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How Dense Would the Little Prince's Planet be?

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Abstract
In the eponymous book by Antoine de Saint-Exupéry, the Little Prince claims he has watched 44 sunsets in a day. In this paper, this piece of information is used to determine his planet's radius, mass, volume, and density. Using a value of 24 min per sunset, it was estimated that the Little Prince's planet radius, mass, volume, and density are 5625.60 m, 4.65×10^{18} kg, 7.458×10^{11} m^3 and 6.235×10^{6} kg m^{-3} respectively. This would mean that the Little Prince's planet is 276 times denser that Osmium, the densest element on Earth.

Introduction

The Little Prince is a 1943 novella in which the narrator, an aviator, encounters a young prince fallen to Earth from another planet. As the narrator gets to know the Little Prince, he learns that the Prince's planet was so small that one day he saw the sunset "forty-four times", and all he had to do was to move his chair "a few steps" [1].

Figure 1 – The Little Prince on his planet [2]

This is a poetic statement, but it can serve as a basis for estimating some characteristics of the Little Prince's planet, such as its radius, volume, mass, and density.

Estimating the number of sunsets in a day

Sunset is defined as the disappearance of the Sun behind the horizon because of the planet's rotation. Assuming that a planet rotates by 6° during a sunset, as it does during civil twilight [3], one could technically see 60 sunsets (360° / 6° = 60 sunsets) in a 24 hour day (24 h × 60 min = 1440 min) if one could teleport to the next sunset location at the end of each sunset. Each sunset would then be 24 minutes long (1440 min / 60 sunsets = 24 min per sunset). The Little Prince only watched 44 sunsets, which less than the 60 theoretically possible. Assuming the Little Prince's planet has a 24 hour day, and each sunset being 24 min, the Little Prince spent 63360 s watching sunsets.

Estimating the radius of the planet

Assuming the Little Prince's planet has a 24 hour day (24 h × 60 min × 60 s = 86400 s), and each sunset being 24 min, it would leave the Little Prince 23040 s (86400 s – 63360 s = 23040 s) to travel between sunsets. The Little Prince is the size of a child, and is described as young in the book [4]. Assuming he is the same age as the intended audience, he is around 6-year old. The walking speed of a 6-year old human is 1.10 m s^{-1} [5]. Thus, the total distance the Little Prince could walk would be:

$$d_t = s \times t \qquad (1)$$

Where d_t is the total distance the Little Prince could walk, s is his speed, and t is the time he has left to walk between the sunsets. It results that the Little Prince could only walk 25,344 m in total. As the Little Prince has to walk 43 times between 44 sunsets, it results in 589 m between each sunset.

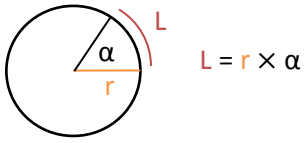

Figure 2 – Relationship between the arc length L of a circle of radius r, to the central angle α in radians

The radius of the planet could be then calculated using this formula derived from Figure 2:

$$r = \frac{L}{\alpha}, \qquad (2)$$

where r is the radius of the planet, L is the maximum distance that the little Prince could walk between each sunset, and α is the 6° angle converted to radians (0.1047 rad). The radius of the little Prince's planet is thus 5625.60 m at most.

Estimating the mass, volume and density of the planet

Assuming the gravity of the Little Prince's planet is the same as the Earth, the gravitation equation can be used to calculate its mass:

$$m_p = \frac{g \times r^2}{G}, \qquad (3)$$

where g is the gravitational acceleration (9.81 m s^{-2}), G is the gravitational constant (6.67×10^{-11} m^3 kg^{-1} s^{-2}), and m_p is the mass of the planet. It results that the Little Prince's planet mass is 4.65×10^{18} kg.

The volume of the planet can be calculated with the equation for the volume of a sphere:

$$V_p = \frac{4 \times \pi \times r^3}{3}, \qquad (4)$$

where V_p is the volume of the planet. It results that the volume of the planet is 7.46×10^{11} m^3.

Finally, the density of the planet can be calculated using the density formula:

$$\rho_p = \frac{m_p}{V_p}, \qquad (5)$$

where ρ_p is the density of the Little Prince's planet. It results that the density of the little Prince's planet is 6.23×10^{6} kg m^{-3}.

Based on these calculations, the Little Prince's planet would be around 1130 times denser than Earth (ρ_E = 5,514 kg m^{-3} [6]) and around 276 times denser than the densest element on Earth (Osmium, ρ_o = 22,587 kg m^{-3} [7]).

If the Little prince's planet had the same density as Earth, and rearranging the equations (3), (4), and (5), the gravitational acceleration would be:

$$g_p = \frac{4 \times \rho_E \times G \times r \times \pi}{3} \qquad (6)$$

Where g_p is the gravitational acceleration on the Little Prince's planet, and ρ_E is the density of Earth. The Little Prince's planet's gravitational acceleration would thus be 8.67×10^{-3} m s^{-2}, which is 1131 times less than Earth gravitational acceleration.

Conclusion

It was estimated that the Little Prince's planet radius, mass, volume, and density are 5625.60 m, 4.65×10^{18} kg, 7.458×10^{11} m^3 and 6.235×10^{6} kg m^{-3} respectively. According to the equations (3), (4), and (5), maximising the radius of the planet minimises its density. Thus, the little Prince's planet is either bigger, and it is impossible to see 44 sunsets by walking just a few steps, or the gravitational acceleration is lower on the little Prince's planet, 1131 times less if the Little Prince's planet had the same density than Earth. But in poetry, as said Saint-Exupéry, "figures are a matter of indifference" [8].

References

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[8] Saint-Exupéry, A. & Howard, R., *The Little Prince* (HMH Books), Chapter 4.