual Goods Money	I	II	III

Explanation:

Allocations Total Subject Groups Payoffs Allocation Criteria Goods Allocations

Problem 1

	A	B	C
I	49	46	5
II	47	48	5

	EML	Econ	Math	Law	%	U1	U2	U3	EF	PO	U	IA	MM
A,B	188	40	113	35	0.70	49	48		EF			IA	
A,BC	62	15	34	13	0.23	49	53			PO	U		MM
AC,B	5	1	3	1	0.02	54	48			PO	U		
B,A	5	0	4	1	0.02	46	47					IA	
BC,A	2	2	0	0	0.01	51	47						
B,AC	1	0	0	1	0.00	46	52						
Valid Total	263	58	154	51	0.99	# Allocations			188	67	67	193	62
invalid	2	0	2	0	0.01	% of Valid Total			0.71	0.25	0.25	0.73	0.24
no value	2	0	2	0	0.01	% of Total			0.70	0.25	0.25	0.72	0.23
Total	267	58	158	51	1.00								

Problem 2

	A	B	C	D
I	5	47	45	3
II	45	5	48	2
III	23	25	32	20

	EML	Econ	Math	Law	%	U1	U2	U3	EF	PO	U	IA	MM
C,A,BD	70	21	40	9	0.26	45	45	45				IA	MM
B,C,AD	70	14	37	19	0.26	47	48	43	EF	PO			
B,A,CD	34	8	18	8	0.13	47	45	52		PO			MM
B,AC,D	24	4	17	3	0.09	47	93	20		PO	U		
B,A,C	21	8	11	2	0.08	47	45	32					
B,A,D	13	2	9	2	0.05	47	45	20					
B,AD,C	10	0	8	2	0.04	47	47	32					
B,C,A	6	0	4	2	0.02	47	48	23					
C,A,B	2	0	2	0	0.01	45	45	25					
B,C,D	2	0	1	1	0.01	47	48	20					
A,B,CD	1	0	1	0	0.00	5	5	52					
AB,C,D	1	0	1	0	0.00	52	48	20		PO			
AD,B,C	1	0	1	0	0.00	8	5	32					
A,B,D	1	0	1	0	0.00	5	5	20					
C,B,AD	1	1	0	0	0.00	45	5	43					
BD,AC,-	1	0	0	1	0.00	50	93	0		PO			
Valid Total	258	58	151	49	0.97	# Allocations			70	130	24	70	104
invalid	8	0	6	2	0.03	% of Valid Total			0.27	0.50	0.09	0.27	0.40
no value	1	0	1	0	0.00	% of Total			0.26	0.49	0.09	0.26	0.39
Total	267	58	158	51	1.00								

Problem 3

	A	B	C	D	E
I	40	2	3	25	30
II	14	26	8	26	26
III	10	26	26	12	26

	EML	Econ	Math	Law	%	U1	U2	U3	EF	PO	U	IA	MM
A,BD,CE	60	24	36		0.28	40	52	52	EF	PO			MM
A,DE,BC	27	8	19		0.13	40	52	52		PO			MM
A,BC,DE	21	6	15		0.10	40	34	38					
A,D,C	17	6	11		0.08	40	26	26	EF				
AE,BD,C	12	2	10		0.06	70	52	26		PO	U		
A,B,C	11	3	8		0.05	40	26	26	EF				
AE,D,C	9	2	7		0.04	70	26	26					
E,CD,AB	6	2	4		0.03	30	34	36					
D,B,C	5	0	5		0.02	25	26	26	EF			IA	
A,CE,BD	5	1	4		0.02	40	34	38					
BD,E,C	4	1	3		0.02	27	26	26				IA	
AE,D,BC	3	0	3		0.01	70	26	52		PO	U		
A,E,C	3	0	3		0.01	40	26	26	EF				
D,B,E	3	0	3		0.01	25	26	26				IA	
A,B,CDE	2	0	2		0.01	40	26	64					
E,B,C	2	0	2		0.01	30	26	26	EF				
A,B,E	2	0	2		0.01	40	26	26	EF				
AB,D,CE	1	0	1		0.00	42	26	52					
A,BE,CD	1	0	1		0.00	40	52	38					
A,CD,BE	1	0	1		0.00	40	34	52					
AB,D,C	1	0	1		0.00	42	26	26					
A,BD,E	1	0	1		0.00	40	52	26					
A,B,CE	1	0	1		0.00	40	26	52					
BD,C,E	1	0	1		0.00	27	8	26					
D,AC,B	1	0	1		0.00	25	22	26					
B,D,A	1	0	1		0.00	2	26	10					
B,D,E	1	0	1		0.00	2	26	26					
D,E,B	1	0	1		0.00	25	26	26				IA	
D,E,C	1	0	1		0.00	25	26	26				IA	
E,D,B	1	0	1		0.00	30	26	26	EF				
ABE,D,C	1	1	0		0.00	72	26	26					
Valid Total	206	56	150		0.95	# Allocations			101	102	15	14	87
invalid	4	0	4		0.02	% of Valid Total			0.49	0.50	0.07	0.07	0.42
no value	6	2	4		0.03	% of Total			0.47	0.47	0.07	0.06	0.40
Total	216	58	158		1.00								

Allocations		Total	Subject Groups				Payoffs			Allocation Criteria					Goods Allocations
Problem 4		EML	Econ	Math	Law	%	U1	U2	U3	EF	PO	U	IA	MM	
C,A,B		172	42	96	34	0.64	32	33	32	EF				MM	
B,C,A		44	7	28	9	0.16	31	31	31				IA		
C,A,BD		12	4	5	3	0.04	32	33	39		PO	U		MM	
CD,A,B		9	0	7	2	0.03	39	33	32		PO	U		MM	
C,AD,B		6	0	5	1	0.02	32	40	32		PO	U		MM	
B,A,C		5	2	3	0	0.02	31	33	30						
C,BD,A		4	2	1	1	0.01	32	36	31						
A,B,C		4	1	3	0	0.01	30	29	30						
BC,A,D		2	0	2	0	0.01	63	33	7		PO				
A,BD,C		2	0	1	1	0.01	30	36	30						
CD,B,A		1	0	1	0	0.00	39	29	31						
A,B,CD		1	0	1	0	0.00	30	29	37						
Valid Total		262	58	153	51	0.98	# Allocations			172	29	27	44	199	
invalid		3	0	3	0	0.01	% of Valid Total			0.66	0.11	0.10	0.17	0.76	
no value		2	0	2	0	0.01	% of Total			0.64	0.11	0.10	0.16	0.75	
Total		267	58	158	51	1.00									

	A	B	C	D
I	30	31	32	7
II	33	29	31	7
III	31	32	30	7

Problem 5		A	B	C	D	E	F
I		5	20	32	3	25	15
II		26	7	23	20	2	22
III		24	17	6	21	30	2

Allocations		Total	Subject Groups				Payoffs			Allocation Criteria					Goods Allocations
Problem 5		EML	Econ	Math	Law	%	U1	U2	U3	EF	PO	U	IA	MM	
BC,AF,DE		108	36	72		0.50	52	48	51	EF	PO	U		MM	
BE,CF,AD		20	3	17		0.09	45	45	45				IA		
CF,AD,BE		18	7	11		0.08	47	46	47	EF					
C,A,E		15	5	10		0.07	32	26	30	EF					
CD,AB,EF		9	1	8		0.04	35	33	32						
C,AB,EF		5	0	5		0.02	32	33	32						
BC,AD,EF		3	0	3		0.01	52	46	32						
EF,CD,AB		3	0	3		0.01	40	43	41	EF					
B,F,D		3	0	3		0.01	20	22	21	EF					
AB,CD,EF		2	0	2		0.01	25	43	32						
DE,BF,AC		2	1	1		0.01	28	29	30						
AC,DF,BE		1	0	1		0.00	37	42	47						
AD,BE,CF		1	0	1		0.00	8	9	8						
B,ACF,DE		1	0	1		0.00	20	71	51		PO				
BC,AE,DF		1	0	1		0.00	52	28	23						
BF,AC,DE		1	0	1		0.00	35	49	51		PO				
BF,CD,AE		1	0	1		0.00	35	43	54	EF	PO				
BCE,AF,D		1	0	1		0.00	77	48	21		PO				
C,AE,BDF		1	0	1		0.00	32	28	40						
D,ABCE,F		1	0	1		0.00	3	58	2						
AC,BF,D		1	0	1		0.00	37	29	21						
BD,C,A		1	0	1		0.00	23	23	24						
C,BF,E		1	0	1		0.00	32	29	30						
C,F,D		1	0	1		0.00	32	22	21						
E,A,C		1	0	1		0.00	25	26	6						
E,C,A		1	0	1		0.00	25	23	24						
AB,CE,DF		1	1	0		0.00	25	25	23						
BC,DF,AE		1	1	0		0.00	52	42	54	EF	PO				
C,AD,EF		1	1	0		0.00	32	46	32						
CD,BF,E		1	1	0		0.00	35	29	30						
Valid Total		207	57	150		0.96	# Allocations			149	113	108	20	108	
invalid		6	0	6		0.03	% of Valid Total			0.72	0.55	0.52	0.10	0.52	
no value		3	1	2		0.01	% of Total			0.69	0.52	0.50	0.09	0.50	
Total		216	58	158		1.00									

Problem 6		A	B	C	D
I		48	4	3	45
II		25	20	40	15
III		2	1	45	52

Allocations		Total	Subject Groups				Payoffs			Allocation Criteria					Goods Allocations
Problem 6		EML	Econ	Math	Law	%	U1	U2	U3	EF	PO	U	IA	MM	
D,AB,C		87	20	51	16	0.33	45	45	45				IA		
A,BC,D		75	20	34	21	0.28	48	60	52	EF	PO			MM	
A,C,D		49	12	32	5	0.18	48	40	52	EF					
A,B,CD		21	2	16	3	0.08	48	20	97		PO	U			
AB,C,D		7	1	5	1	0.03	52	40	52		PO				
A,BD,C		7	2	2	3	0.03	48	35	45						
A,B,C		5	0	4	1	0.02	48	20	45						
A,C,BD		1	0	1	0	0.00	48	40	53		PO				
A,D,BC		1	0	1	0	0.00	48	15	46						
AC,B,D		1	0	1	0	0.00	51	20	52						
AD,B,C		1	0	1	0	0.00	93	20	45		PO				
A,D,C		1	0	1	0	0.00	48	15	45						
C,D,B		1	0	1	0	0.00	3	15	1						
D,C,AB		1	0	1	0	0.00	45	40	3						
D,C,B		1	0	1	0	0.00	45	40	1						
A,B,D		1	1	0	0	0.00	48	20	52						
D,C,A		1	0	0	1	0.00	45	40	2						
Valid Total		261	58	152	51	0.98	# Allocations			124	105	21	87	75	
invalid		4	0	4	0	0.01	% of Valid Total			0.48	0.40	0.08	0.33	0.29	
no value		2	0	2	0	0.01	% of Total			0.46	0.39	0.08	0.33	0.28	
Total		267	58	158	51	1.00									

Allocations	Total	Subject Groups				Payoffs			Allocation Criteria					Goods Allocations
A,C,BD,2,2,2	1	0	1	0	0.00	47	42	5						
A,B,CD,7,0,0	1	0	1	0	0.00	52	20	47						
A,B,C,0,0,0	1	0	1	0	0.00	45	20	45						
A,B,D,7,0,0	1	0	1	0	0.00	52	20	2						
A,C,D,0,0,0	1	0	1	0	0.00	45	40	2						
A,D,C,0,0,0	1	0	1	0	0.00	45	25	45						
BD,C,A,1,5,1	1	0	1	0	0.00	53	45	53						
BD,C,A,0,3,5,3,5	1	0	1	0	0.00	52	44	56		PO				
B,C,D,7,0,0	1	0	1	0	0.00	11	40	2						
B,C,D,0,0,0	1	0	1	0	0.00	4	40	2						
C,BD,A,3,5,3,5,0	1	0	1	0	0.00	7	49	52						
C,B,D,2,3,2	1	0	1	0	0.00	5	23	4						
D,AB,C,0,7,0	1	0	1	0	0.00	48	42	45						
D,BC,A,2,5,2,2,5	1	0	1	0	0.00	51	62	55	EF	PO				
D,BC,A,1,0,5	1	0	1	0	0.00	49	60	57						
D,BC,A,0,0,0	1	0	1	0	0.00	48	60	52	EF					
D,B,AC,5,2,0	1	0	1	0	0.00	53	22	97		PO	U			
D,B,AC,4,2,1	1	0	1	0	0.00	52	22	98		PO	U			
D,B,AC,3,2,2	1	0	1	0	0.00	51	22	99		PO	U			
D,B,AC,2,3,4,6,0	1	0	1	0	0.00	50	25	97						
D,B,C,1,2,4	1	0	1	0	0.00	49	22	49						
D,C,AB,4,3,0	1	0	1	0	0.00	52	43	53	EF					
D,C,AB,3,3,0	1	0	1	0	0.00	51	43	53	EF					
D,C,A,4,2,1	1	0	1	0	0.00	52	42	53	EF					
D,C,A,2,4,1	1	0	1	0	0.00	50	44	53	EF					
D,C,A,2,2,2	1	0	1	0	0.00	2	2	2	EF			IA		
D,BC,A,3,5,3,5,0	1	1	0	0	0.00	52	64	52	EF	PO				
A,BD,C,2,1,4	1	1	0	0	0.00	47	46	49						
D,C,A,2,5,1,3,5	1	1	0	0	0.00	51	41	56	EF					
D,C,A,0,5,2	1	1	0	0	0.00	48	45	54	EF					
D,C,A,0,6,1	1	1	0	0	0.00	48	46	53	EF					
BD,C,A,0,0,7	1	1	0	0	0.00	52	40	59		PO				
D,BC,A,2,2,3	1	0	0	1	0.00	50	62	55	EF	PO				
D,BC,A,2,33,2,33,2,33	1	0	0	1	0.00	50	62	54	EF	PO				
D,BC,A,5,2,0	1	0	0	1	0.00	53	62	52	EF	PO				
D,BC,A,4,1,2	1	0	0	1	0.00	52	61	54	EF	PO				
D,BC,A,4,0,0	1	0	0	1	0.00	52	60	52	EF					
D,BC,A,3,0,3	1	0	0	1	0.00	51	60	55	EF					
A,BD,C,3,2,2	1	0	0	1	0.00	48	47	47						
D,B,AC,3,4,0	1	0	0	1	0.00	51	24	97		PO	U			
D,B,AC,1,6,0	1	0	0	1	0.00	49	26	97		PO	U			
D,B,AC,3,3,0	1	0	0	1	0.00	51	23	97						
BC,A,D,0,0,7	1	0	0	1	0.00	7	15	9						
C,D,A,7,0,0	1	0	0	1	0.00	10	25	52						
Valid Total	260	57	152	51	0.97	# Allocations			148	98	20	49	2	
invalid	5	0	5	0	0.02	% of Valid Total			0.57	0.38	0.08	0.19	0.01	
no value	2	1	1	0	0.01	% of Total			0.55	0.37	0.07	0.18	0.01	
Total	267	58	158	51	1.00									

Problem 9

	A	B	C	D
I	23	40	20	17
II	2	43	1	54
III	49	4	4	43

	EML	Econ	Math	Law	%	U1	U2	U3	EF	PO	U	IA	MM
BC,D,A	91	27	45	19	0.34	60	54	49	EF	PO			MM
AC,B,D	80	14	49	17	0.30	43	43	43				IA	
B,D,A	47	13	29	5	0.18	40	54	49	EF				
C,BD,A	11	1	7	3	0.04	20	97	49		PO	U		
B,D,AC	6	3	1	2	0.02	40	54	53		PO			
B,A,D	3	0	3	0	0.01	40	2	43					
C,D,A	3	0	3	0	0.01	20	54	49					
CD,B,A	3	0	2	1	0.01	37	43	49					
C,B,A	2	0	1	1	0.01	20	43	49					
A,B,D	2	0	0	2	0.01	23	43	43					
AB,C,D	1	0	1	0	0.00	63	1	43		PO			
A,B,CD	1	0	1	0	0.00	23	43	47					
BD,C,A	1	0	1	0	0.00	57	1	49					
B,AC,D	1	0	1	0	0.00	40	3	43					
B,CD,A	1	0	1	0	0.00	40	55	49	EF	PO			
B,A,C	1	0	1	0	0.00	40	2	4					
B,D,C	1	0	1	0	0.00	40	54	4					
C,D,AB	1	0	1	0	0.00	20	54	53					
D,BC,A	1	0	1	0	0.00	17	44	49					
D,B,A	1	0	1	0	0.00	17	43	49					
-,AB,CD	1	0	1	0	0.00	0	45	47					
-,D,A	1	0	1	0	0.00	0	54	49					
AD,C,B	1	0	0	1	0.00	40	1	4					
Valid Total	261	58	152	51	0.98	# Allocations			139	110	11	80	91
invalid	3	0	3	0	0.01	% of Valid Total			0.53	0.42	0.04	0.31	0.35
no value	3	0	3	0	0.01	% of Total			0.52	0.41	0.04	0.30	0.34
Total	267	58	158	51	1.00								

Allocations		Total	Subject Groups				Payoffs			Allocation Criteria					Goods Allocations		
		EML	Econ	Math	Law	%	U1	U2	U3	EF	PO	U	IA	MM	EML	%	
C,B,A,0,9,0	57	7	35	15	0.21	44	45	44			PO				C,B,A	198	0.76
C,B,A,0,8,0	50	10	35	5	0.19	44	44	44					IA		A,B,C	31	0.12
C,B,A,9,0,0	23	7	11	5	0.09	53	36	44			PO				AC,B,-	11	0.04
C,B,A,1,8,0	15	7	6	2	0.06	45	44	44			PO				C,A,B	9	0.03
C,B,A,3,3,3	9	0	9	0	0.03	47	39	47			PO				A,C,B	5	0.02
A,B,C,0,0,9	9	0	6	3	0.03	53	36	34			PO				ABC,-,-	1	0.00
C,B,A,0.33,8.33,0.33	9	9	0	0	0.03	44	44	44			PO		IA	MM	B,A,C	1	0.00
C,B,A,4,3,2	5	0	4	1	0.02	48	39	46			PO				-B,A	1	0.00
C,A,B,3,3,3	5	0	4	1	0.02	47	38	33							B,C,-	1	0.00
AC,B,-,0,0,9	4	0	3	1	0.01	97	36	9			PO	U			A,B,-	1	0.00
A,B,C,0,2,7	4	0	2	2	0.01	53	38	32			PO				-,-,-	1	0.00
C,B,A,0,0,0	4	2	2	0	0.01	44	36	44									
C,B,A,5,2,2	4	3	1	0	0.01	49	38	46			PO						
A,B,C,9,0,0	3	0	3	0	0.01	62	36	25			PO						
A,B,C,3,3,3	3	1	2	0	0.01	56	39	28			PO						
C,B,A,6,3,0	3	1	1	1	0.01	50	39	44			PO						
AC,B,-,0,3,6	3	1	0	2	0.01	97	39	6			PO	U					
A,B,C,0,0,0	2	0	2	0	0.01	53	36	25									
A,B,C,2,2,5	2	0	1	1	0.01	55	38	30			PO						
A,B,C,1,3,5	2	0	1	1	0.01	54	39	30			PO						
A,B,C,0,9,0	2	1	1	0	0.01	53	45	25			PO						
C,B,A,0.5,8,0.5	2	1	1	0	0.01	45	44	45			PO						
C,B,A,4,5,0	2	1	0	1	0.01	48	41	44			PO						
A,B,C,0,1,8	2	1	0	1	0.01	53	37	33			PO						
ABC,-,-,9,0,0	1	0	1	0	0.00	109	0	0			PO						
AC,B,-,0,4,5	1	0	1	0	0.00	97	40	5			PO	U					
AC,B,-,0,2,7	1	0	1	0	0.00	97	38	7			PO	U					
AC,B,-,0,1,8	1	0	1	0	0.00	97	37	8			PO	U					
A,B,C,8,0,1	1	0	1	0	0.00	61	36	26			PO						
A,C,B,2,4,3	1	0	1	0	0.00	55	33	33			PO						
A,C,B,1,3,5	1	0	1	0	0.00	54	32	35			PO						
A,C,B,0,9,0	1	0	1	0	0.00	53	38	30									
A,C,B,0,5,4	1	0	1	0	0.00	53	34	34									
A,B,-,0,0,9	1	0	1	0	0.00	53	36	9									
B,A,C,0,9,0	1	0	1	0	0.00	3	44	25									
B,C,-,0,0,9	1	0	1	0	0.00	3	29	9									
C,A,B,9,0,0	1	0	1	0	0.00	53	35	30									
C,A,B,2,1,6	1	0	1	0	0.00	46	36	36									
C,B,A,6,0,0	1	0	1	0	0.00	50	36	44									
C,B,A,3.5,2,3.5	1	0	1	0	0.00	48	38	48			PO						
C,B,A,1,7,1	1	0	1	0	0.00	45	43	45			PO						
C,B,A,1,6,1	1	0	1	0	0.00	45	42	45									
C,B,A,1,4,3	1	0	1	0	0.00	45	40	47									
C,B,A,0,0,9	1	0	1	0	0.00	44	36	53			PO						
-B,A,9,0,0	1	0	1	0	0.00	9	36	44									
-,-,-,3,3,3	1	0	1	0	0.00	3	3	3			EF		IA				
C,B,A,8.33,0.33,0.33	1	1	0	0	0.00	52	36	44			PO						
C,B,A,7,2,0	1	1	0	0	0.00	51	38	44			PO						
C,B,A,5,0,4	1	1	0	0	0.00	49	36	48			PO						
C,B,A,3,6,0	1	1	0	0	0.00	47	42	44			PO						
C,B,A,2,7,0	1	1	0	0	0.00	46	43	44			PO						
AC,B,-,1,3,5	1	1	0	0	0.00	98	39	5			PO	U					
C,B,A,4.2,5,2.5	1	0	0	1	0.00	48	39	47			PO						
C,B,A,0,5,0	1	0	0	1	0.00	44	41	44									
C,B,A,5,4,0	1	0	0	1	0.00	49	40	44			PO						
C,B,A,3,4,2	1	0	0	1	0.00	47	40	46			PO						
A,B,C,1,1,7	1	0	0	1	0.00	54	37	32			PO						
C,A,B,0,9,0	1	0	0	1	0.00	44	44	30									
C,A,B,0,0,0	1	0	0	1	0.00	44	35	30									
A,C,B,0,4,5	1	0	0	1	0.00	53	33	35									
Valid Total	260	58	152	50	0.97	# Allocations			1	183	11	60	9				
invalid	6	0	5	1	0.02	% of Valid Total			0.00	0.70	0.04	0.23	0.03				
no value	1	0	1	0	0.00	% of Total			0.00	0.69	0.04	0.22	0.03				
Total	267	58	158	51	1.00												

Problem 10

	A	B	C
I	53	3	44
II	35	36	29
III	44	30	25

M=9

	EML	%
C,B,A	198	0.76
A,B,C	31	0.12
AC,B,-	11	0.04
C,A,B	9	0.03
A,C,B	5	0.02
ABC,-,-	1	0.00
B,A,C	1	0.00
-B,A	1	0.00
B,C,-	1	0.00
A,B,-	1	0.00
-,-,-	1	0.00
Total	260	1.00