Supply chain risk management considering put options and service level constraints

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Abstract: This paper considers a supply chain composed of a supplier and a retailer who commits to a service level to make end-users happy and promote sales. To reduce the losses resulting from the high demand volatility, the retailer purchases put options from the supplier to adjust its initial order. The optimal ordering and production policies with and without put options under the service level constraint are derived. We find that, in the two cases, the expected profits of the retailer are non-increasing in the service level constraint while that of the supplier are non-decreasing in it. Model comparison reveals that with put options, the retailer will offer higher service level and earn more profit than without; such effect is more salient when the demand is more variable. However, the put option contract will not always benefit the supplier especially when the service constraint is high. We also find that put option contract can effectively improve the decentralized system's performance, but this only happens when the service constraint is low. In addition, we find that put option contract have no better capability than wholesales price contract in coordinating the supply chain in the presence of a service level constraint.

Keywords: Risk management; supply chain management; put option contract; service level constraints; operations-finance-marketing interfaces.

1 Introduction

Perishable products, such as fashion clothing, fresh food, newspapers and air tickets, are also known as seasonal products or short life cycle products. Apart from their short life cycle, perishable products are also characterized by their long production or ordering lead-time and high demand uncertainty (Burnetas and Ritchken, 2005; Li *et al.*,2014; Chen *et al.*, 2019; Wang *et al.*, 2019). Today, with the increasing competition, the fast changing consumer preference and the rapidly advancing technology, an increasing number of goods have the traits of perishable products (Hu *et al.*, 2014). This phenomenon is particularly true in high-tech industry. One example is that an IC chip is likely to lose 60% of its value within only the first 6 months of its lifecycle (Mallik and Harker, 2004); another example we may find is that the demand volatility of a state-of-the-art semiconductor might be as high as 80% deviation from the forecast (Wu, 2005).

On the other hand, in order to gain and maintain competitive advantage in existing or new markets, particularly for today's customer-oriented market, an increasing number of companies are promising a high service level (the probability of meeting the customer demand) to satisfy their customers and promote sales (Chen *et al.*, 2015; Taleizadeh *et al.*, 2018). Some companies have gone so far as to promise a 100% service level. E.g., Dayton Hudson (an apparel company) commits a 100% service level to its customers (Sethi *et al.*, 2007). Costless Express, one of Canada's largest business products catalogue retailers, commits 100% to order fill rate and next-business-day delivery to customers (Costless Express 2006). However, whether 100% service level is the best choice for all companies? In fact, the service level marks a trade-off between opportunity costs and inventory costs. The high service level helps companies to collect more revenue, but it is usually accompanied by higher inventory risks. Schalit and Vermorel (2014) found that for most retailers, to increase the service level from 95% to 97% is vastly more expensive than from 85% to 87%, which means the higher the service level the retailer wants to achieve, the more cost it has to bear. Therefore, setting an appropriate service level target is important for a company to balance customer satisfaction and expected profit. However, it is usually complex and challenging, especially for companies with perishable products. One of the typical strategies recognized by scholars and practitioners is to provide retailers with more flexibility in ordering so as to respond to the ever changing market demand.

In recent years, a well-known financial derivative, the real option has become prevalent in supply chain to facilitate flexible ordering. The real option was firstly designed to aid capital investment decision-making to amplify good fortune or mitigate loss (Lander and Pinches, 1998). Due to its intrinsic flexibility, the real option is acknowledged as a powerful tool to evaluate uncertain projects and has interested numerous researchers. Various option-type decision-making frameworks have been developed to model and value real option. However, such models are usually complex and require substantial mathematical techniques to solve. Besides, many of the required modeling assumptions are often violated in a real option application. Due to such reasons, these models were not widely used by practitioners then (Lander and Pinches, 1998). Fortunately, such research has been carried on and a number of more practical frameworks/models and solutions have been proposed. Examples include, among others, mobile phone operator (Cassimon, et al., 2011), energy economics (Feng and Ryan 2013), biopharmaceutical industry (Nigro, et al. 2014) and natural resources extraction (Alonso-Ayuso et al. 2014). What's more, the development of computing technology has gradually removed the technical limitations and little by little the real option theory has gained its popularity in capital investment. For this reason, both scholars and practitioners have adopted real options to hedge the risks of supply chains (Chen *et al*. 2014). Essentially, real option (here after referred to as option) is a special right: by pre-paying a fee, the buyer (retailer) gets the right (not the obligation) to reorder from or return goods to the seller (supplier) at a predetermined price on a future date. Based on different choices, option contracts can be divided into call option (reorder) contract and put option (return) contract (Burnetas and Ritchken, 2005), which are frequently adopted in practice. E.g., to hedge their risks, Hewlett-Packard Company (Nagali *et al*., 2008) and China Telecom Corporation Limited (Chen *et al*., 2014) adopt call option contracts while Enron (Chen and Parlar, 2007) adopts put option contracts. Among these examples, the most commonly acknowledged one is the risk management program launched by Hewlett-Packard in 2000. By adopting call option contracts to tackle the joint uncertainty in demand, cost and availability for key components such as memory chips and flat-panels, the company is reported to have achieved approximately $425 million in cumulative cost savings during the period of 2000-2006 (Nagali et al., 2008). Such examples suggest that option contracts are of great applicability and there's no doubt that they will be popular in the future. In the literature, the issues on supply chains management with call option contracts have been well-studied. As is shown in §2, the role of put option contracts in supply chain management has been examined from different perspectives by some researchers, but none of them have considered the service level constraint. Motivated by these observations, our study addresses the following questions in the context of a supply chain composed of a supplier and a retailer with a service level constraint:

(1) What are optimal operational decisions (e.g., the retailer's order quantity and the supplier's production quantity) with put option contract?

(2) How does the service level constraint influence the supply chain members' operational decisions and performances?

(3) What impact do put options have on the decisions and performances of the supply chain?

(4) Can the supply chain be coordinated under the put option contract?

Our study offers three contributions. First, we extend the supply chain models to incorporate both put option contract and the service level constraint. We derive the optimal ordering policies as well as the optimal production policies with and without put options. Besides, we discuss the effect of the service level constraint on the members' performance. It is shown that, in two cases, the expected profits of the retailer are non-increasing in the service level constraint while that of the supplier are non-decreasing in it. Finally, for each part, we examine the value of the put option. We find that with put options, the retailer will offer higher service level and earns more profit than without them, such effect is more salient when the demand is more variable. However, the put option contract will not always benefit the supplier especially when the service constraint is high. We also find that put option contract can effectively improve the decentralized system's performance, but this only happens when the service constraint is low. In addition, we find that the put option contract do not have any superiority as compared to the wholesale price contract in coordinating the supply chain in the presence of a service level constraint.

After a brief literature review in §2, we detail model assumptions and formulation in §3. In §4 we develop model to derive and analyze the retailer's optimal ordering policy and the supplier's production policy. In §5, the effects of the service level constraint as well as put option contract on the supply chain is discussed. In §6, the supply chain coordination is discussed. §7 summarizes our findings and gives suggestion on future research directions. All the proofs are relegated to the Appendix.

2 Literature review

This work is associated to several research streams in the supply chain management literature. Here, we focus on two most relevant streams: service level constraints and put option contracts, which are elaborated in the following discussion.

The supply chains management with service level constraints has drawn much attention from the academic circle. Several authors have incorporated service level constraints into the classic multi-echelon inventory control model. Usually, they derive the optimal stock levels for each stocking location in the presence of service level constraints by adopting base-stock policies and developing optimal or heuristic procedures (Bollapragada *et al.*, 2004;Tarim *et al.*, 2011;Wang *et al.,* 2013; Tunc *et al.,* 2014; Woerner *et al.,* 2018). For more detailed literature review on this, readers may refer to the works of Bijvank and Vis (2012) and Woerner *et al.* (2018). There is a rapidly growing body of literature focusing on solving optimization problems with service level constraints in supply chain context, which is more relevant to the issue that we are addressing. Ernst and Powell (1998) studied a distribution system in which a manufacturer provides financial incentives to the retailer to improve its service level. Sethi *et al.* (2007) investigated a two-level supply chain with demand forecast updates. The buyer has a replenishment opportunity and commits to a service level after demand forecast updates upon the observation of the market signal. They found that both the optimal order quantity of the first-stage and the maximum expected profit are monotone with the target service level. Finally, they extended their analysis to the situation when an order cancellation is allowed and the channel coordination issue. Elena *et al*.(2008) considered a supplier who delivers goods to a retailer through stocking. The supplier commits to achieve a minimum fill rate (service level agreement) over a specified time horizon. They concentrated on the impact of the magnitude of the bonus for meeting or exceeding the service level target and the length of the review period. Li *et al.* (2011) proposed a price discount mechanism to coordinate a supply chain composed of a vendor and a buyer who faces service level constraints. Jha and Shanker (2013) considered a context in which a vendor supplies production to a set of buyers. They presented an integrated production-inventory model which includes service level constraints corresponding to each buyer to find the optimal order quantity, lead time and safety factor of the buyers simultaneously. Considering disruption risk and risk-aversion in a supply chain, Sawik (2016) adopted two different service-level measures (the expected worst-case order fulfillment rate and the demand fulfillment rate) to study the worst-case optimization of service level. All the studies reviewed above do not take option contracts into consideration.

The literature on supply chain with option contracts is abundant but mainly focuses on call option contracts, such as Barnes-Schuster *et al.*, (2002), Nagali *et al.*, (2008), Fu *et al.*, (2012), Hu *et al.*, (2014), Wang and Chen (2015, 2017), Wang *et al.*, (2017), Luo and Chen (2017), Wan and Chen (2018), Benyong *et al.*, (2018) and Chen and Wan (2019). Here the review mainly concentrates on the studies on supply chains with put option contracts. Burnetas and Ritchken (2005) investigated the pricing of put option in a supply chain consisting of a manufacturer and a retailer with a downward sloping demand curve. They focused on how the wholesale price and strike price adjust after the introduction of put option contracts. They found that the wholesale price will not readjust in such case if there's the range in which the strike prices are curtailed. They also found that in some cases the put option contracts may hurt the retailer. Liu *et al.* (2013) examined the value of put option contracts in a two-echelon container shipping service chain with capacity and order constraints. Recently, considering the case that both demand and cost are uncertainty, Nosoohi and Nookabadi (2016) investigated the outsourcing model with put option. Wang and Chen (2018) studied the ordering and retail price policies of fresh products under put option contracts. In addition, Chen and Parlar (2007) studied the value of a put option in a single-period inventory model, in which the risk-averse newsvendor not only chooses the order quantity but also determines the "strike quantity" and/or the "strike price" of the put option. They found that the optimal order quantity will not change with or without the put option. They also found that the news vendor's maximum expected profit are not affected by the strike price and strike quantity which however do affect profit variance. It's worth noting the major differences between their work and our paper. On the hand, the put option in their model can enable the buyer to make profit only when the actual demand is smaller than the strike quantity (option order quantity). On the other hand, in their study, the part which can be resold to the option writer or be compensated by the option writer is the gap between the demand and the strike quantity. In contrast, in our paper, the part that can be resold to the supplier can be up to the strike quantity whatever the relationship of the demand and the strike quantity is. In a nutshell, the previous researchers have studied the role of put option contracts in a supply chain from different situations.

The above review shows that both service level constraints and put option contracts have been extensively studied in the supply chain management literature. However, most related studies investigate these two important issues in separate contexts. Different to those studies on put options in supply chain (Liu et al., 2013; Nosoohi and Nookabadi; 2016; Wang and Chen 2018), we incorporate both put option contract and the service level constraint in our modelling and find that the effort of put option contracts on the supply chain members' performance is subject to the service level constraint. Different to the studies (Chen and Shen 2012; Chen *et al.*, 2017) that consider option contracts in the presence of service level constraints such as Chen and Shen (2012) focused on the call option and Chen et al. (2017) concentrated on the bidirectional option, we mainly deal with put option contracts. Moreover, we also discuss the coordination of the supply chain under put option contracts in the presence of service level constraints.

3 Model assumptions and formulation

This paper considers a two-echelon supply chain composed of a supplier and a retailer. The retailer orders products from the supplier and sells to end-user sunder a service level constraint in a single selling period. In addition to placing an initial firm order, the retailer can purchase put options from the supplier. The put option is characterized by two parameters, namely, the option price and the exercise price. Each put option gives the retailer the right but not the obligation to return one unit to the supplier at the exercise price after demand is observed. In this paper, we focus on how the service level constraint and put options affect the retailer's ordering decision and the supplier's production decision. Therefore, all cost and profit parameters are assumed as exogenous. This assumption is reasonable, especially when selling/contract parameters are determined by supply chain firms in advance and ordering/production quantities and delivery conditions are negotiated later (see Barnes-Schuster *et al.*, 2002; Li *et al*, 2011; Hu*et al*, 2014). Besides, considering the current market environment where many retailers play a more dominant role than their suppliers in the supply chain, we assume the retailer is the leader and the supplier is the follower.

The sequence of events will be as follows. Before the beginning of the production period, the retailer determines how many products to order initially and how many put options to purchase according to the preliminary demand forecast and the service level constraint. During the production period, the supplier produces according to the retailer's initial firm order. At the beginning of the selling period, the supplier delivers all the ordered products to the retailer. During the selling period, the actual demand is realized and the retailer manages to meet it with products on hand. After the selling period, the retailer excises the put options, and any leftover for the retailer and the supplier can be salvaged at the same specific price. The main model notations in this paper are summarized in Table 1.

Table 1. The notations

D	Random variable representing market demand during the selling period, $D \geq 0$;
$f(x)$	Probability density function of D;
$F(x)$	Cumulative distribution function of D;
w_1	Unit wholesale price ($);
w_2	Unit option exercise price($);
b	Unit option price ($);
q	Firm order quantity of the retailer;
q_1	Put option order quantity of the retailer, $q_1 < q$;
α	Retailer's service level commitment, $0 < \alpha \leq 1$;
Q	Production quantity of the supplier;
c	Unit production cost($);
p	Unit retail price ($);
s	Unit salvage value after the selling period($);

We further assume that both the retailer and the manufacturer are rational and self-interested and each firm is risk-neutral. Meanwhile symmetric information is assumed, i.e. at the beginning of the game, both firms hold the same information, which means all parameters and rules are known by each firm. Besides, to avoid an unrealistic case, we require that $q_1 < q, w_2 > b + s, p > w_1 + b$ and $w_1 + b + s > w_2 + c$.The first condition avoids the situation that the returned product quantity exceed the retailer's purchase. The second condition ensures the retailer to make profit. The third condition assures the incentive for the retailer to purchase put options. Similarly the fourth condition assures the supplier's profit and there is, otherwise, no incentive for the supplier to produce the product or accept put option contract. For clarity, superscripts 's' (supplier) and 'r' (retailer) are adopted to differentiate between the profit of the supplier and that of the retailer. (notation $[z]^+ = \max\{z, 0\}$).

4 Optimal ordering policy and production policy with put options

In this section, we develop models to analyze the retailer's optimal ordering policy and the supplier's optimal production policy under put option contract in the presence of a service level constraint. In this context, the retailer has two decision variables: q(the quantity of firm order) and q_1 (the quantity of put option). The expected profit of the retailer, denoted $\pi_r(q, q_1)$, is

$$\pi_r(q, q_1) = pE[\min(q, D)] + w_2 E\min[q_1, (q-D)^+] + sE[(q - q_1 - D)^+] - w_1 q - b q_1$$

The first three terms above refer to the expected revenue from selling the products to customers, from exercising put options and from salvage of the leftover, respectively. The last two terms capture the costs of firm order and put options purchase, respectively. Then

$$\pi_r(q, q_1) = (p - w_1)q - (p - w_2)\int_0^q F(x)\,dx - (w_2 - s)\int_0^{q-q_1} F(x)dx - bq_1 \quad (1)$$

The retailer solves the following problem under put option contract in the presence of the service level constraint α:

$$\max_{q_1 > 0, q > q_1} \pi_r(q, q_1),$$

$$\text{s.t.}\, Pr\{q \geq D\} \geq \alpha \qquad (2)$$

Equation (2) indicates that $q > F^{-1}(\alpha)$ and $q^\alpha \equiv F^{-1}(\alpha)$. It's clear that q^α is increasing in α. Noting that in the above model, we use a service level constraint instead of the shortage cost because the product shortage affects the service level, which results in lost sales cost; furthermore, practically customers will not ask the firms to provide a 100% service level though a certain service level is required. Thus, we have the following lemma.

Lemma 1. ***With put options, the retailer's optimal firm order quantity q^* and option order quantity q_1^* in the presence of the service level constraint satisfies:***

$$q^* = \begin{cases} q^\gamma & if\,\alpha < \gamma \\ q^\alpha & if\,\alpha \geq \gamma \end{cases}. (3)$$

$$q_1^* = q^* - F^{-1}\left(\frac{b}{w_2 - s}\right). (4)$$

where $\gamma = \frac{p - w_1 - b}{p - w_2}$ and $q^\gamma = F^{-1}\left(\frac{p - w_1 - b}{p - w_2}\right)$ are the maximum service level and the optimal firm order quantity of the retailer with put options in the case of without any constraints, respectively.

This lemma characterizes the optimal ordering policy of the retailer with the service level constraint and put options. It is shown that, if α is lower than γ, then the service level constraint is not binding and the retailer will order to achieve its maximum profit. If α is higher than γ, then the service level constraint is binding and the retailer should order quantity as the service level constraint (q^{α}). Moreover, it's worth noting that$q_1^{\gamma} > 0$is equivalent to $b < \frac{(p-w_1)(w_2-s)}{p-s}$, which shows that the retailer will order no options at all if the option price $b$is too high.

Assuming that the demand is normally distributed and the standarddeviationis σ, we have the following corollary.

Corollary 1. ***q_1^* is non-increasing in w_1, decreasing in b, and increasing in w_2 and σ.***

Corollary 1 reveals that when the wholesale price w_1 decreases, the retailer will maintain or increases its option order according to the service level constraint. If the option price b decreases, the option exercising price$w_2$increases or the variance of the demand increases, the retailer will purchase more options.

Corollary 2. ***$\pi_r(q^*, q_1^*)$ is decreasing in w_1 and b, and increasing in w_2.***

This corollary shows that with put options, the retailer's maximum expected profit will decrease if the wholesale price or the option price increases. However, the maximum expected profit of the retailer will increase if the option exercising price increases. Corollary 1 and Corollary 2 show that our study can give significant managerial insights into pricing (wholesale price, option price and exercising price) though all price parameters are assumed exogenous in this paper.

In what follows, we consider the supplier's optimal production policy. Although the retailer may exercise part of or all the put options (return the leftover to the supplier), the supplier will deliver the quantity of the retailer orders at the beginning of the selling season, which means that the supplier has to produce the exact quantity the retailer orders. Therefore, we have the following proposition.

Lemma 2. ***With put options, the supplier's optimal production quantity in the presence of a service level constraint is***

$$Q^* = q^* = \begin{cases} q^{\gamma} & if \alpha < \gamma \\ q^{\alpha} & if \alpha \geq \gamma \end{cases}. \quad (5)$$

According to Lemma 2, the maximum expected profit of the supplier with put options, denoted$\pi_s(Q^*)$, is

$$\pi_s(Q^*) = w_1 q^* + b q_1^* + sE\min[q_1^*, (q^* - D)^+] - cq^* - w_2 E\min[q_1^*, (q^* - D)^+].$$

The first three terms are the expected revenue from firm orders, option sales and salvage of the returned products from the retailer, respectively. The fourth term is the production cost and the last term is the expense for the option exercised by the retailer. Then

$$\pi_s(Q^*) = (w_1 - c)q^* + bq_1^* - (w_2 - s)\int_{q^*-q_1^*}^{q^*} F(x)\,dx. (6)$$

5 The effect of service level constraint and put option contract

The previous subsection derives the retailer's optimal ordering policy and the supplier's optimal production policy as well as their maximum expected profit with service level and put options. In this section, we examine the effects of the service level constraint and put options on the supply chain. Firstly, in order to establish a performance benchmark, we consider the base case of without put options (wholesale price contract). Assume that the retailer places a firm order q_0 from the supplier before the beginning of the production season, and then we can know that he expected profit of the retailer, denoted $\pi_r(q_0)$,

$$\pi_r^0(q_0) = (p - w_1)q_0 - (p - s)\int_0^{q_0} F(x)dx. \tag{7}$$

Thus, the retailer solves the following problem in the presence of service level constraints (α):

$$\max_{q_0 > 0} \pi_r^0(q_0),$$

$$\text{s.t. } Pr\{q_0 \geq D\} \geq \alpha. \tag{8}$$

Equation (8) indicates that $q_0 > F^{-1}(\alpha)$and $q^\alpha \equiv F^{-1}(\alpha)$. It's clear that q^α is increasing in α. From Equation (7), we can get the optimal order quantity of the retailer without the service constraint is

$$q_0^\beta = F^{-1}\left(\frac{p-w_1}{p-s}\right). \tag{9}$$

Set $\beta = \frac{p-w_1}{p-s}$, which is the maximum service level without any constraints, then $q_0^\beta \equiv F^{-1}(\beta)$. Therefore, without put options the optimal order quantity of the retailer with a service level α is $q_0^* = \max(q_0^\beta, q^\alpha)$, that is,

$$q_0^* = \begin{cases} q_0^\beta & if \alpha < \beta \\ q^\alpha & if \alpha \geq \beta \end{cases}. \tag{10}$$

It is shown that, if α is lower than β, then the service level constraint is not binding and the retailer will order to achieve its maximum profit. If α is higher than β, then the service level constraint is binding and the retailer should order quantity as the service level constraint (q^α).

From Equation (7), we find that the retailer's maximum expected profit is $\pi_r^0(q_0^*) = (p - w_1)q_0^\beta - (p - s)\int_0^{q_0^\beta} F(x)dx$ for the case $\alpha < \beta$, for the case $\alpha \geq \beta$, the retailer's maximum expected profit$\pi_r^0(q_0^*)$ is $\pi_r^0(q_0^*) = (p - w_1)\, q^\alpha - (p - s)\int_0^{q^\alpha} F(x)dx$. In addition, the supplier's optimal production quantity (denoted as Q_0^*) is $Q_0^* = q_0^*$. The maximum expected profit of the supplier (denoted as $\pi_s^0(Q_0^*)$) is $\pi_s^0(Q_0^*) = (w_1 - c)q_0^*$.

5.1The effect of a service level constraint

Here, we consider the effect of the service level constraint on the optimal policies and the expected profit of the supply chain.

Proposition1. ***q^*, q_1^* and q_0^* are non-decreasing in α, but $\pi_r(q^*, q_1^*)$ and $\pi_r^0(q_0^*)$ are non-increasing in α.***

Proposition1states that as the service level constraint α increases, the retailer's optimal order quantity with and without put options will not decrease, which is consistent with the intuition that a higher service level constraint requires more initial order of the retailer and meanwhile the option order will increase. However, increasing the service level can improve customer satisfaction and promote sales, while incur a higher inventory risk. From this proposition, we can see that the retailer's expected profit will not increase as the service level constraint α increases, instead, it will decrease especially when the service level constraint is binding ($\alpha \geq \gamma$ for the case with put options, $\alpha \geq \beta$ for the case of without). On the one hand, a high service level improves customer satisfaction and therefore increases customer demand. On the other hand, a high service level requires the retailer to held sufficient inventory and therefore to increase the quantities of firm order and put options. Meanwhile, the large quantities of firm order and put option orders push up the cost and therefore reduce the profit margin of the retailer. The proposition demonstrates that there is a trade-off between customer satisfaction and operational cost. It is important to strike a balance between the two when retailers make the strategic decision on service level and operationalize the strategy.

Proposition 2. ***Q^* and Q_0^* are non-decreasing in α, $\pi_s(Q^*)$ and $\pi_s^0(Q_0^*)$ are non-decreasing in α.***

From Proposition2, we find that as the service level constraint increases, the optimal production quantity and the expected profit of the supplier will not decrease in both cases (with and without put options). This proposition reveals that the service level constraint is always beneficial to the supplier because the high service level constraint requires the retailer to order larger quantity of products and increases the supplier's profit. Furthermore, a high service level also helps to stimulate the demand of end consumers and contributes to the expansion of market share. Therefore, from the supplier's perspective, a high service level increases customer demand and enhances its competitiveness.

5.2 The effect of put option contract

In this subsection, we examine the effect of put option contract on the optimal ordering policies and the expected profit of the supply chain. Comparing the models between the cases of with and without put option contract, we get the following propositions.

Proposition3. ***If*** $\boldsymbol{\alpha < \gamma}$***, then*** $\boldsymbol{q^* > q_0^*}$***;if*** $\boldsymbol{\alpha \geq \gamma}$***, then*** $\boldsymbol{q^* = q_0^*}$.

Proposition 3 indicates that if the service level constraint α is lower than γ, the firm order quantity with put options will be higher than that of without, whereas if α is higher than γ or equal to γ, there is no difference in the retailer's firm order quantity between the two cases. It reveals that when the service level constraint is low ($\alpha < \gamma$), the put option contract can induce the retailer to order more and provide a higher service level than without. This occurs because, by purchasing put options, the retailer obtains the right to return some unsold products to the supplier. Therefore, the retailer will place a larger firm order to avoid loss from product shortage and can also reduce the cost of excessive inventory by exercising the put options. Nevertheless, when the service level constraint is high ($\alpha \geq \gamma$), the optimal order quantity is the same under the two cases.

Proposition4. For any $\boldsymbol{\alpha, \pi_r(q^*, q_1^*) > \pi_r^0(q_0^*)}$***, and*** $\boldsymbol{\Delta = \pi_r(q^*, q_1^*) - \pi_r^0(q_0^*)}$ ***is increasing in*** $\boldsymbol{\sigma}$**.**

Proposition 4 suggests that whatever the service level constraint is, the maximum expected profit of the retailer with put option contract is larger than that of without. It shows that with put options, the retailer will always earn more profits than without, and this is not affected by the constraint of the service level. Combining Propositions 3 and 4, it is clear that put option contract cannot only help the retailer improve the service level but also increase the profit. It further reveals that by adopting put options, the retailer can effectively deal with the risk of demand uncertainty and simultaneously achieve the profits and service level. More importantly, the value of put options will increase as the demand variability increases. That is, the retailer will benefit more from put options when the demand is more volatile.

Now we look at the effect of put option contract on the supplier's optimal production policy and expected profit. By comparing the equilibrium of the two models, we have the following two propositions.

Proposition 5. ***If*** $\boldsymbol{\alpha < \gamma}$***, then*** $\boldsymbol{Q^* > Q_0^*}$***; if*** $\boldsymbol{\alpha \geq \gamma}$***, then*** $\boldsymbol{Q^* = Q_0^*}$.

From Proposition 5, we can see that the supplier's optimal production quantity with put option contract is equal to that of without if the service level constraint α is higher than γ or equal to γ, whereas if α is lower than γ, the optimal production quantity with put option contract will be higher than that of without. It follows that supplier's optimal production quantity with put options contract is always no less than that of without. Proposition 3 shows that the production quantity increases due to the increased order quantity.

Proposition6. ***There is*** $\boldsymbol{\alpha_0 \in (\beta, \gamma)}$***, if*** $\boldsymbol{\alpha < \alpha_0}$***, then*** $\boldsymbol{\pi_s(Q^*) > \pi_s^0(Q_0^*)}$***; if*** $\boldsymbol{\alpha \geq \alpha_0}$***, then*** $\boldsymbol{\pi_s(Q^*) \leq \pi_s^0(Q_0^*)}$**.**

Proposition 6 states there is a threshold value α_0 within (β, γ) (γand $\beta$are the maximum service level with and without put option contract in the case of without any constraints, respectively), if the service level constraint α is lower than α_0 or equal to α_0, then the maximum expected profit of the supplier with put option contract is larger than that of without, but if α is higher than α_0, then the maximum expected profit of the supplier with put option contract is smaller than that of without. It suggests that with put option contract, the supplier will not always earn more profits than without, and whether the supplier can benefit from the put option contract depends on the service level constraint α. Only when the service level constraint is low ($\alpha \leq \alpha_0$), the supplier under put option contract will be better off than without. Under this context, the supplier will be willing to apply/accept the put option contract; otherwise, the supplier will not embrace it. In other words, the service level constraint (α) the retailer promised is a key factor that determines whether the supplier should adopt put option contract.

Furthermore, we look at the impact of the put option contract on the total profit of the supply chain and have the following proposition.

Proposition 7. ***If*** $\boldsymbol{\alpha < \gamma}$***, then*** $\boldsymbol{\pi_r(q^*, q_1^*) + \pi_s(Q^*) > \pi_r(q_0^*) + \pi_s(Q_0^*)}$***; if*** $\boldsymbol{\alpha \geq \gamma}$***, then*** $\boldsymbol{\pi_r(q^*, q_1^*) + \pi_s(Q^*) = \pi_r(q_0^*) + \pi_s(Q_0^*)}$**.**

Proposition 7 shows that if the service level constraint α is lower than γ, the maximum total expected profit of the whole supply chain with put option contract is larger than that of without it; if the service level constraint α is not lower than γ, the maximum total expected profit of the whole supply chain with put option contract is equivalent to that of without it. It implies that whether the performance of the whole supply chain can be improved by the put option contract also depends on the service level constraint α. When the service level constraint is lower than the threshold ($\alpha < \gamma$), the whole supply chain can benefit from put option contract, otherwise ($\alpha > \gamma$), the put option contract make no difference.

Further, combining with Propositions 4, 6, and 7 we can conclude that when the service level constraint is lower than the threshold ($\alpha < \gamma$), the win-win situation can be achieved by put option contract if the retailer is willing to compensate the supplier in the case that the supplier is worse off from the put option contract ($\alpha \geq \alpha_0$).However, under the case ($\alpha \geq \gamma$) that the put option contract do not make any difference, the best contract for the supply chain is the wholesale price contract due to its simplicity.

6 Supply chain coordination

This section focuses on supply chain coordination in the presence of a service level constraint. We first discuss the centralized solution of the integrated supply chain. To optimize the system-wide expected profit for the supply chain, we take the supply chain system as a centralized entity. Assume that the production quantity of the centralized entity is Q_I, and then the expected profit of the integrated supply chain, denoted $\Pi_I(Q_I)$, is

$$\Pi_I(Q_I) = pE[\min(Q_I, D)] + sE[(Q_I - D)^+] - cQ_I.$$

The first two terms are the expected revenue and expected salvage value, respectively. The last term is the production cost. Then

$$\Pi_I(Q_I) = (p-c)Q_I - (p-s)\int_0^{Q_I} F(x)dx. \quad (11)$$

The decision problem faced by the centralized entity is

$$\max_{Q_I} \Pi_I(Q_I),$$

$$\text{s.t. } Pr\{Q_I \geq D\} \geq \alpha. \quad (12)$$

From Equation (12), we get $q > F^{-1}(\alpha)$and $q^{\alpha} \equiv F^{-1}(\alpha)$. Clearly, $q^{\alpha}$is increasing in α.

Equation (11) shows that$\frac{d\Pi_I(Q_I)}{dQ_I} = (p-c) - (p-s)F(Q_I)and\frac{d^2\Pi_I(Q_I)}{dQ_I^2} = -(p-s)f(Q_I) < 0$. Thus, $\Pi_I(Q_I)$is concave in Q_I. Set$\frac{d\Pi_I(Q_I)}{dQ_I} = 0$, we can get the optimal production quantity of the centralized entity without service level constraints is

$$Q_I^{\tau} = F^{-1}\left(\frac{p-c}{p-s}\right). \quad (13)$$

Now set $\tau = \frac{p-c}{p-s}$which is the maximum service level of the integrated supply chain without any constraints, then $Q_I^\tau \equiv F^{-1}(\tau)$. Therefore, the optimal production quantity of the integrated supply chain with a service level α is $Q_I^* = \max(Q_I^\tau,\ q^\alpha)$, that is,

$$Q_I^* = \begin{cases} Q_I^\tau & if \alpha < \tau \\ q^\alpha & if \alpha \geq \tau \end{cases} .(14)$$

It is shown that, if α is lower than τ, then the service level constraint is not binding and the integrated supply chain will production to achieve the maximum profit. If α is higher than τ, then the service level constraint is binding and the integrated supply chain should production quantity as the service level constraint (q^α).

As shown in Equation (14), when $\alpha < \tau$, $Q_I^* = Q_I^\tau$, which implies that$\Pi_I(Q_I)$ is constant in α. When $\alpha \geq \tau$, $Q_I^* = q^\alpha$, then we can see that $\Pi_I(Q_I)$ is decreasing in the service constraints α. It follows that the maximum expected profit of the integrated supply chain is non-increasing in the service level α.

From the analysis above, we can obtain the following proposition.

Proposition8. ***If $\alpha < \tau$,the supply chain cannot be coordinated with and without put options; if $\alpha \geq \tau$, the supply chain can be coordinated in both cases***.

Proposition 8suggeststhat the service level constraint is the key determinant of the supply chain coordination. If the service level constraint$\alpha$is lower than τ (the maximum service level of the integrated supply chain without any constraints), the coordination of the supply chain cannot be achieved with or without put option contract. However, if the service level constraint is higher than τ, the coordination of the supply chain can be achieved in both cases. It reveals that with a service level constraint, the put option contracts do not have any superiority in coordinating the supply chain than the wholesale price contract.

7 Numerical examples

To illustrate the developed model numerically, we assume that the demand D is normally distributed with μ =100 and σ =30 during the selling season. Other parameters are as follows: w_1 =20\$, b =4\$, w_2 =18\$, p =40\$, s =3\$ and c =4\$. The effects of the service level constraint α on the retailer's optimal order quantity (q^*, q_0^*) and the expected profit of each parties (π_r, π_s, Π) are illustrated by Figure 1.

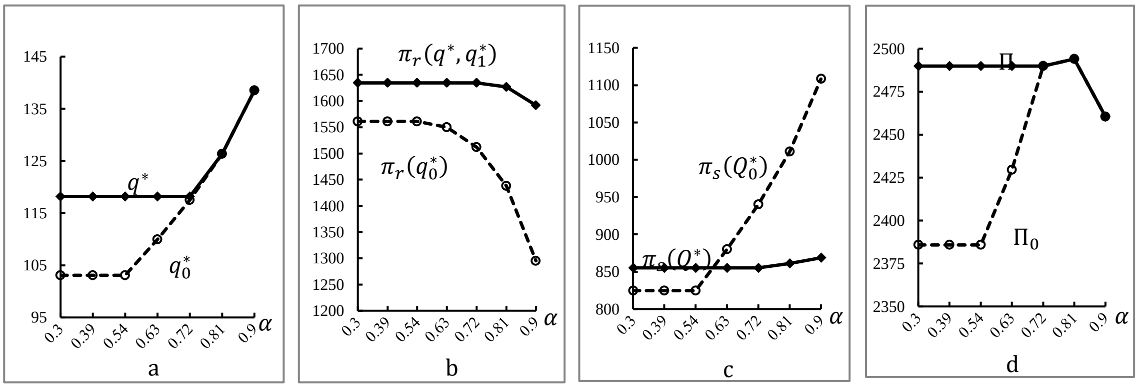

Figure. 1. Effects of service level constraint α on decision and profits.

The maximum service level with and without options in the context of without any constraints is 0.72 (γ) and 0.54 (β), respectively. Figure 1a confirms what were discussed in Proposition 1 and 5, namely, both the retailer's optimal order quantity q^* and q_0^* are non-decreasing in α, specially, if α is lower than 0.72, then q^* will be higher than q_0^*, whereas if α is equal to or beyond 0.72, there is no difference between $q^*$and q_0^*. Figure 1b confirms what were discussed in Proposition 1 and 6, that is, both the retailer's expected profit $\pi_r(q^*, q_1^*)$ and $\pi_r^0(q_0^*)$ are non-increasing in α, and the expected profit of the retailer with put options is larger than that of without.

Figure 1c confirms what were discussed in Proposition 2 and 8, the expected profit of the supplier $\pi_s(Q^*)$ and $\pi_s(Q_0^*)$ are non-decreasing in α. Moreover, there is a threshold value 0.59, if α is lower than it, then $\pi_s(Q^*)$ is larger than $\pi_s(Q_0^*)$, but if α is higher than 0.59, then $\pi_s(Q^*)$ is smaller than $\pi_s(Q_0^*)$. Figure 1d confirms what were discussed in Proposition 7, that is, if the service level constraint α is lower than 0.72, the total expected profit of the whole supply chain with put options Π is larger than that of without Π_0; if the service level constraint α is beyond or equal to 0.72, Π is equivalent to Π_0.

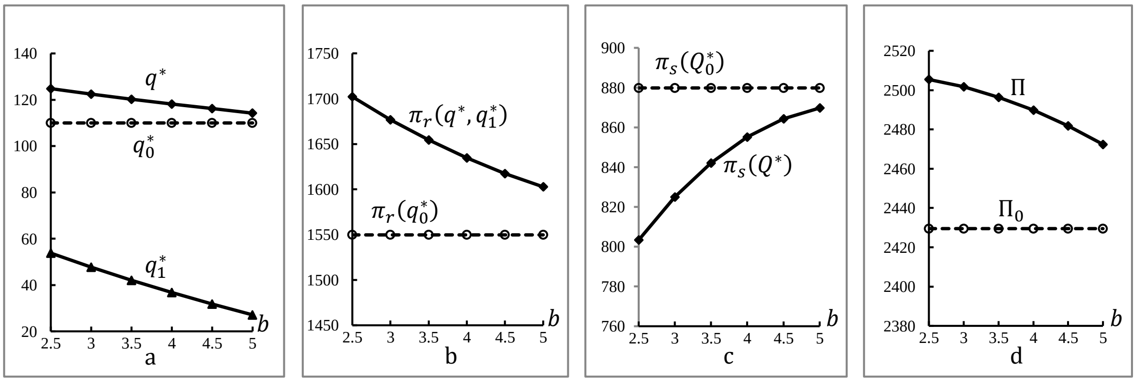

Figure. 2. Effects of option price b on decision and profits.

We assume that the retailer sets its initial (non-optimal) service level at α =0.7 and further run experiments to study the effects of option price b and exercise price w_2 on the decision and profits of the supply chain by setting b ={2.5$, 3$, 3.5$, 4$, 4.5$, 5$} and w_2 ={15$, 16$, 17$, 18$, 19$, 20$}, respectively. The results are illustrated by Figure 2 and Figure 3.

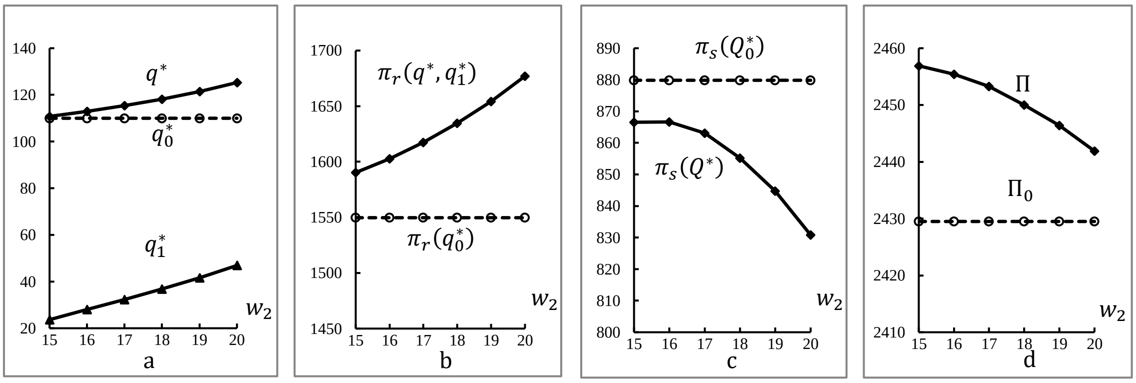

Figure. 3. Effects of exercise price w_2 on decision and profits.

Figure 2a (3a) and 2b (3b) confirm what were discussed in Corollary 1 and 2, that is, as the option price b increases (or exercise price w_2 decreases), the retailer will decrease its option order q_1^* and firm order q^*, which will eventually result in less profit. However, the firm order quantity and expected profit of the retailer with options are still larger than that of without. The decline in the retailer's firm order will certainly lead to the decrease in the production of the supplier. Therefore, from Figure 2c (3c), it can be observed that the expected profit of the supplier $\pi_s(Q^*)$ will decrease if the option price b increases (or exercise price w_2 decreases).

Figure 2d (3d) reveals that, from the supply chain perspective, the high option price b (or high exercise price w_2) is unfavorable. Therefore, when maximizing its own profit by setting a reasonable option price or/and exercise price, the retailer should take the profit of the supply chain into full consideration so as to minimize the loss of the supply chain's performance. In addition, Figure 2c (3c) and 2d (3d) show that under such circumstance (0.59< α <0.72), although the expected profit of the supplier with options $\pi_s(Q^*)$ is lower than that of without $\pi_s(Q_0^*)$, the whole supply chain with options will be better off than without. It confirms what were discussed in Proposition 7.

Options contracts including put option are increasingly employed by firms across various industrial sectors such as energy, commodities, telecommunication and technology to manage demand uncertainty and hedge against the associated risks. Although we do not have a specific industry in mind when the proposed model is developed, the insights derived from our analytical and numerical results offer important managerial implications that can be utilized as strategic and operational guidance for firms to implement put option contract. For instance, the China Telecom Corporation Limited has recently considered applying the methodology to its purchasing strategy management of printed circuit board assembly. For the China Telecom Corporation Limited, the contract parameters of printed circuit board assembly are determined in advance, and the ordering quantities are negotiated with the supplier later. Furthermore, the delivery related service level (e.g., conditions and time) are critical regarding the supply of printed circuit board assembly. In this case, the China Telecom Corporation Limited can use the proposed models to optimize their supply chain decisions by setting the model parameters according to their specific circumstance.

8 Conclusions and suggestions for further research

We study a supply chain that consists of a supplier and a retailer who commits to a service level (α) to ensure customer service and customer demand. In order to reduce its down-ward risk, the retailer can purchase put options from the supplier. In this paper, we focus on the value of put option contacts on the supply chain management. To the best of our knowledge, this is the first study to consider both put option contacts and the service level constraint in the supply chain. Our research provides several interesting observations.

Observation 1: With the service level constraint, there are unique optimal solutions for both the retailer's order policies and supplier's production policies with and without put option contacts. Particularly, when the service level constraint (α) is lower than the maximum service level corresponding to the retailer's optimal order quantity (β for the case without option contracts and γ for the case with option contracts), the service level constraint is not binding and the retailer orders to achieve its maximum profit. Otherwise, the retailer should order quantity as per service level constraints (q^{α}) in both cases. This observation provides some insights into the ordering strategy of the retailer and the production strategy of the supplier under the put option contact in the presence of the service level constraint.

Observation 2: In both cases (with and without the put option contract), the optimal order quantity of the retailer is non-decreasing in the service level constraint but the expected profit of the retailer is non-increasing in it, while both the optimal production quantity and the expected profit of the supplier are non-decreasing in the service level constraint. This finding shows that high service level can always benefit the supplier. Therefore, the supplier would also prefer the retailer with a higher service level. However, for the retailer, it needs to balance the trade-off between the high customer satisfaction and the low expected profit. This observation offers significant insights into the retailer's service level strategy when there're simultaneous challenges to reduce down-ward risk and increase service level.

Observation 3: With the put option contract, the retailer will offer higher service level and earn more profit than without it, such effect is more salient when the demand is more volatile. However, whether the supplier can benefit from the put option contract depends on the service level constraint α. Only when the service level constraint is lower than the critical threshold ($\alpha \leq \alpha_0$), the supplier with the put option contract will be better off than without it. We also find that put option contract can effectively improve the decentralized system's performance, but this only applies to the scenario that the service constraint is not higher than the threshold ($\alpha < \gamma$).Under this case, the win-win situation can be achieved by the put option contract if the retailer is will to redistribute the profit gained from the put option contract. However, in the case of ($\alpha \geq \gamma$), the best solution for the whole supply chain system is stick with the wholesale price contract due to its simplicity. Moreover, our study also shows that with a service level constraint, the put option contract does not demonstrate any superiority in coordinating the supply chain as compared to the conventional wholesale price contract. This study provides broad opportunities for future research. First, a natural extension of our work is to consider more general supply chains, such as multi-suppliers and/or multi-retailers models (Choi, 2016.). Second, both the retailer and the supplier are assumed to be risk-neutral in our model. One possible extension of this work is to include other attitudes toward risks (such as loss aversion) of the decision maker. Finally, the study only considers the demand uncertainty. Another future extension is to incorporate the supply uncertainty in the modeling.

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Appendix

Proof of Lemma 1: Equation (1) shows that $\frac{\partial\pi_r(q,q_1)}{\partial q} = (p - w_1) - (p - w_2)F(q) - (w_2 - s)F(q - q_1)$, $\frac{\partial\pi_r(q,q_1)}{\partial q_1} = -b + (w_2 - s)F(q - q_1)$, $\frac{\partial^2\pi_r(q,q_1)}{\partial q^2} = -(p - w_2)f(q) - (w_2 - s)f(q - q_1) < 0$, $\frac{\partial^2\pi_r(q,q_1)}{\partial q_1^2} = -(w_2 - s)f(q - q_1) < 0$ and $\frac{\partial^2\pi_r(q,q_1)}{\partial q\partial q_1} = \frac{\partial^2\pi_r(q,q_1)}{\partial q_1\partial q} = (w_2 - s)f(q - q_1)$. Then $\begin{vmatrix} \frac{\partial^2\pi_r(q,q_1)}{\partial q^2} & \frac{\partial^2\pi_r(q,q_1)}{\partial q\partial q_1} \\ \frac{\partial^2\pi_r(q,q_1)}{\partial q_1\partial q} & \frac{\partial^2\pi_r(q,q_1)}{\partial q_1^2} \end{vmatrix} = (p - w_2)(w_2 - s)f(q - q_1) > 0$, so $\pi_r(q, q_1)$ is jointly concave in q and q_1. Let $\frac{\partial\pi_r(q,q_1)}{\partial q} = \frac{\partial\pi_r(q,q_1)}{\partial q_1} = 0$, then $q^\gamma = F^{-1}\left(\frac{p-w_1-b}{p-w_2}\right)$ and $q_1^\gamma = F^{-1}\left(\frac{p-w_1-b}{p-w_2}\right) - F^{-1}\left(\frac{b}{w_2-s}\right)$. Set $\gamma = \frac{p-w_1-b}{p-w_2}$, then $q^\gamma \equiv F^{-1}(\gamma)$. Therefore, $q^* = \max(q^\gamma, q^\alpha)$, that is, $q^* = \begin{cases} q^\gamma & if\alpha \le \gamma \\ q^\alpha & if\alpha > \gamma \end{cases}$, and $q_1^* = q^* - F^{-1}\left(\frac{b}{w_2-s}\right)$ ■

Proof of Corollary 1: Assume that the demand is normally distributed and the standard deviation is σ. Denote the probability density function and cumulative distribution function of the standard normal distribution as $\emptyset$ and Φ, respectively. (1) When $\alpha \le \gamma$, Lemma 1 implies that $q_1^* = q^\gamma - F^{-1}\left(\frac{b}{w_2-s}\right)$. Then $\frac{dq_1^*}{dw_1} = \frac{dq_1^\gamma}{dw_1} = -\frac{1}{f(q^\gamma)}(p - w_2) < 0$, $\frac{dq_1^*}{db} = -\frac{1}{f(q^\gamma)(p-w_2)} - \frac{1}{f\left[F^{-1}\left(\frac{b}{w_2-s}\right)\right](w_2-s)} < 0$ and $\frac{dq_1^*}{dw_2} = \frac{1}{f(q^\gamma)(p-w_2)^2} +$

$\frac{b}{f\left[F^{-1}\left(\frac{b}{w_2-s}\right)\right](w_2-s)} > 0$. In addition, Let $z^{\gamma} = \Phi^{-1}\left(\frac{p-w_1-b}{p-w_2}\right)$, we obtain $q^{\gamma} = \mu + \sigma z^{\gamma}$, where z^{γ} is the optimal quantile. Let $z^{1} = \Phi^{-1}\left(\frac{b}{w_2-s}\right)$, from the Equation (3), we have $q_1^* = \sigma(z^{\gamma} - z^{1})$. Thus, $\frac{dq_1^*}{d\sigma} = (z^{\gamma} - z^{1}) = \Phi^{-1}\left(\frac{p-w_1-b}{p-w_2}\right) - \Phi^{-1}\left(\frac{b}{w_2-s}\right) > 0$. So q_1^* is decreasing in w_1 and b, and increasing in w_2 and σ. (2) When $\alpha > \gamma$, Lemma 1 implies that $q_1^* = q^{\alpha} - F^{-1}(b/(w_2-s))$. We get $\frac{dq_1^*}{dw_1} = 0$, $\frac{dq_1^*}{db} = -\frac{1}{f\left[F^{-1}\left(\frac{b}{w_2-s}\right)\right](w_2-s)} < 0$ and $\frac{dq_1^*}{dw_2} = \frac{b}{f\left[F^{-1}\left(\frac{b}{w_2-s}\right)\right](w_2-s)^2} > 0$. So q_1^* is not affected by w_1 but decreasing in b, and increasing in w_2.

Proof of Corollary 2: (1) When$\alpha \le \gamma$, Lemma 1 implies that $q^* = q^{\gamma}$ and $q_1^* = q_1^{\gamma}$. From Equation (1), we have $\frac{d\pi_r(q^*,q_1^*)}{dw_1} = -q^{\gamma} + (p-w_2)\left[\frac{p-w_1-b}{p-w_2} - F(q^{\gamma})\right]\frac{dq^{\gamma}}{dw_1} = -q^{\gamma} < 0$, $\frac{d\pi_r(q^*,q_1^*)}{db} - q^{\gamma} + F^{-1}\left(\frac{b}{w_2-s}\right) < 0$ and $\frac{d\pi_r(q^*,q_1^*)}{dw_2} = \int_{F^{-1}\left(\frac{b}{w_2-s}\right)}^{q^{\gamma}} F(x)dx > 0$. So $\pi_r(q^*,q_1^*)$ is decreasing in w_1 and b, and increasing in w_2. (2) When $\alpha > \gamma$, Lemma 1 means that $q^* = q^{\alpha}$ and $q_1^* = q^{\gamma} - F^{-1}\left(\frac{b}{w_2-s}\right)$. From Equation (1), we get $\frac{d\pi_r(q^*,q_1^*)}{dw_1} = -q^{\alpha} < 0$, and $\frac{d\pi_r(q^*,q_1^*)}{db} = -q^{\alpha} + F^{-1}\left(\frac{b}{w_2-s}\right) < 0$, $\frac{d\pi_r(q^*,q_1^*)}{dw_2} = \int_{F^{-1}\left(\frac{b}{w_2-s}\right)}^{q^{\alpha}} F(x)dx > 0$. It also shows $\pi_r(q^*,q_1^*)$ is decreasing in w_1 and b, and increasing in w_2.■

Proof of Proposition 1: (1) When $\alpha \le \gamma$, $q^* = q^{\gamma}$. Equation (1) shows that $\pi_r(q^*,q_1^*)$ has nothing to do with α. (2) When $\alpha > \gamma$, $q^* = q^{\alpha}$ and $q_1^* = q^* - F^{-1}\left(\frac{b}{w_2-s}\right)$. From Equation (1), we have $\pi_r(q^*,q_1^*) = (p-w_1)q^{\alpha} - (p-w_2)\int_0^{q^{\alpha}} F(x)\,dx - (w_2-s)\int_0^{F^{-1}\left(\frac{b}{w_2-s}\right)} F(x)dx - b\left[q^{\alpha} - F^{-1}\left(\frac{b}{w_2-s}\right)\right]$. Then $\frac{d\pi_r(q^*,q_1^*)}{d\alpha} = \frac{d\pi_r(q^*,q_1^*)}{q^{\alpha}}\frac{dq^{\alpha}}{d\alpha} = (p-w_2)[F(q^{\gamma}) - F(q^{\alpha})]\frac{dq^{\alpha}}{d\alpha}$. Since $p > w_2$, $q^{\gamma} < q^{\alpha}$ and $dq^{\alpha}/d\alpha > 0$, so we get $\frac{d\pi_r(q^*,q_1^*)}{d\alpha} < 0$, it implies that $\pi_r(q^*,q_1^*)$ is decreasing in α. Therefore, $\pi_r(q^*,q_1^*)$ is non-increasing in the service constraints α. ■

Proof of Proposition 2: (1) When $\alpha \le \gamma$, $Q^* = q^* = q^{\gamma}$. Equation (6) shows that $\pi_s(Q^*)$ has nothing to do with α. (2) When $\alpha > \gamma$, $Q^* = q^* = q^{\alpha}$. From Equation (6), we have $\pi_s(Q^*) = (w_1 - c)q^{\alpha} + b\left[q^{\alpha} - F^{-1}\left(\frac{b}{w_2-s}\right)\right] - (w_2-s)\int_{F^{-1}\left(\frac{b}{w_2-s}\right)}^{q^{\alpha}} F(x)dx$. Then $\frac{d\pi_s(Q^*)}{d\alpha} = \frac{d\pi_s(Q^*)}{dq^{\alpha}}\frac{dq^{\alpha}}{d\alpha} = (w_2-s)\left[\frac{w_1+b-c}{w_2-s} - F(q^{\alpha})\right]\frac{dq^{\alpha}}{d\alpha}$. From $w_2 + s + b > w_2 + c$ and $s > 0$, we can get $w_2 + b - c > w_2 - s$. Since $w_2 + b > c$ and $w_2 > s$, so it can be seen that $\frac{w_2+b-c}{w_2-s} > 1$ and $\frac{w_2+b-c}{w_2-s} > \alpha = F(q^{\alpha})$. Thus, we can see that $\frac{d\pi_s(Q^*)}{d\alpha} > 0$, it implies that $\pi_s(Q^*)$ is increasing in α. Therefore, $\pi_s(Q^*)$ is non-increasing in the service constraints α. ■

Proof of Proposition 3: (1) If $\alpha \le \beta$, Lemma 1 and Equation (10) show that $q^* = q^\gamma$ and $q_0^* = q_0^\beta$. Because $b < \frac{(p-w_1)(w_2-s)}{p-s}$, then we get $\frac{\gamma}{\beta} = \left(1 - \frac{b}{p-w_1}\right)\frac{p-s}{p-w_2} > \left(1 - \frac{w_2-s}{p-s}\right)\frac{p-s}{p-w_2} = 1$. It implies that $\gamma > \beta$. Therefore, we get $q^\gamma > q_0^\beta$, that is, $q^* > q_0^*$. (2) If $\beta \le \alpha < \gamma$, we see that $q^* = q^\gamma$ and $q_0^* = q^\alpha$. Since $\alpha < \gamma$, then $q^* > q_0^*$. (3) If $\alpha \ge \gamma$, it can be seen that $q^* = q^\alpha$ and $q_0^* = q^\alpha$, which indicates that $q^* = q_0^*$.■

Proof of Proposition 4: When $\alpha \le \beta$, $q^* = q^\gamma$ and $q_0^* = q_0^\beta$. From Equation (1) and (7), we get $\pi_r(q_0^\beta, q_1) = (p-w_1)q_0^\beta - (p-w_2)\int_0^{q_0^\beta} F(x)\,dx - (w_2 - s)\int_0^{q_0^\beta - q_1} F(x)\,dx - bq_1$ and $\pi_r^0(q_0^\beta) = (p-w_1)q_0^\beta - (p-s)\int_0^{q_0^\beta} F(x)\,dx$. Then, we obtain $\Delta(q_1) = \pi_r(q_0^\beta, q_1) - \pi_r^0(q_0^\beta) = -(p-w_2)\int_0^{q_0^\beta} F(x)\,dx - (w_2 - s)\int_0^{q_0^\beta - q_1} F(x)\,dx - bq_1 + (p-s)\int_0^{q_0^\beta} F(x)\,dx$. From $\Delta(0) = 0$ and $\frac{d\Delta(q_1)}{dq_1}|_{q_1=0} = (w_2 - s)F(q_0^\beta) - b = (w_2 - s)\left[F(q_0^\beta) - \frac{b}{w_2-s}\right] > 0$, we know that $\Delta(q_1) > 0$, i.e. $\pi_r(q_0^\beta, q_1) > \pi_r^0(q_0^\beta)$. Because of $\pi_r(q^\gamma, q_1) > \pi_r(q_0^\beta, q_1)$, so we get $\pi_r(q^\gamma, q_1) > \pi_r^0(q_0^\beta)$. Therefore, we conclude that $\pi_r(q^*, q_1^*) > \pi_r^0(q_0^*)$. In addition, from $\frac{dq_1^*}{d\sigma} > 0$, we know that $\frac{d\Delta(q_1^*)}{d\sigma} = \frac{dq_1^*}{d\sigma}[(w_2 - s)F(q_0^\beta) - b] > 0$, so we get $\Delta\pi$ is increases in σ. Similarly, when $\beta < \alpha < \gamma$ and $\alpha \ge \gamma$, we have the same results. ■

Proof of Proposition 5: Proposition 3 shows that if $\alpha < \gamma$, $q^* > q_0^*$. Since $Q^* = q^*$ and $Q_0^* = q_0^*$, we can get $Q^* > Q_0^*$. However, it can be seen that $q^* = q_0^*$ if $\alpha \ge \gamma$, which indicates that $Q^* = Q_0^*$. ■

Proof of Proposition 6: (1) When $\alpha \le \beta$, $Q^* = q^* = q^\gamma$ and $Q_0^* = q_0^* = q_0^\beta$. From Equation (6), and $\pi_s^0(Q_0^*) = (w_1 - c)q_0^*$, let $\Delta(w_1) = \pi_s(Q^*) - \pi_s^0(Q_0^*)$, we get $\Delta(w_1) = (w_1 - c)(q^\gamma - q_0^\beta) + b\left[q^\gamma - F^{-1}\left(\frac{b}{w_2-s}\right)\right] - (w_2 - s)\int_{F^{-1}\left(\frac{b}{w_2-s}\right)}^{q^\gamma} F(x)\,dx$. Set $w_1^0 = p - \frac{b(p-s)}{w_2-s}$. If $w_1 = w_1^0$, then $q^\gamma = q_0^\beta = F^{-1}\left(\frac{b}{w_2-s}\right)$. It follows that $\Delta(w_1^0) = 0$. In addition, $\frac{d\Delta(w_1)}{d\,w_1}|_{w_1=w_1^0} = \frac{(w_1^0-c)(s-w_2)}{f\left[F^{-1}\left(\frac{b}{w_2-s}\right)\right](p-s)(p-w_2)}$. From $w_1^0 > c$, $w_2 > s$ and $f\left[F^{-1}\left(\frac{b}{w_2-s}\right)\right] > 0$, we can get $\frac{d\Delta(w_1)}{d\,w_1}|_{w_1=w_1^0} < 0$. Since $w_1 < w_1^0$, we obtain that $\Delta(w_1) > 0$, i.e. $\pi_s(Q^*) > \pi_s^0(Q_0^*)$.

(2) When $\beta < \alpha < \gamma$, $Q^* = q^* = q^\gamma$ and $Q_0^* = q_0^* = q^\alpha$. In this case, we get $\Delta(w_1) = (w_1 - c)(q^\gamma - q^\alpha) + b\left[q^\gamma - F^{-1}\left(\frac{b}{w_2-s}\right)\right] - (w_2 - s)\int_{F^{-1}\left(\frac{b}{w_2-s}\right)}^{q^\gamma} F(x)\,dx$, and $\frac{d\Delta(w_1)}{d\alpha} = -(w_1 - c)\frac{dq^\alpha}{d\alpha}$. Since $\frac{dq^\alpha}{d\alpha} > 0$, so we

see that $\frac{d\Delta(w_1)}{d\alpha} < 0$, it indicates that $\Delta(w_1)$ is decreasing in α. According to case (1), when $\alpha = \beta$, we have $\Delta(w_1) > 0$. Furthermore, let $\Gamma(w_1) = b\left[q^\gamma - F^{-1}\left(\frac{b}{w_2-s}\right)\right] - (w_2 - s)\int_{F^{-1}\left(\frac{b}{w_2-s}\right)}^{q^\gamma} F(x)\,dx$. Then $\Delta(w_1) = (w_1 - c)(q^\gamma - q^\alpha) + \Gamma(w_1)$ and $\frac{d\Gamma(w_1)}{dw_1} = (w_2 - s)\left[\frac{b}{w_2-s} - F(q^\gamma)\right]\frac{dq^\gamma}{dw_1}$. From equation (9) and $b < \frac{(p-w_1)(w_2-s)}{p-s}$, we get $F(q_0^\beta) > \frac{b}{w_2-s}$. Besides, from equation (2) and $q^\gamma > q_0^\beta$, we have $F(q^\gamma) > F(q_0^\beta)$. Therefore, we get $F(q^\gamma) > \frac{b}{w_2-s}$; Since $w_2 > s$, $\frac{dq^\gamma}{dw_1} < 0$, so we see that $\frac{d\Gamma(w_1)}{dw_1} > 0$, it implies that $\Gamma(w_1)$ is increasing in w_1. From $\Gamma(w_1) = 0$ and $w_1 < w_1^0$, we can see that $\Gamma(w_1) < 0$. If $\alpha = \gamma$, it's easy to get that $\Delta(w_1) = \Gamma(w_1) < 0$, while if $\alpha = \beta$, then $\Delta(w_1) > 0$. Therefore, it can be concluded that there is $\alpha_0 \in (\beta, \gamma)$, if $\alpha = \alpha_0$, then $\Delta(w_1) = 0$, i.e. $\pi_s(Q^*) = \pi_s^0(Q_0^*)$. Since $\Delta(w_1)$ is decreasing in α, so if $\beta < \alpha < \alpha_0$, then $\Delta(w_1) > 0$ i.e. $\pi_s(Q^*) > \pi_s^0(Q_0^*)$. However, we get that $\Delta(w_1) < 0$ if $\alpha_0 < \alpha < \gamma$, i.e. $\pi_s(Q^*) < \pi_s^0(Q_0^*)$.

(3) When $\alpha \geq \gamma$, $Q^* = q^* = q^\alpha$ and $Q_0^* = q_0^* = q^\alpha$, and $\Delta(w_2) = b\left[q^\alpha - F^{-1}\left(\frac{b}{w_2-s}\right)\right] - (w_2 - s)\int_{F^{-1}\left(\frac{b}{w_2-s}\right)}^{q^\alpha} F(x)\,dx$. Set $w_2^0 = \frac{b}{F(q^\alpha)} + s$, if $w_2 = w_2^0$, then $q^\alpha = F^{-1}\left(\frac{b}{w_2-s}\right)$ It follows that $\Delta(w_2^0) = 0$. Besides, $\frac{d\Delta(w_2)}{d\,w_2}\Big|_{w_2=w_2^0} = -\int_{F^{-1}\left(\frac{b}{w_2-s}\right)}^{q^\alpha} F(x)\,dx\Big|_{w_2=w_2^0} = 0$ and $\frac{d^2\Delta(w_2)}{dw_2^2} = \frac{b^2}{(w_2-s)^3 f\left[F^{-1}\left(\frac{b}{w_2-s}\right)\right]} < 0$. So $\Delta(w_2)$ is concave in w_2 and a unique optimal solution exists, that is, $\Delta(w_2) = 0$. Since $w_2 > w_2^0$, so we know that $\Delta(w_2) < 0$, i.e. $\pi_s(Q^*) < \pi_s^0(Q_0^*)$. ■

Proof of Proposition 7: Set $\Delta\pi_r = \pi_r(q^*, q_1^*) - \pi_r^0(q_0^*)$ and $\Delta\pi_s = \pi_s(Q^*) - \pi_s^0(Q_0^*)$, we will discuss the value of $\Delta\pi_r + \Delta\pi_s$ in three different cases below. *Case 1:* When $\alpha \leq \alpha_0$, Proposition 4 and 6 show that $\pi_r(q^*, q_1^*) > \pi_r^0(q_0^*)$ and $\pi_s(Q^*) > \pi_s^0(Q_0^*)$. It is safe to confirm that $\Delta\pi_r + \Delta\pi_s > 0$. *Case 2:* When $\alpha_0 < \alpha < \gamma$, Lemma 1 indicates that $q^* = q^\gamma$, and Equation (9) shows that $q_0^* = q^\alpha$. From Equation (1) and (7), we get $\Delta\pi_r = (p - w_1 g)(q^\gamma - q^\alpha) + (p - s)\int_0^{q^\alpha} F(x)\,dx - (p - w_2)\int_0^{q^\gamma} F(x)\,dx - (w_2 - s)\int_0^{F^{-1}\left(\frac{b}{w_2-s}\right)} F(x)dx - b\left[q^\gamma - F^{-1}\left(\frac{b}{w_2-s}\right)\right]$. Because $Q^* = q^* = q^\gamma$, $Q_0^* = q_0^* = q^\alpha$ and $\pi_s(Q_0^*) = (w_1 - c)q_0^*$, then from Equation (6), we have $\Delta\pi_s = (w_1 - c)(q^\gamma - q^\alpha) + b\left[q^\gamma - F^{-1}\left(\frac{b}{w_2-s}\right)\right] - (w_2 - s)\int_{F^{-1}\left(\frac{b}{w_2-s}\right)}^{q^\gamma} F(x)dx$. So, $\Delta\pi_r + \Delta\pi_s = (p - c)(q^\gamma - q^\alpha) + (p - s)\int_{q^\gamma}^{q^\alpha} F(x)\,dx$ and $\frac{d(\Delta\pi_r + \Delta\pi_s)}{d\alpha} = (p - s)\left[F(q^\alpha) - \frac{p-c}{p-s}\right]\frac{dq^\alpha}{d\alpha}$. From $F(q^\alpha) < (p - c)/(p - s)$, $p > s$, and $dq^\alpha/d\alpha > 0$, we have $\frac{d(\Delta\pi_r + \Delta\pi_s)}{d\alpha} < 0$. When $\alpha = \gamma$, it's observed that $\Delta\pi_r +$

$\Delta\pi_s = 0$. Therefore, $\Delta\pi_r + \Delta\pi_s > 0$ when $\alpha_0 < \alpha < \gamma$. *Case 3:* When $\alpha \geq \gamma$, we can see that $q^* = q_0^* = q^\alpha$. In the same way as case 2, we can get $\Delta\pi_r + \Delta\pi_s = 0$. The analysis above indicates that when $\alpha < \gamma$, $\pi_r(q^*, q_1^*) + \pi_s(Q^*) > \pi_r^0(q_0^*) + \pi_s^0(Q_0^*)$; when $\alpha \geq \gamma$, $\pi_r(q^*, q_1^*) + \pi_s(Q^*) = \pi_r^0(q_0^*) + \pi_s^0(Q_0^*)$. ■

Proof of Proposition 8: Because $b < \frac{(p-w_1)(w_2-s)}{p-s}$, then we get $\frac{\gamma}{\beta} = \left(1 - \frac{b}{p-w_1}\right)\frac{p-s}{p-w_2} > \left(1 - \frac{w_2-s}{p-s}\right)\frac{p-s}{p-w_2} = 1$, which implies that $\beta < \gamma$. In addition, from $w_1 + b + s > w_2 + c$, we get $s - c > w_2 - w_1 - b$, so $\tau - \gamma = \frac{(s-c)(p-w_2)-(w_2-w_1-b)(p-s)}{(p-s)(p-w_2)} > \frac{(w_1-w_2+b)((w_2-s)}{(p-s)(p-w_2)} > 0$, which implies that $\gamma < \tau$. Therefore, we have $\beta < \gamma < \tau$. From Lemma 1, Equation (5), (10) and (14), we get (1) if $\alpha \leq \beta$, $Q_0^* = q^\beta$, $Q^* = q^\gamma$ and $Q_I^* = q^\tau$. (2) if $\beta < \alpha \leq \gamma$, $Q_0^* = q^\alpha$, $Q^* = q^\gamma$ and $Q_I^* = q^\tau$. (3) if $\gamma < \alpha < \tau$, $Q_0^* = Q^* = q^\alpha$ and $Q_I^* = q^\tau$. (4) if $\alpha \geq \tau$, $Q_0^* = \mathrm{Q}^* = Q_I^* = q^\alpha$. The analysis above show that $Q_0^* < Q^* < Q_I^*$ if $\alpha < \tau$, which means that the optimal production quantity of the supplier in two cases (with and without put option contract) are always less than that of the integrated supply chain. It further indicates that the supply chain coordination cannot be achieved in both cases. However, if $\alpha \geq \tau$, we can see that $Q_0^* = Q^* = Q_I^\tau$, which means that the optimal production quantity of the supplier in two cases are always equal to that of the integrated supply chain. It further indicates that the supply chain coordination can be achieved in both cases. ■