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NILAI TOTAL KETIDAKTERATURAN-H PADA GRAF C_n x P_3

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Abstrak

Penentuan nilai total ketidakteraturan dari semua graf belum dapat dilakukan secara lengkap. Penelitian ini bertujuan untuk menentukan nilai total ketidakteraturan-H pada graf C_n x P_3 untuk $n \geq 3$. Penentuan nilai total ketidakteraturan-H pada graf C_n x P_3 dengan menentukan batas bawah terbesar dan batas atas terkecil. Batas bawah dianalisis berdasarkan sifat-siat graf dan teorema pendukung lainnya. Sedangkan batas atas dianalisa dengan pemberian label pada titik dan sisi pada graf C_n x P_3.Berdasarkan hasil penelitian ini diperoleh nilai total ketidakteraturan-H pada graf (C_n x $P_{3,}$ C_4)=$\left\lceil\frac{2n+7}{8}\right\rceil$ dengan $n \geq 3$.

Kata kunci : Selimut-H, Nilai total ketidakteraturan-H

1. Pendahuluan

Graf adalah pasangan dua buah himpunan yaitu himpunan titik dan himpunan sisi, dinotasikan dengan $G = (V, E)$ di mana V menyatakan himpunan titik yang tak kosong dan E adalah himpunan sisi yang merupakan pasangan tak terurut dari titik-titik V. Pelabelan graf merupakan suatu topik dalam teori graf. Objek kajiannya berupa graf yang secara umum direpresentasikan dengan titik dan sisi, dan himpunan bagian bilangan yang disebut label. Pelabelan graf adalah suatu fungsi dengan domain himpunan titik dan himpunan sisi atau keduanya dengan *range*nya bilangan riil.

Salah satu jenis pelabelan tidak teratur lainnya adalah pelabelan total tidak teratur$-H$(H-*Irreguler total labelling*). Pelabelan jenis ini melibatkan konsep selimut graf. Pelabelan ini di perkenalkan oleh Baca dkk. Secara umum pelabelan total tidak teratur-H dari suatu graf di definisikan sebagai berikut. Misal G adalah suatu graf, selimut sisi dari G (*edge covering*) adalah koleksi subgraf $H_1, H_2, \ldots, H_t$ dari graf G sedemikian sehingga setiap sisi dari G termuat dalam paling sedikit satu subgraf H_i, dimana $i = 1,2, \ldots, t$. Dalam hal ini, maka G disebut memiliki selimut (sisi)-$(H_1, H_2, \ldots, H_t)$ ($(H_1, H_2, \ldots, H_t)$-(*edgecovering*). Jika setiap subgraf H_i isomorfik dengan suatu graf H, maka dikatakan G memiliki suatu selimut-H(H *covering*).

Terdapat beberapa hasil penelitian terkait dengan pelabelan total H tidak teratur antara lain Bača dkk (2017) telah menentukan nilai total sisi (ehs) dan nilai total titik (vhs) pada graf kipas, lintasan dan ladder. Bača dkk (2017) telah menentukan nilai total L_n dan F_n tidak teratur (ths) pada graf planar, dan Bača dkk (2017) telah menentukan nilai total P_m tidak teratur (ths) pada graf P_n dan nilai total C_m tidak teratur (ths) pada graf L_n dan C_m merupakan graf lingkaran dengan m titik $ths(F_n, C_3)$ dimana F_n merupakan graf Kipas dengan n titik dan C_3 graf Lingkaran dengan 3 titik. Hesti Agustin dkk (2017) telah menentukan $ths(Amal(C_3, v, n)$ dimana $Amal(C_3, v, n)$ merupakan banyaknya graf C_3 yang di amalgamasi, dan $ths(Shack(C_m, v, n)$.

2. Hasil dan Pembahasan

Himpunan titik dan himpunan sisi graf $C_n \times P_3$, untuk n bilangan bulat positif $n \geq 3$ adalah sebagai berikut :

$V(C_n \times P_3) = \{x_i, y_i, z_i | i = 1,2, \dots, n\}$

$${}_i z_{i+1}, z_1 z_n \,| i = 1,2, \dots, n\}$$

$$E(C_n \times P_3) = \{x_i x_{i+1}, x_i x_n \cup y_i y_{i+1}, y_n \cup z$$

$$\cup \{x_i y_i \cup y_i z_i \,| i = 1,2, \dots, n\}$$

Dengan menggunakan himpunan titik dan himpunan sisi tersebut, maka didefinisikan himpunan titik dan himpunan sisi dari subgraf $H_i \cong C_4$ di $C_n \times P_3$ sebagai berikut:

Untuk

$$V(H_i) = \{x_i, y_i, x_{i+1}, y_{i+1}\}$$

$$E(H_i) = \{x_i y_i,\ x_i x_{i+1}, x_{i+1} y_{i+1}, y_i y_{i+1}\}$$

$$V(H_n) = \{x_1, y_1, x_n, y_n\}$$

$$E(H_n) = \{x_1 x_n, y_1 y_n, x_n y_n, x_1 y_1\}$$

Untuk dengan $j = 1, \cdots, n-1$

$$V(H_i) = \{y_i, z_i, y_{i+1}, z_{i+1}\}$$

$$E(H_i) = \{y_i z_i,\ y_i y_{i+1}, y_{i+1} z_{i+1}, z_i z_{i+1}\}$$

$$V(H_{2n}) = \{y_1, z_1, y_n, z_n\}$$

$$E(H_{2n}) = \{y_1 z_1, y_1 y_n, y_n z_n, y_1 z_1\}$$

$i = 1, \cdots, n-1$

$i = n + j$

Teorema 2.6.2 *Misalkan* $C_n \times P_3$ *adalah graf dengan* $n \geq 3$ *maka,*

$$ths\,(C_n \times P_3, C_4) \leq \left\lceil \frac{2n+7}{8} \right\rceil.$$

Bukti :

Untuk membuktikan bahwa ths$(\mathrm{C_n} \times \mathrm{P_3}, \mathrm{C_4}) \leq \left\lceil \frac{2\mathrm{n}+7}{8} \right\rceil$ maka akan dibuktikan ths $(\mathrm{C_n} \times \mathrm{P_3}, \mathrm{C_4}) \geq \left\lceil 1 + \frac{\mathrm{t}-1}{|V(H)|+|E(H)|} \right\rceil$ dan ths$(\mathrm{C_n} \times \mathrm{P_3}, \mathrm{C_4}) \leq \left\lceil \frac{2\mathrm{n}+7}{8} \right\rceil$.

Untuk membuktikan ths$(\mathrm{C_n} \times \mathrm{P_3}, \mathrm{C_4}) \leq \left\lceil \frac{2\mathrm{n}+7}{8} \right\rceil$, maka digunakan Teorema 2.6.1 yaitu ths$(\mathrm{C_n} \times \mathrm{P_3}, \mathrm{C_4} \geq \left\lceil 1 + \frac{\mathrm{t}-1}{|V(H)|+|E(H)|} \right\rceil$ dengan t adalah banyaknya subgraf H. Perhatikan bahwa banyaknya subgraf C_4 dari $C_n \times P_3$ adalah 2n dan $|V(C_4)| = 4$ dan $|E(C_4)| = 4$. Jadi ths$(\mathrm{C_n} \times \mathrm{P_3}, \mathrm{C_4}) \leq \left\lceil 1 + \frac{2\mathrm{n}-1}{8} \right\rceil = \left\lceil \frac{2\mathrm{n}+7}{8} \right\rceil$. Selanjutnya akan ditentukan bahwa $(\mathrm{C_n} \times \mathrm{P_3}, \mathrm{C_4}) \leq \left\lceil \frac{2\mathrm{n}+7}{8} \right\rceil$. Untuk membuktikan bahwa $(\mathrm{C_n} \times \mathrm{P_3}, \mathrm{C_4}) \leq \left\lceil \frac{2\mathrm{n}+7}{8} \right\rceil$, maka akan dikonstruksi suatu fungsi pelabelan ketidakteraturan C_4 total pada $\mathrm{C_n} \times \mathrm{P_3}$ sebagai berikut:

$$f(x_i) = \begin{cases} \left\lceil \frac{i}{2} \right\rceil; & i = 1, 2, .., \left\lfloor \frac{n+9}{3} \right\rfloor \\ \left\lceil \frac{n+2-i}{2} \right\rceil; & i = \left\lfloor \frac{n+12}{3} \right\rfloor, \dots, n \end{cases}$$

$$f(z_i) = \begin{cases} \left\lceil \frac{i+1}{2} \right\rceil; & i = 1, 2, .., \left\lfloor \frac{n+9}{3} \right\rfloor \\ \left\lceil \frac{n+3-i}{2} \right\rceil; & i = \left\lfloor \frac{n+12}{3} \right\rfloor, \dots, n \end{cases}$$

$$f(x_i y_i) = \begin{cases} \left\lceil \frac{i+1}{2} \right\rceil; & i = 1, 2, .., \left\lfloor \frac{n+9}{3} \right\rfloor \\ \left\lceil \frac{n+2-i}{2} \right\rceil; & i = \left\lfloor \frac{n+12}{3} \right\rfloor, \dots, n \end{cases}$$

$$f(y_i z_i) = \begin{cases} \left\lceil \frac{i+1}{2} \right\rceil; & i = 1, 2, .., \left\lfloor \frac{n+9}{3} \right\rfloor \\ \left\lceil \frac{n+2-i}{2} \right\rceil; & i = \left\lfloor \frac{n+12}{3} \right\rfloor, \dots, n \end{cases}$$

- Untuk ganjil

$$f(x_i x_{i+1}) = \begin{cases} \left\lceil \frac{i}{2} \right\rceil; & i = 1, 2, .., \left\lceil \frac{n}{2} - 1 \right\rceil \\ \left\lceil \frac{n+2-i}{2} \right\rceil; & i = \left\lceil \frac{n}{2} \right\rceil \\ \left\lceil \frac{n+1-i}{2} \right\rceil; & i = \left\lceil \frac{n+12}{3} \right\rceil, \dots, n-1 \\ f(x_n x_1) = 1 \end{cases}$$

$$f(y_i y_{i+1}) = \begin{cases} \left\lceil \frac{i+1}{2} \right\rceil; & i = 1, 2, .., \left\lceil \frac{n+9}{3} \right\rceil \\ \left\lceil \frac{n+2-i}{2} \right\rceil; & i = \left\lceil \frac{n+12}{3} \right\rceil, \dots, n \\ y_n y_1 = 1 \end{cases}$$

$$f(z_i z_{i+1}) \begin{cases} \left\lceil \frac{i}{2} \right\rceil; & i = 1, 2, .., \left\lceil \frac{n}{2} - 1 \right\rceil \\ \begin{cases} \left\lfloor \frac{i+1}{2} \right\rfloor; \\ \left\lceil \frac{n}{4} \right\rceil + 1; \end{cases} & \begin{matrix} i = \left\lceil \frac{n}{2} \right\rceil genap \\ i = \left\lceil \frac{n}{2} \right\rceil ganjil \end{matrix} \\ \left\lceil \frac{n+1-i}{2} \right\rceil; & i = \left\lfloor \frac{n+12}{3} \right\rfloor, \dots, n-1 \\ f(z_n z_1) = 1 \end{cases}$$

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- Untuk n genap

$$f(x_i x_{i+1}) = \begin{cases} \left\lceil \frac{i}{2} \right\rceil; & i = 1, 2, .., \left\lceil \frac{n}{2} - 1 \right\rceil \\ \begin{cases} \left\lceil \frac{n}{4} \right\rceil; \\ \left\lceil \frac{n}{4} \right\rceil + 1; \end{cases} & \begin{matrix} i = \left\lceil \frac{n}{2} \right\rceil\ genap \\ i = \left\lceil \frac{n}{2} \right\rceil ganjil \end{matrix} \\ \left\lceil \frac{n+1-i}{2} \right\rceil; & i = \left\lceil \frac{n+12}{3} \right\rceil, \dots, n-1 \\ f(x_n x_1) = 1 \end{cases}$$

$$f(y_i y_{i+1}) = \begin{cases} f(y_1 y_2) = 1 \\ \left\lceil \frac{i+2}{2} \right\rceil; & i = 2, .., \left\lceil \frac{n}{2} \right\rceil \\ \left\lceil \frac{n+2-i}{2} \right\rceil; & i = \left\lceil \frac{n+1}{2} \right\rceil, \dots, n \\ y_n y_1 = 1 \end{cases}$$

$$f(z_i z_{i+1}) = \begin{cases} \left\lceil \frac{i}{2} \right\rceil; & i = 1, 2, .., \left\lceil \frac{n}{2} - 1 \right\rceil \\ \begin{cases} \left\lceil \frac{n}{4} \right\rceil; \\ \left\lceil \frac{n}{4} \right\rceil + 1; \end{cases} & \begin{matrix} i = \left\lceil \frac{n}{2} \right\rceil genap \\ i = \left\lceil \frac{n}{2} \right\rceil ganjil \end{matrix} \\ \left\lceil \frac{n+1-i}{2} \right\rceil; & i = \left\lfloor \frac{n+12}{3} \right\rfloor, \dots, n-1 \\ f(z_n z_1) = 1 \end{cases}$$

selanjutnya akan ditunjukkan bahwa bobot setiap subgraf berbeda untuk pada $C_n \times P_3$ sebagai berikut :

1. Untuk genap

$$W_1 = W_t(H_n) = \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n-1}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n-1}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n-1}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil + \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{1}{2}\right\rceil$$

$$< \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil + \left\lceil\frac{n+3-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil +$$

$$\left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{1}{2}\right\rceil + \left\lceil\frac{3}{2}\right\rceil + 1 + 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil = W_t(H_{2n})$$

$$W_2 = W_t(H_{2n}) = \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n-1}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n-1}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil +$$

$$\left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n-1}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{1}{2}\right\rceil + \left\lceil\frac{3}{2}\right\rceil + 1 + 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil$$

$$< \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + 1 + \left\lceil\frac{i+2}{2}\right\rceil$$

n

$$= \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{3}{2}\right\rceil + \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{3}{2}\right\rceil = w_t(H_1)$$

$$W_3 = W_t(H_1) = \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + 1 + \left\lceil\frac{i+2}{2}\right\rceil$$

$$= \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{3}{2}\right\rceil + \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{3}{2}\right\rceil$$

$$< \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil + 1 + \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{3}{2}\right\rceil + 1 + \left\lceil\frac{3}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{3}{2}\right\rceil = W_t(H_{n+1})$$

$$W_4 = W_t(H_{n+1}) = \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil + 1 + \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{3}{2}\right\rceil + 1 + \left\lceil\frac{3}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{3}{2}\right\rceil$$

$$< \left(\left\lceil\frac{\left\lceil\frac{n}{2}\right\rceil+1}{2}\right\rceil\right) + \left(\left\lceil\frac{\left\lceil\frac{n}{2}\right\rceil+1}{2}\right\rceil\right) + \left(\left\lceil\frac{\left\lceil\frac{n}{2}\right\rceil+1}{2}\right\rceil\right) + \left(\left\lceil\frac{\left\lceil\frac{n}{2}\right\rceil+1}{2}\right\rceil\right)$$

$$+ \left(\left\lceil\frac{\left\lceil\frac{n}{2}\right\rceil+1}{2}\right\rceil\right) + \left(\left\lceil\frac{n}{4}\right\rceil\right) + \left(\left\lceil\frac{\left\lceil\frac{n}{2}\right\rceil+1}{2}\right\rceil\right) + \left(\left\lceil\frac{\left\lceil\frac{n}{2}\right\rceil+1}{2}\right\rceil\right) = W_t(H_{3n})$$

2. Untuk ganjil

$$W_1 = W_t(H_n) = \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n-1}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n-1}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil + \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{1}{2}\right\rceil$$

$$< \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil + \left\lceil\frac{n+3-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{1}{2}\right\rceil + \left\lceil\frac{3}{2}\right\rceil + 1 + 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil = W_t(H_{2n})$$

$$W_2 = W_t(H_{2n}) = \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil + \left\lceil\frac{n+3-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil + \left\lceil\frac{n+2-n}{2}\right\rceil + \left\lceil\frac{n+1-n}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil + \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{1}{2}\right\rceil$$

$$< \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil$$

$$= \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{3}{2}\right\rceil + \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{3}{2}\right\rceil = w_t(H_1)$$

$$W_3 = W_t(H_1) = \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil$$

$$= \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{3}{2}\right\rceil + \left\lceil\frac{1}{2}\right\rceil + 1 + 1 + \left\lceil\frac{3}{2}\right\rceil$$

$$< \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{3}{2}\right\rceil + 1 + \left\lceil\frac{3}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{3}{2}\right\rceil = W_t(H_{n+1})$$

$$W_4 = W_t(H_{n+1}) = \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+2}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil + \left\lceil\frac{i+1}{2}\right\rceil$$

$$= 1 + \left\lceil\frac{3}{2}\right\rceil + 1 + \left\lceil\frac{3}{2}\right\rceil + 1 + \left\lceil\frac{1}{2}\right\rceil + 1 + \left\lceil\frac{3}{2}\right\rceil$$

n

$$< \left\lceil\frac{n}{2}\right\rceil \quad \left\lceil\frac{n}{2}\right\rceil \quad \left\lceil\frac{n}{2}\right\rceil \quad \left\lceil\frac{n}{2}\right\rceil \quad \left\lceil\frac{n}{2}\right\rceil$$

$$\left\lceil\frac{n}{2}\right\rceil \quad \left\lceil\frac{n}{2}\right\rceil \quad \left\lceil\frac{n}{2}\right\rceil \quad = W_t(H\ _n)$$

Berdasarkan w_1, w_2, w_3, w_4 maka diperoleh bahwa bobot setiap subgraf berbeda.

Daftar Pustaka

[1] Agustin, I.H. 2017. *On H-Irregulerity Strengh of Graph:A New Notion*. *Journal of Physic*: Confrens Series 855.

[2] Ahmad, A. 2014. Irregular Total Labeling of Disjoint Union of Prisms and Cycles. *Australasian Journal of Combinatorics*. 59 : 98-106.

[3] Ashraf, F., Ba ˇa, M., Kim ́kov ́, Z., and Semani ˇov ́-Fe ˇovčíkov ́, A. (2016). On Vertex and Edge H-Irregularity Strength of Graphs, *Discrete Math. Algorithms and Applications. 8.* 13 pages

[4] Bača, M., dkk. 2017. *On H-Irregulerity Strengh of Graph*. *Discucsiones Mathematics*. Graph Theory 37(2017) 1067-1078.

[5] Baca, M., Jendrol., Miller, M., dan Ryan, J. 2007. On Irregular total Labelings. *Discrere Mathematics*. 307 : 1378-1388.

[6] Indriati D, Widodo, Indah IE, Sugeng KA. 2015. On Total Irregularity Strength of Double-Star and Related Graphs. *Procedia Computer Science 74* Hal 118-123. Elsevier: Indonesia.

[7] Ramdani R dan Salman ANM. 2013. On The Total Irregularity Strength of Some Cartesians Product Graphs. Int.J.Graphs Comb.,10 No.2.Page 199-209. AKCE. Bandung : Indonesia.

[8] Tarawneh I, Hasni R, Ahmad A. 2016. On the Edge Irregularity Strength of Corona Product of Cyccle with Isolated Vertices. *AKCE International Journal of Graphs and Combinatorics 13.Page 213-217*. Elsevier. Mathematics Science.