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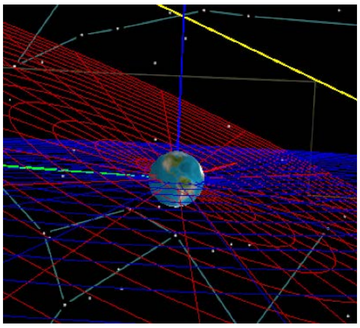

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Case study

Active Learning in Physics, Astronomy and Engineering with NASA's General Mission Analysis Tool

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Abstract

Astrodynamics is the study of the motion of artificial satellites and spacecraft, subject to both natural and artificially induced forces. It combines celestial mechanics, attitude dynamics and aspects of positional astronomy to describe spacecraft motion and enable the planning and analysis of missions. It is of significant interdisciplinary interest with relevance to physics, astronomy and spaceflight engineering, but can be challenging to deliver in an effective, engaging manner because of the often abstract nature of some concepts, the four-dimensional nature of the problems, and the computation required to explore realistic astrodynamics behaviour. The University of Leicester has adopted NASA's General Mission Analysis Tool (GMAT) as a core resource to support active learning in this subject for students at Level 6 (BSc) and Level 7 (MSc). This paper describes our approach to the implementation of GMAT as an essential element of teaching and learning in the subject.

Keywords: Teaching Practice; Flipped Classroom Teaching; Physics & Astronomy; Astrodynamics; Simulations; Celestial Mechanics; Visualisation.

Introduction

Astrodynamics is the study of the motion of artificial satellites, subject to both natural and artificially induced forces (Griffin & French, 1991, quoted in Vallado, 2013). Aside from its obvious relevance in courses related to space exploration and spaceflight systems engineering, it provides opportunities to demonstrate the application of fundamental concepts such as gravitation, Newton's law of motion, and Kepler's laws; it is as applicable to a description of the motion of natural bodies in the universe, as it is to the planning of space missions.

Learners and instructors face several challenges when exploring the subject. At High School level, a simple treatment of orbits based on the balancing of gravitational and centripetal forces can lead to familiar results such as Kepler's laws, which can be appreciated in two dimensions. But the subject is intrinsically four dimensional, involving time-varying parameters in three spatial dimensions; extension of the theory to three or more bodies leads to rapidly increasing complexity, and this intricate, multi-dimensional problem is poorly served by the traditional 2D learning environment of blackboard and paper. While generations of successful flight dynamicists had their first encounters with the topic in those traditional teaching environments, the availability of sophisticated and validated software tools developed for the research and industrial space community is allowing a step change in the way astrodynamics and related subjects can be taught, using approaches which are aligned with current pedagogies to improve the effectiveness of the experience for both learner and facilitator.

The Constructivist theory of learning states that acquiring knowledge is not simply a matter of the teacher transmitting information to the learner; such an approach can lead to rote learning and inert knowledge (Bruer, 1993; Perkins, 1992). Instead, Constructivism holds that the learner constructs knowledge with their own activities, building on what they already know (Biggs & Tang, 2011). As discussed concisely by Dori & Belcher (2005), "Such ownership enables the learner to understand the knowledge in an intimate way that cannot be achieved by mere memorization". Active learning takes place when new information arrives to challenge the existing mental framework; in this situation, the learner takes an active role in the learning process, adjusting their cognitive framework so that it is consistent with the new information. This fosters meaningful learning and deep understanding of physical phenomena.

Considered in these Constructivist terms, the role of simulations and models such as GMAT is to provide the information which challenges the existing cognitive framework, in an effective and engaging way. The use of visualisation as a tool to facilitate this active process and to support learning is discussed widely in the literature. Notable examples include Zhang & Linn (2011), who describe a study into the use of dynamic visualisations to support science learning, specifically chemistry. Dori & Belcher (2005) discuss a technology-enabled active learning environment incorporating visualisation and simulation technologies to support student learning in electromagnetism, aligned with the philosophy of social constructivism. Several reviews of the subject have also been carried out. Rutten, Van Joolingen, & Van der Veen (2012) consider research in the use of computer simulations in science education, finding that all reviewed studies report positive results where simulations were used to replace or enhance traditional lectures, but that it was necessary to provide learners with instructional support during the use of the simulations to assist in e.g. hypothesis generation, investigation planning, and monitoring of learning activities (reporting work by Alfieri, Brooks, Aldrich, & Tenenbaum, 2011). Rutten et al. note findings by Windschitl and Andre (1998) that traditional objectivist approaches to instruction can reduce the use of simulations to "cookbook" treatments which deprive the learner of the opportunity to create, test and evaluate their own hypotheses. In terms of fundamental pedagogy, this observation is consistent with Bruner (1966), who proposes the theory of "discovery learning", closely related to

Constructivism, which holds that the learner is more likely to recall and understand concepts which they have discovered independently, than those which have been taught directly. Appropriately designed and applied software simulation offers a “discovery space” within which these explorations can take place - particularly in subjects that are challenging to support through more practical experimental activities. However, when designing learning activities with the implementation of this paradigm in mind, criticisms of discovery learning and related approaches must be considered; Kirschner, Sweller, & Clark (2006) put forward evidence from studies in human cognition and practical implementations of “Problem Based Learning”, suggesting that unguided or minimally guided instruction is ineffective, arguing strongly for the superiority of guided instruction.

The design of the GMAT Astrodynamics Workshop described in this paper attempts to capture the opportunities which evidence suggests are offered by appropriate simulation, allowing students the freedom to explore their own ideas particularly in the latter stages of the activity, while providing a support framework that avoids the “cookbook” approach and the deficiencies identified by Kirschner et al. (2006).

The General Mission Analysis Tool

The General Mission Analysis Tool (GMAT) is open-source software developed in a partnership between NASA, industry, and public/private contributors, and is available to download without charge (http://gmatcentral.org). GMAT is designed to support the design and analysis of space missions, and has an extensive range of capabilities. It permits orbits to be modelled, analysed and visualised in detail, orbital perturbations to be studied and manoeuvres planned, propulsion system requirements to be determined, and mission lifetimes to be estimated. Though its focus is on the modelling of spacecraft orbits, it contains a detailed representation of the solar system and can be used to visualise coordinate systems, orbits, and other natural phenomena (such as axial tilts giving rise to seasons, or the phases of the moon; see, for example, the representation of the Vernal Equinox in Figure 1). GMAT also interfaces with external platforms such as MATLAB and Python, providing an extensible architecture for future expansion both internally and external to the core code (Hughes, 2016). Significantly, GMAT is not a dedicated educational tool: it has been validated against real missions, and has been used to support the planning and analysis of programmes including MAVEN (Jakosky et al., 2015), Lunar Reconnaissance Orbiter (Tooley et al., 2010), and OSIRIS-REx (Beshore et al., 2015). Hughes (2016) cites 13 commercial firms who describe their use of GMAT in the literature, as well as international organisations such as ESA and NASA who use the code. This number is growing. Thus, while the focus of this paper is the exploitation of GMAT as a tool to facilitate learning, there is a significant additional benefit to its adoption in Higher Education institutes, by equipping graduates with experience in the use of a system that is becoming widely adopted in the space community.

Workshop Philosophy

Buchberger (1990) discusses the use of learning technologies in the context of symbolic computation software systems for mathematics courses, and his philosophy is directly applicable to the subject of the current paper. Buchberger proposes the “White-Box/Black Box Principle for Using Symbolic Computation Software in Math Education”. As interpreted by Hoyles & Lagrange (2009), a learning technology is being used as a “white box” when students are aware of the mathematics they are using the technology to perform, and is a “black box” when they have no conceptual understanding of the mathematics being implemented. Cedillo & Kieran, 2003 (cited in Hoyles & Lagrange, 2009)

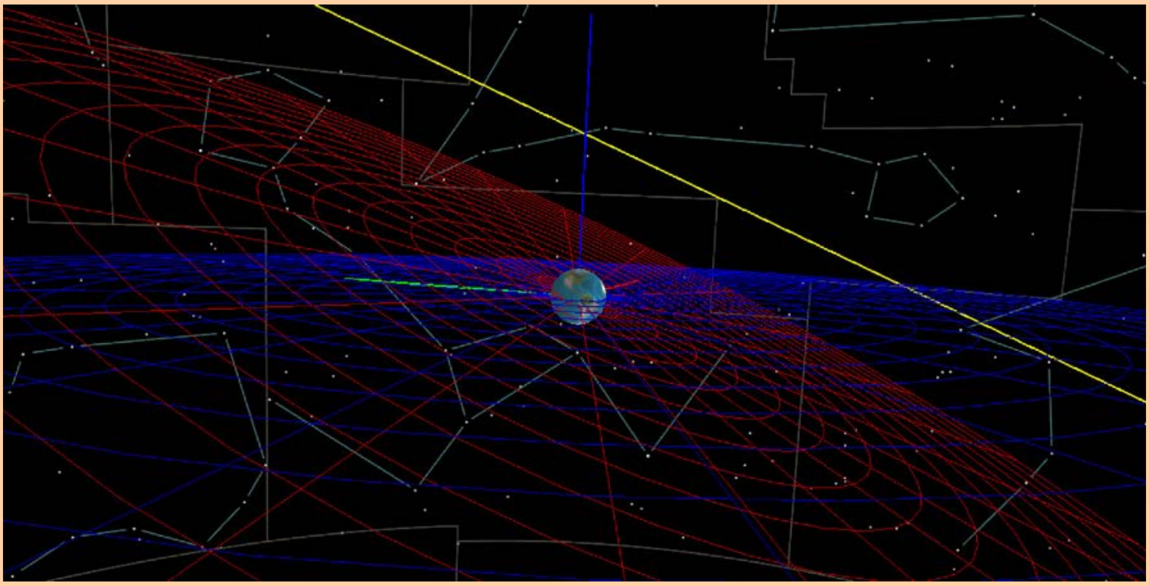

Figure 1: An equinox represents the point at which the Earth's equatorial plane passes through the centre of the Sun; there are two equinoxes each year, familiar as the Vernal (Spring) equinox, at which the sub-solar point moves from the southern to the northern hemisphere, and the Autumn equinox, when the subsolar point moves into the southern hemisphere. This frame from a GMAT scenario shows the position of the Vernal Equinox as the vector (red) lying along the intersection of the equatorial (blue) and ecliptic (red) planes; the Sun's apparent path is shown in yellow.

propose that the metaphor be extended to include a "grey box" approach, which "intertwines both the white and black boxes", used at appropriate points during the learning. Neill & Maguire (2006) review these ideas in the specific context of Computer Algebra Systems, but their statement that the tool "should only be used within a pedagogically sound framework" is equally true in the context of a code that solves the algebra underpinning celestial mechanics and astrodynamics.

GMAT's open source code makes it possible to treat the entire system as a white box: students can examine the code to see how the physics has been implemented, although given the complexity of the supporting code, this approach is unlikely to be efficient or to bring clarity to the learning. Conversely, the presence of a sophisticated Graphical User Interface can shield the student from the detailed operation of the code, with their interaction reduced to entering randomly chosen parameters for the initial state of an orbit and pressing "play". This mode of interaction results in GMAT propagating the initial state through a gravitational model and displaying whatever trajectory results, more closely representing Buchberger's "black box" approach. However, the principle of "garbage in, garbage out" applies to astrodynamics simulations – without sensibly defined initial values for orbital parameters, useful orbits with specific properties are unlikely to be generated, and many simulations will simply fail. Hence, the "hands-on" elements of the workshop which exploit the capabilities of GMAT, are embedded within a wider set of activities designed to consider, question and explore the physical principles of astrodynamics, aligning the activity with Cedillo & Kieran's "grey box" approach to the use of learning technologies.

The workshop is associated with a third year undergraduate module comprising 12 conventional lectures and supporting screencasts providing an introduction to astrodynamics. The lectures begin with a derivation of key concepts, including solutions to the two body equation of motion, leading to fundamental results that describe the essential features of orbits in the presence of a single gravitating body with a spherically symmetrical gravitational field. GMAT is used to produce visual demonstrations which are played to the class as movies during the lectures, or recorded as screencasts. The lecture module then builds on this framework to help students understand and analyse problems such as "time-of-flight" (relating to the prediction of satellite position as a function of time), perturbations (how disturbing forces introduce deviations from the idealised situations considered at the beginning of the module), examples of specific orbit types (e.g. Sun-synchronous and Geostationary), and interplanetary trajectories.

The lecture module is not a prerequisite for the workshop; student's attention is drawn to the complementarity of lecture course and workshop, and approximately 80% of students taking the workshop also take the lecture module, but the workshop is designed to be accessible to those who have had no exposure to the theory of orbits beyond a simple first-year undergraduate treatment of gravity. While it is arguably easier to design an effective workshop assuming knowledge from the taught module, the applicability of the concepts beyond the immediate astrodynamics topic makes it desirable to offer a self-contained practical activity in this area. Thus, the workshop begins with a review of conic sections and the "restricted two body" equation of motion, which leads quickly to the important results summarised in Figure 2. Appealing to the concept of the scaffolding of learning (Wood, Bruner, & Ross, 1976), these "cornerstone topics" are appropriate points from which to develop an exploration of basic orbital motion, including the effects of forces deliberately applied to the spacecraft through the application of propulsive manoeuvres to change the orbital parameters. It is therefore essential that these topics are considered by workshop participants at the start of the activity, and before work with simulations begins, to avoid a "black box" approach. Hence, some preparation is needed on the part of the student before the workshop begins.

$$r = \frac{p}{1 + e\cos\nu} \quad (1)$$

$$\frac{V^2}{2} - \frac{GM}{r} = -\frac{GM}{2a} \quad (2)$$

$$e = \frac{a-b}{a+b} \quad (3)$$

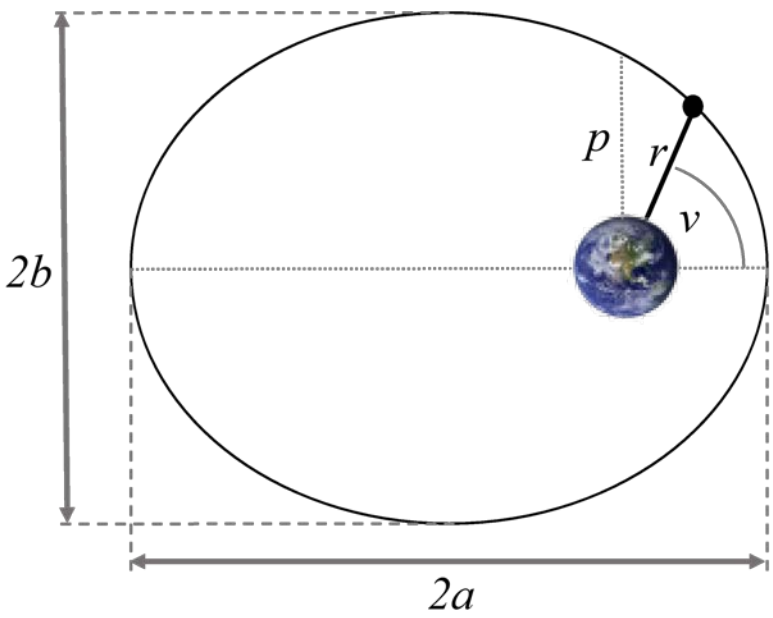

Figure 2: The trajectory equation (1) the vis-viva equation (2), and the equation for eccentricity (3), together with a definition of the basic geometry of a closed orbit (right) provide the basis for an in-depth exploration of orbital dynamics in GMAT. In this diagram, a is the semimajor axis of the orbit, b is the semiminor axis, p is the semiparameter, r is the radial distance between the satellite (moving at a velocity V) and the centre of the gravitating body of mass M, and v is the true anomaly of the satellite at a given instant, measured from the point of closest approach to the gravitating body (the periapsis). Hence p is seen to be the radial distance of the satellite from the centre of the gravitating body when the true anomaly v = 90°. The point furthest from the gravitating body is the apoapsis.

As noted elsewhere (e.g. Sappington, Kinsey, & Munsayac, 2002; Bishop & Verleger, 2013), noncompliance with reading assignments is widespread, with some studies reporting that over 70% of students rarely read required material by the due date. Thus, while student support notes for the workshop cover the development of these core concepts, and students are advised to study the notes before the first session, there is no assumption of compliance. Evidence in the literature (e.g. Stelzer, Gladding, Mestre, & Brookes, 2009; Falconer, DeGrazia, Medlin,& Holmberg, 2009) indicates that the use of multimedia content can increase significantly the level of student engagement in preparatory assignments, and these findings are supported by a recent internal study conducted within the Department of Engineering at the University of Leicester (Williams, private communication - https://screencastsinengineering.wordpress.com). Consequently, screen casts have been produced to cover the cornerstone topics of the 8 workshops, and are offered to students in the two weeks prior to the first session via the workshop area of the Institute's Virtual Learning Environment. Additionally, the first 90 minutes of each 3 hour workshop, are used to cover cornerstone topics and engage in an interactive discussion with the students about the key concepts.

Workshop Structure

GMAT has been made available on all University of Leicester computers and is freely installable on personal machines, so students can work on problems outside scheduled contact sessions. The package has an extensive set of help documentation and is supported by online resources including an active user forum and YouTube videos. In the December 2015 pilot workshop, students were instructed to follow an introductory tutorial leading to the production of a simple orbital model. However, student feedback from the pilot indicated that an academic-led approach was strongly preferred at this early stage, and so following a review of the cornerstone topics, the workshop now includes an introductory "walk-through". Students and the academic tutor begin by creating a blank GMAT scenario. The tutor shows their GMAT session on the projector screen, and talks the students through the definition, entry and demonstration of a closed, elliptical orbit around Earth, which they replicate on their own machines as the walk-through progresses – allowing exploration of the concepts shown in Figure 2. This approach has the advantage of allowing the tutor to explain the architecture of the user interface in the context of a practical example, while encouraging real-time exploration of the physical parameters which are being defined in the scenario. At this stage, the emphasis is on inviting open discussion: questions such as "*how might we expect this orbit to change if we increase the argument of periapsis by 90°?"* elicit conversations that can be used to highlight misconceptions and demonstrate meaning through the exploitation of GMAT's 3D visualisation capabilities.

The training environment is an important element in the success of the workshop. Because extensive use is made of the tutor's display to present walk-through support and demonstrate aspects of the simulations and associated physical concepts, it is appropriate to adopt a row-based configuration in which students have good line of sight to the main projector screen while looking at their own display. However, it is essential that students can share the results of their models, or show some aspect of their calculations to the wider group, and so the IT infrastructure has been configured to allow any student to send their GMAT session to the main screen at the invitation of the tutor via a network-connected projector. This feature finds its greatest use (leading to typically lively interactions and exchanges between students, as well as with the tutor), when students observe unexpected effects in their simulations, due either to errors in the calculations used to configure the model, or to the influence of a physical effect which the student had not considered. Deviations from "expected" behaviour are a rich seam of discussion, and students are encouraged to ask questions, highlight interesting observations, and investigate small modifications to the walk-through parameters, to foster this discussion. In almost all cases where mistakes are made in configuring a simulation, the resulting behaviour of the spacecraft reveals interesting and important aspects of the physics that merit some attention.

Introducing Simple Celestial Mechanics

Following the walk-through, the workshop presents a number of problem scenarios (referred to as "missions") which students must model and analyse. In the first instance, students are tasked with implementing a simple scenario in which a spacecraft is in a polar Earth orbit with specific requirements on the size, shape and orientation of the orbit. This orbit is then "propagated" (the initial conditions are passed through an algorithm which includes a force model describing how factors such as gravity, atmospheric drag and radiation pressure affect the motion of the satellite) such that a simulation covering one day of mission elapsed time is generated. The motion of the satellite can be visualised in 3 dimensions on an accelerated timescale to give students an appreciation of fundamental orbit behaviour described by Kepler's laws, and the task can be achieved entirely using the knowledge acquired during the walk-through.

The "cookbook" approach is avoided at all stages by posing questions as part of each mission. This begins with a set of well-structured questions, but as the learner progresses through increasingly sophisticated scenarios with less prescriptive summaries in the supporting text, the questions move to a more "ill-structured" form where problems and the expected pathways to solution are less prescriptive (Simon, 1973; Jonassen, 1997). In the case of the simple polar orbit problem, students are asked to verify that the characteristics of the orbit shown in the simulation, agree with theory. As an example of this approach, Analysis Case Study 1 shows how learners can answer the following challenge: "How can you establish, quantitatively, that the velocity of the satellite at apoapsis is consistent with the predictions of theory?"

Analysis Case Study 1: Verifying apoapsis velocity in a simple closed orbit

GMAT offers data reporting tools including custom-designed tables and plots to access the numerical results which underpin the graphical simulation. The simplest interrogation method is the "Command Summary", which displays values for fundamental parameters of the orbit and the state of the spacecraft at the end of the simulated period. An example output for the polar orbit exercise is shown in Figure 3. The sections titled "Cartesian State" and "Keplerian State" contain the coordinates and fundamental orbital parameters of the satellite at the end of the simulation period. Other sections report time-dependent properties of the satellite at the final instant of the simulated period (such as *VMAG*: the magnitude of the satellite's velocity) or summarise fundamental properties of the resulting orbit (such as *Orbit Period*).

One of the early mission tasks requires students to verify that the velocity of the spacecraft at apoapsis (the point in the orbit which is furthest from the planet) is in accordance with predictions. The student can use a variety of approaches to address this question. For example, in "Other Orbit Data" the apoapsis velocity is reported as *VelApoapsis* = 3.783 km/s. This can be compared with the prediction of Equation 2, which requires knowledge of semimajor axis a (reported as *SMA* by GMAT), and the radial distance at a specific point in the orbit, r, (reported as *RMAG*). Astrodynamics problems are commonly soluble using a variety of approaches and one way of addressing this

```
Time System   Gregorian                     Modified Julian
----------------------------------------------------------------------
UTC Epoch:    01 Jan 2000 17:04:08.516      21545.2112096753
TAI Epoch:    01 Jan 2000 17:04:40.516      21545.2115800456
TT  Epoch:    01 Jan 2000 17:05:12.700      21545.2119525456
TDB Epoch:    01 Jan 2000 17:05:12.700      21545.2119525449

Cartesian State                       Keplerian State
---------------------------           --------------------------------
X  =  -0.0090921506305 km             SMA  =  15000.021473907 km
Y  =  -19500.045373677 km             ECC  =  0.3000019555741
Z  =   21.521321873013 km             INC  =  89.999620994477 deg
VX =   0.0000250200828 km/sec         RAAN =  89.999973703386 deg
VY =  -0.0041747649839 km/sec         AOP  =  359.93676512396 deg
VZ =  -3.7826755591279 km/sec         TA   =  180.00000000000 deg
                                      MA   =  180.00000000000 deg
                                      EA   =  180.00000000000 deg

Spherical State                       Other Orbit Data
---------------------------           --------------------------------
RMAG =   19500.057249733 km           Mean Motion        =   3.436616465e-04 deg/sec
RA   =  -90.000026714905 deg          Orbit Energy       =  -13.286662362230 km^2/s^2
DEC  =   0.0632347457959 deg          C3                 =  -26.573324724460 km^2/s^2
VMAG =   3.7826778629580 km/s         Semilatus Rectum   =   13650.001941005 km
AZI  =   179.99962099425 deg          Angular Momentum   =   73762.434884977 km^2/s
VFPA =   90.000000055822 deg          Beta Angle         =   10.593761427751 deg
RAV  =  -89.656620674839 deg          Periapsis Altitude =   4121.8493980805 km
DECV =  -89.936764174228 deg          VelPeriapsis       =   7.0250033672391 km/s
                                      VelApoapsis        =   3.7826778629580 km/s
                                      Orbit Period       =   18283.056520209 s

Planetodetic Properties
---------------------------
LST       =   269.99998180260 deg
MHA       =   356.70576235807 deg
Latitude  =   0.0648764342238 deg
Longitude =  -86.705780555470 deg
Altitude  =   13121.920977043 km


Spacecraft Properties
------------------------------
Cd                    =  2.200000
Drag area             =  15.00000 m^2
Cr                    =  1.800000
Reflective (SRP) area =  1.000000 m^2
Dry mass              =  850.00000000000 kg
Total mass            =  850.00000000000 kg
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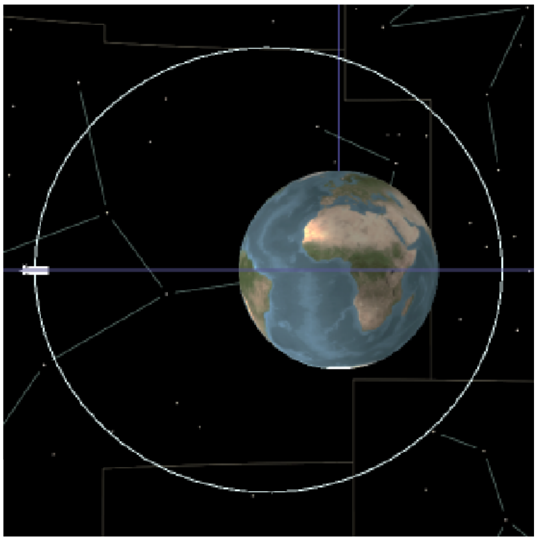

Figure 3: Part of the Command Summary for the simple polar orbit exercise (graphical output shown inset). The initial state for the orbit is shown in the Keplerian State section of the report. The sections "Spherical State", "Other Orbit Data" and "Planetodetic Properties" reflect the status of the satellite at the end of the simulated period.

question is to use the Trajectory Equation (Equation 1) to solve for *r* at apoapsis, recognising that apoapsis corresponds to a value of v = 180°, and using the GMAT-reported parameters *ECC* (for eccentricity *e*), and *Semilatus Rectum* (an alternative term for semiparameter, *p*). Substitution of these values into Equation 1 results in an identical apoapsis value of 19500 x 10^3 m.

Then, assuming values of $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2} for the gravitational constant and $M = 5.972 \times 10^{24}$ kg for the mass of Earth, the student can use the *vis-viva* equation to give

$$v = \sqrt{\frac{2GM}{19500 \times 10^3} - \frac{GM}{15000 \times 10^3}} = 3782.550 \text{ m s}^{-1}$$

This value differs from the value reported by GMAT by only 13 cm s^{-1} (a difference attributed to the higher levels of internal precision with which GMAT performs calculations); hence, the student may conclude that the results of the simulation are consistent with the predictions of the restricted two body problem illustrated in Figure 2.

Increasing Detail: Perturbations

Analysis Case Study 1 uses the Restricted Two Body Problem (R2BP) to help students gain familiarity with GMAT while demonstrating the value of numerical simulation and visualisation in astrodynamics, using tractable closed form solutions to explore the application and implications of the most fundamental aspects of the subject. The R2BP assumes that the spacecraft is subject only to the force of gravity, that there is a single source of gravity, and that the gravitational field is perfectly radial (i.e. the gravitating body is treated as a point mass). One of the key results arising from the R2BP approach is that the motion of the satellite is confined to a plane, which is easy to represent in 2D treatments. In reality, spacecraft are subject to other forces such as atmospheric drag; there are typically several significant sources of gravity (for example, the Earth and the Moon), and because real planets are not perfectly spherical or uniformly dense, their gravitational fields are not perfectly radial. These factors lead to spacecraft behaviour which deviates from the predictions of the R2BP; these deviations, referred to as perturbations, lead to spacecraft motion which more closely represents “real” spacecraft behaviour. The GMAT Astrodynamics Workshop allows students to explore these concepts and the relationships between the mathematical descriptions of the environment and the resultant orbital motion.

Sun-synchronous orbits are used to illustrate how relaxation of the R2BP assumptions leads to more complexity. The assumption of a point mass gravitational source gives way to a description of the gravitating body as an extended object that is not perfectly spherical. Screencasts and discussions in teaching sessions are used to show how a mathematical description of this non-ideal shape can be constructed using the technique of “spherical harmonics”.

One of the most significant consequences of the Earth’s non-sphericity on a spacecraft orbit is a perturbation caused by the fact that the Earth is *oblate* – meaning that the equatorial diameter is larger than the polar diameter, due to the planet’s rotation. Using a simplified representation of the Earth as a perfectly spherical body surrounded by a discrete band of material, students appreciate how this shape introduces non-radial forces on the satellite, which lead to a torque being exerted on the orbit (Figure 4). References to topics from core mechanics modules in the first and second years of the course, and demonstration of how a torque applied to a gyroscope leads to the familiar axial “wobble” of a spinning top, can be used to support the learning (e.g. McGlynn, 2007) so that students make the link between the applied torque, and the effect of *precession* (rotation of the angular momentum vector – or equivalently, the rotation axis).

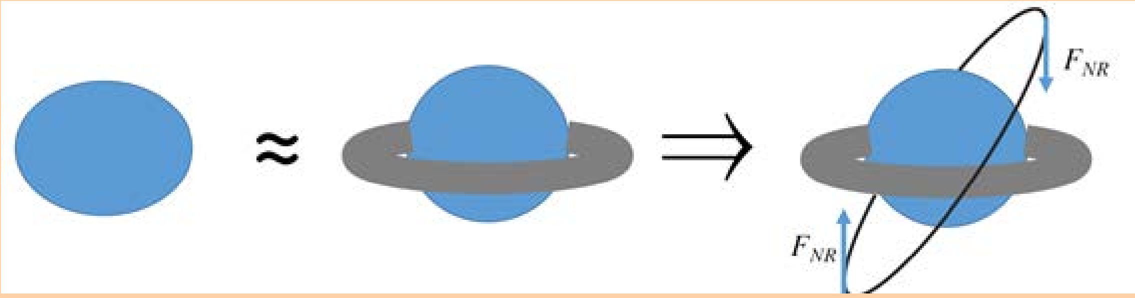

Figure 4: Earth's oblateness (left) can be approximated by a spherical gravitating body surrounded by a band of additional mass in the equatorial plane (middle). This leads to the introduction of a non-radial

This leads to an expression for the rate of orbit plane precession, $\dot{\Omega}_{J2}$ (radians per second) for a circular orbit of radius *r* and inclination *i,* described by Equation (4).

$$\dot{\Omega}_{J_2} = -\frac{3}{2}\sqrt{\frac{GM}{r^3}}J_2\left(\frac{R_\oplus}{r}\right)^2 \cos i. \qquad (4)$$

Here, J_2 is an experimentally determined coefficient with a value of 1.081874 x 10^{-3} and is related to the magnitude of the perturbation, while $R_\oplus$ is the Earth's radius. Analysis Case Study 2 illustrates how perturbations are introduced in the workshop by considering Sun Synchronous orbits.

Analysis Case Study 2: Exploring perturbations with Sun synchronous orbits

Perturbations can be appreciated by studying how the behaviour of a satellite is modified as the gravitational model used in the simulations is changed, from a simple spherical representation, through the oblate spheroid of Figure 4, to increasingly complex representations that include other localised deviations from the ideal spherical case. This study begins with the polar orbiting satellite from Analysis Example 1 which, when simulated over a period of several days or weeks, is seen to occupy an orbit that is fixed in the inertial frame (i.e. with respect to the distant stars) – consistent with the prediction of the R2BP that orbital motion is confined to a plane. The satellite can then be "copied" so that two identical spacecraft are represented in the simulation. The student adjusts the level of detail in the force model applied to the second satellite, so that it includes the Earth's oblateness - the most significant source of gravitational perturbation. Re-running the simulation, the non-radial component of force results in a torque which causes precession as expected.

Investigations begin with a qualitative examination of the resulting orbits displayed in the 3D Orbit View. A more quantitative approach is enabled using custom-defined reports and plots to examine parameters as they vary with time. Work from one student (Student A) is shown in Figure 5. The student chose an orbital radius *r*, of 6778.14 km, and used Equation 4 to calculate that an inclination *i* = 97.03° would produce an orbit whose rate of precession matches the rate at which the Earth orbits the Sun (and hence the rate at which the Sun's apparent position changes, as measured against the inertial frame represented by the distant stars). This rate is equal to 360°/365.24 mean solar days, or 1.991 x 10^{-7} radians sec^{-1}, and an orbit with a precession rate matching this value is known as a "Sun-synchronous orbit", because the orientation of the orbit remains fixed with respect to a line drawn between the Earth and the Sun.

An orbital parameter known as the *Right Ascension of the Ascending Node* (Ω) can be used to

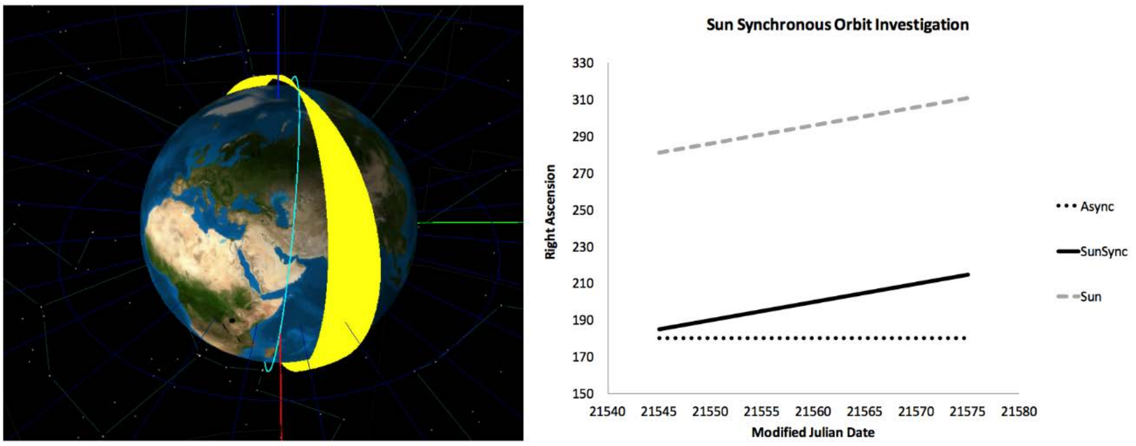

Figure 5: Left: a student's simulation of two satellites initial orbit parameters that are identical apart from a small difference (5°) in ascending node value to make them distinguishable on the plot. The motion of one satellite (cyan path) is predicted using a force model which assumes a spherical Earth. The motion of the other satellite (yellow) is calculated using a gravity model accounting for Earth's oblateness. The simulation covered 30 days of motion, and the precession of the second satellite is clearly visible in the orbit trace. Right: a graph generated using data exported from GMAT, showing that the Right Ascension of the Ascending Node remains constant for the first satellite, but varies with the second at a rate that matches that of the Sun's apparent position.

describe the orientation of the satellite orbit. Ω is the angle, measured in the plane of the celestial equator, between the origin of the celestial coordinate system analogous to latitude and longitude on Earth, and the point where the satellite crosses the plane of the Earth's equator moving from the southern to the northern hemisphere. When no precession is experienced, this angle remains constant. But with precession included, the location of that crossing point (or ascending node) changes. Student A plotted the value of Ω for the two satellites, and showed that the purely spherical potential resulted in no precession (a fixed value for Ω), while including the Earth's oblateness caused Ω to change over time. In this case, the student took the study one step further, and researched the Right Ascension of the Sun at the start and end dates of the simulation. Using a simple linear relationship, they added a third line to the graph in figure 5, showing that the gradient of the Sun-synchronous orbit matches the gradient of the Sun's position as it changes throughout the year, providing confirmation of the expected behaviour.

Manoeuvres & Targeting

The topics of Analysis Case Studies 1 and 2 are equally applicable to celestial mechanics and astrodynamics. Propulsive manoeuvres, however, are generally the preserve of spaceflight dynamics, and can be represented in GMAT. Manoeuvres demonstrate another application of numerical simulation: the systematic exploration of parameter space to identify solutions satisfying one or more user requirements.

Propulsive manoeuvres are enabled in GMAT by adding "Hardware" (fuel tanks and thrusters) to the spacecraft, and then adding "burns" which use the hardware to change velocity. Thruster representation can be detailed, with chemical and electric propulsion options available, and thrust levels, directions and reference frames definable. This level of detail means that the GMAT can be used to support courses in space propulsion and spacecraft systems engineering. But to maintain the workshop's focus on astrodynamics, chemical thrusters with the default settings are generally used, and most, though not all, propulsive manoeuvres are assumed to be “impulsive” – i.e. the change in velocity is assumed to be instantaneous rather than taking place over a finite period of time.

A simple manoeuvre typically used as an introduction to the topic, is the Hohmann Transfer which uses an elliptical orbit to transfer a spacecraft from one circular orbit to another in the same plane (Figure 6). The apoapsis of the transfer orbit is coincident with the higher circular orbit, while the periapsis matches the radius of the lower circular orbit. The transfer is achieved by performing two burns, one at apoapsis and one at periapsis, to increase or decrease velocity. The first manoeuvre causes the spacecraft to leave the initial orbit and join the transfer, the second one allows the spacecraft to join the destination orbit.

The mathematics of the transfer use the vis-viva equation (Equation 2). Four velocities are calculated: the velocity of the spacecraft in the two circular orbits, and the velocity in the elliptical orbit at periapsis and at apoapsis. The differences between the circular and elliptical orbit velocities at apoapsis and periapsis represent the velocity changes or ΔV (“delta-vee”) which the propulsion system must provide to achieve the transfer. To transfer from high altitude to low altitude, the first burn is ΔV_a, and is negative, indicating a reduction in velocity (thrust applied in a direction opposite to the spacecraft motion). With this first manoeuvre complete, the spacecraft would continue to follow the elliptical transfer, and so to join the lower altitude circular orbit, a second burn, also a velocity reduction, is required at periapsis. Orbit raising is conducted in the same way, but the periapsis burn is performed first, and both periapsis and apoapsis burns increase the spacecraft velocity. Two approaches can be taken to investigate the Hohmann transfer: (1) definition of the initial orbit and manual calculation of ΔV magnitudes and directions which are then used by GMAT to arrive at a new orbit whose radius is checked by the student to ensure it agrees with the expected value, or (2) definition of the initial and final orbits, and use of a “target” module in GMAT to vary ΔV and find the value which achieves the required transfer. In neither case does GMAT operate as a

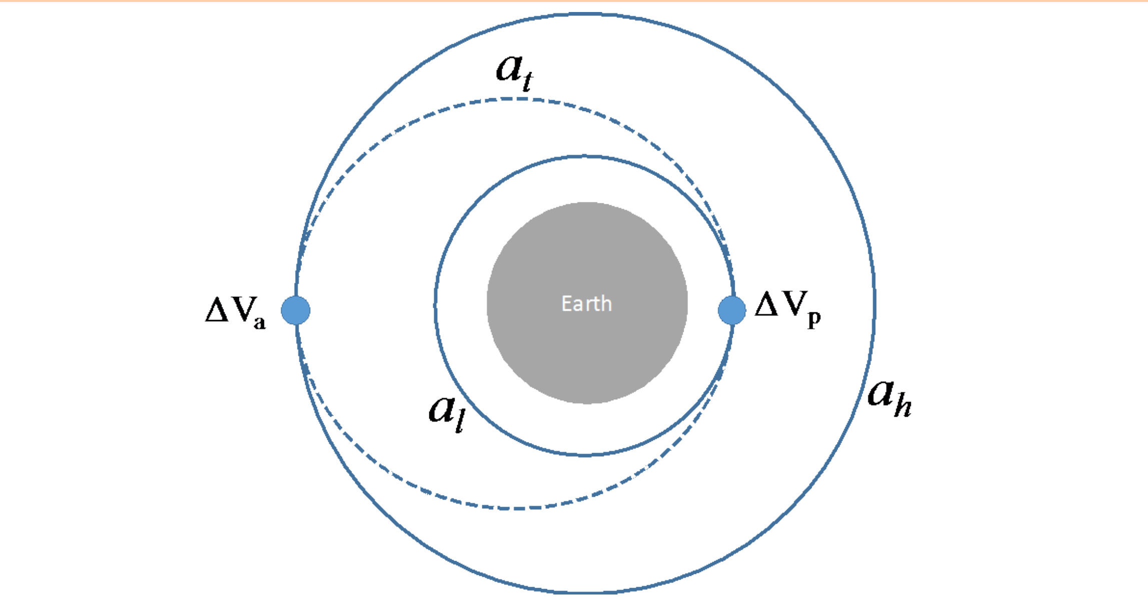

Figure 6: Geometry of the Hohmann transfer. The elliptical transfer orbit is shown as the dashed ellipse and is used to raise or lower a spacecraft between two other, co-planar orbits which, for a simple introductory treatment, are assumed to be circular. The semimajor axis of the higher altitude circular orbit is a_h, that of the lower circular orbit a_l, and the semimajor axis of the elliptical transfer is a_t which, from geometry, is simply $0.5(a_h + a_l)$. The propulsive manoeuvres generating a change in velocity take place at apoapsis (velocity change ΔV_a), and periapsis (ΔV_p).

Black Box – success requires the learner to apply key concepts, but the simulation can aid in the process of making meaning, by showing the student the effects of changing parameters, and proving conjecture (mirroring the experience of Furinghetti & Paola, 2003). Students explore both approaches in the workshop.

The GMAT tutorials available online include a Hohmann transfer, and the reader is directed to that resource for further information on how this type of manoeuvre can be represented in GMAT. Instead, here we will show how a student approached a different manoeuvre problem: an orbital inclination change. The inclination, *i*, of an orbit is the angle between the plane of the orbit and the Earth's equatorial plane. Beginning with the situation of circular orbits, simple vector geometry (figure 7) shows that the velocity change required to adjust the inclination of a circular orbit through an angle Δi while keeping other parameters fixed, is

$$\Delta V = 2V \sin\left(\frac{\Delta i}{2}\right), \qquad [5]$$

where the magnitude of the velocity in the initial orbit, V_i , and the final orbit, V_f , is identical (because the two circular orbits differ only in their orientation) hence $|V_i| = |V_f| = V$. Experience in previous years of the astrodynamics course shows that students understand this concept and can estimate ΔV magnitudes, but they often overlook two important points: first, the manoeuvre must be undertaken at a point which is common to the initial and final orbits, and second, that the propulsive manoeuvre has a direction which is not perpendicular to the orbit plane. Analysis Case Study 3 summarises how one student investigated inclination changes during the 2016 workshop.

Analysis Case Study 3: Plane Change in a Circular Orbit

Student B was presented with a mission in which a satellite began in a circular Earth orbit of radius 7200 km and inclination 20°, and required the inclination to be reduced to 0° (an orbit lying in the Earth's equatorial plane). The student began by setting up the initial orbit in GMAT, and performing

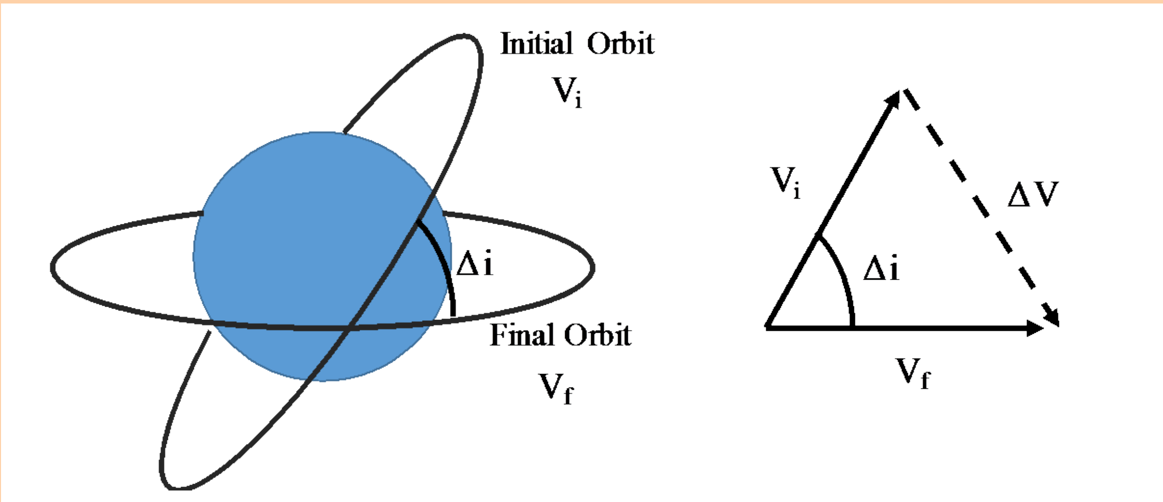

Figure 7: Geometry of the simple plane change. A spacecraft in a circular orbit with velocity V_i begins with some initial inclination, and a manoeuvre is required to change to a new inclination while keeping all other orbital parameters fixed. The total change in inclination is Δi, and the initial and final velocities are equal in magnitude, differing only in their direction. The total change in velocity required for the change is ΔV.

a manual calculation to find the circular orbit velocity $V = \sqrt{GM/r}$ = 7439 m/s, concluding that a ΔV of 2584 m/s would be required for the plane change (Equation 5). They recognised that the plane change should be performed at the intersection of the initial and final orbits, which corresponds to the celestial equator, and hence propagated the orbit in GMAT until the spacecraft reached a point in its orbit where it was over the Equator. They then implemented a motor burn perpendicular to the orbit plane, and configured GMAT to find the ΔV which resulted in a 0° inclination. GMAT reported successful solving (convergence) of the problem, but examination of the 3D view showed that while the new orbit had the required 0° inclination, the semimajor axis had changed. Reviewing the command summary revealed a 1100 km increase in *a,* and a ΔV = 2706 m/s for the manoeuvre, significantly more than predicted (figure 8, top panel). The workshop leader asked the student to consider whether a perpendicular burn was consistent with the geometry of the plane change. Reflecting on the vector sum in figure 7 and the corresponding internal angles, the student recognised that the ΔV had components in the perpendicular and velocity directions. In their second attempt, the student configured GMAT to perform a burn with components in these two directions, and added a new condition that the final orbit must have a semimajor axis matching the original. This time the model resulted in a final orbit identical to the initial one but in the equatorial plane. The burn parameters were 451 m/s in the anti-velocity direction and 2551 m/s perpendicular to the orbit plane, giving a total magnitude of 2590 m/s, only 6 m/s different from the manual calculation, and showing that the angle between the velocity vector and the burn direction was 90°- $\tan^{-1}$ (-451/2551) = 79.97°. The student then used manual resolution of the vector triangle to confirm that this result was consistent with the base angles in an isosceles triangle with vertex angle 20°. Outputs from Student B's work are shown in figure 8.

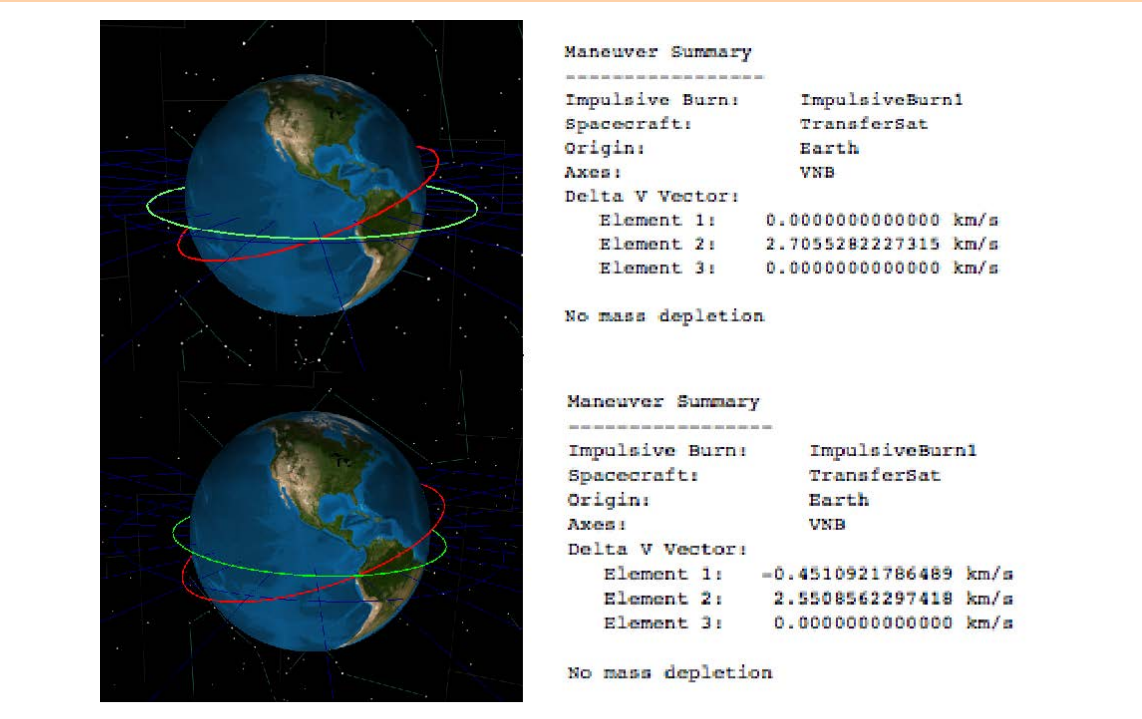

Figure 8: Top panel: student's first attempt showing initial orbit (red), and final orbit (green) with required inclination but increased semimajor axis. Bottom panel: second attempt with directed burn showing pure inclination change. Command summary outputs for propulsive manoeuver shown on the right, indicating the correct burn has components in both velocity and orbit plane normal axes.

Advancing Beyond Earth

Much of the current workshop focuses on Earth-orbit, but the final phases introduce topics relevant to interplanetary missions, including the use of different coordinate frames to target planetary fly-bys and orbit insertions. GMAT includes position and physical property data for the major planets along with Pluto and Earth's moon; other bodies can be added using data from NASA's SPICE system (Acton, 1996). A wide variety of concepts can be explored within this solar system model, including deep space missions that use gravitational assist (GA) manoeuvres to reach the outer planets.

GMAT is not designed to calculate launch windows or to identify multiple GA trajectories; such computationally intensive studies are best solved by alternative methods. Tools such as NASA's Trajectory Browser (Foster, 2013), or the European Space Agency's Global Trajectory Optimisation Problems Database and associated codes Pykep and PyGMO (Izzo, 2010) can help in this regard, while pre-computed trajectories for specific targets are published (e.g. George & Kos, 1998). Alternatively, course facilitators may consider introducing undergraduate projects on relevant concepts. For example, solving *Lambert's problem* can lead to the identification of launch windows, directions and energy requirements (generating the so-called "pork chop plot") to provide initial state vectors for GMAT. See, for example, Vallado, 2013, for a discussion of Lambert's problem and a range of algorithms for its solution.

Despite the considerable preparation needed before complex interplanetary trajectories can be built in GMAT, exploring relevant concepts is within the scope of the workshop. Spacecraft initial states can be given in terms of the arrival direction, energy and distance of closest approach in the frame of the destination planet. Hence, students can investigate the geometry and energetics of a gravity assist manoeuvre, without the need to calculate a trajectory from the Earth to their chosen planet. Missions at this stage in the workshop are less prescriptive than they are at the beginning, with students having developed in-depth familiarity with the system over the course of the activity. In the specific case of gravitational assist manoeuvres, students are asked to carry out an investigation of planetary encounters, using GMAT to demonstrate underpinning concepts and verifying the results quantitatively. Analysis Case Study 4 summarises one student's exploration of gravity assist manoeuvres in the workshop.

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Analysis Case Study 4: Gravity Assist Manoeuvres

Student C chose Jupiter for an investigation of Gravity Assist manoeuvres, creating a scenario in which Jupiter was the central body, with the Sun and Earth included. He defined multiple spacecraft with initial state vectors describing the radius, energy and direction of the incoming trajectory in the reference frame of the target planet, varying the parameters to explore their influence. Two particular spacecraft were given similar arrival conditions except that one spacecraft arrived on the side of the planet facing into the direction of the planet's orbital motion around the Sun, while the other passed by the trailing hemisphere. The student generated reports for each spacecraft, outputting spacecraft energy and velocity in the Jovian and Sun-centred frames, the direction of the spacecraft incoming and outgoing trajectory, the radius of periapsis r_p (i.e. the distance of closest approach), and the eccentricity of the orbit. The student sought to validate several relationships developed in screencasts and contact sessions:

- The angle ϕ through which the trajectory was rotated during the encounter (the "turning angle") is related to eccentricity by $\sin(\varphi/2) = 1/e$ [6]
- The eccentricity of the swingby trajectory is related to r_p , the arrival velocity v_∞ and the mass of Jupiter M_J by $e = 1 + \frac{v_\infty^2 r_p}{GM_J}$ [7]
- The energy in the frame of the arrival planet is unchanged, while the energy in the heliocentric (Sun-centred) frame increases/decreases when the swingby is on the trailing/leading hemisphere respectively.

A selection of outputs from the student's investigation are shown figure 9. The family of flyby trajectories are shown in the top left panel; all but one arrive at Jupiter from the trailing hemisphere, while the red orbit approaches from the leading hemisphere; the middle left panel shows this trajectory and its trailing hemisphere counterpart in more detail. In the lower left panel, the results of those two encounters are seen to produce a reduction in orbit energy during the leading hemisphere flyby, resulting in a bound heliocentric orbit, while the trailing hemisphere encounter increases the energy of the trajectory in the heliocentric frame, leading to the escape of the spacecraft from the Jovian system. The lower right panel shows the same flybys in terms of trajectory energy in the heliocentric frame. The upper and middle right panels illustrate how the student tested equations [6] and [7], showing results from the simulation as data points, plotted on top of the predicted behaviour (solid lines). By exploring these fundamental relationships, completion of this investigation enabled the student to begin designing gravitational assist manoeuvres which achieved specific outcomes in terms of turning angle and spacecraft energy.

Workshop Assessment & Final Challenge

The preceding discussion highlights selected examples of mission problems covering basic orbits, perturbations and manoeuvres. The current version of the workshop contains eleven exercises or "missions" which students follow in a progression from exploration of basic orbital parameters to an introduction to deep space mission techniques. Each mission is worth a number of marks, depending on the level of complexity of the problem. The set of current missions is summarised table 1, along with the available marks for each.

Summative feedback is provided to students throughout the workshop in the form of group discussions and one-to-one conversations with the facilitators. Formative assessment is provided in two phases. First, a student's work on each of the missions is assessed in real time, in conversations between the student and the facilitator. The student demonstrates their solutions, and discusses their approach to and understanding of the problems with the facilitator. These marks (up to a total of 40) contribute 70% of the overall workshop mark.

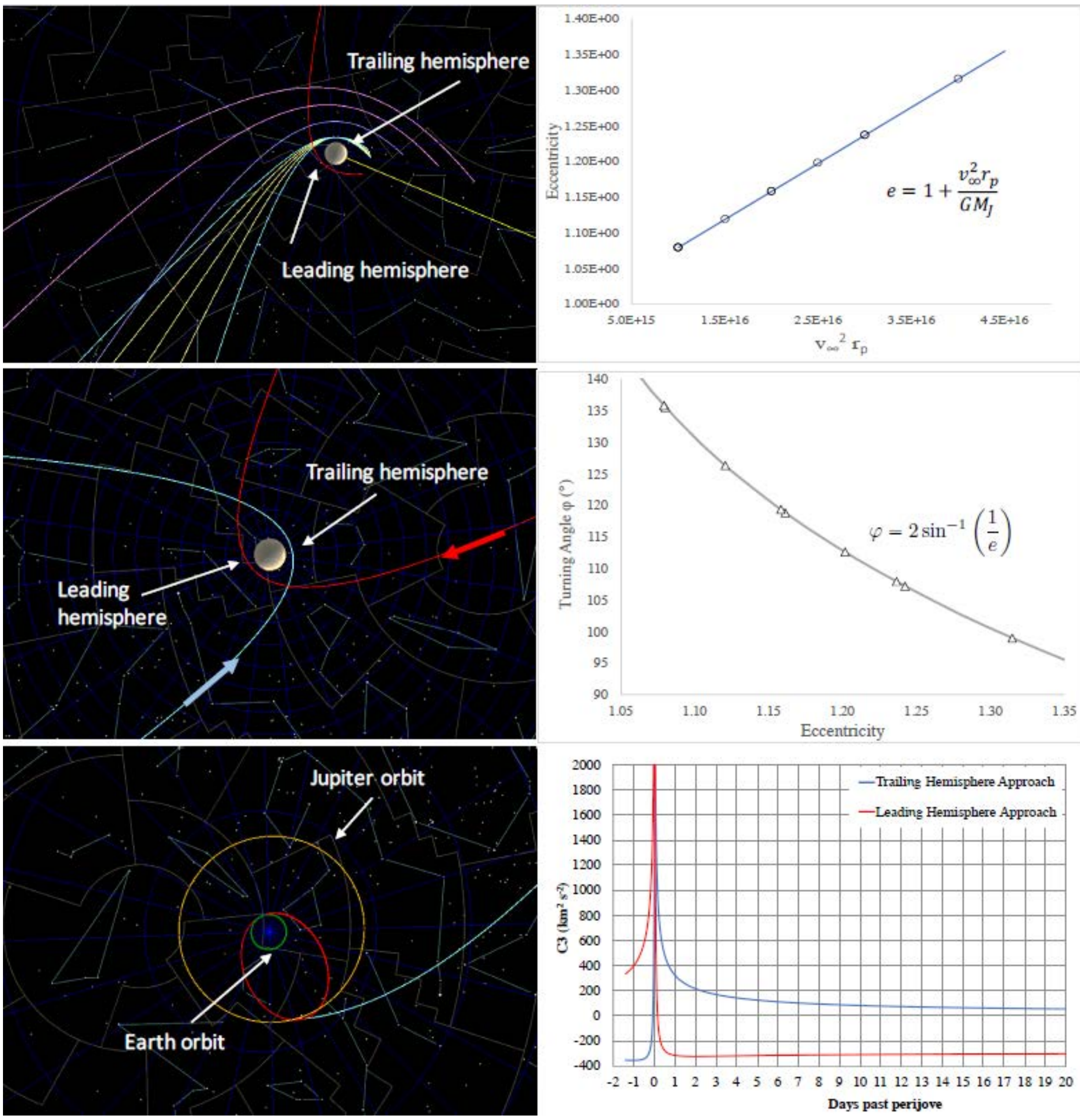

Figure 9: Student's investigation of gravitational assist manoeuvres. Top left: multiple spacecraft approaching Jupiter (view direction onto the North Pole). Middle left: two "matched" spacecraft in similar orbits but approaching from different hemispheres. Lower left: View centred on the solar North Pole, showing Jupiter's orbit and the result of the two matched flybys, with an escape trajectory resulting from the trailing hemisphere encounter and a bound solar orbit from the leading hemisphere encounter. Top right: confirming the relationship between eccentricity, arrival velocity and periapsis radius. Middle right: testing the relationship between turning angle and eccentricity. Lower right: the energy of the two matched spacecraft (in the heliocentric frame) before, during and after the encounter, showing the increased energy of the trailing hemisphere encounter and the reduced (negative, bound) energy of the spacecraft in the leading hemisphere encounter.

The second element of assessment takes the form of a "final challenge" in which the cohort is divided into groups of 4-5 students. A briefing session is held after the last formal workshop session, in which students are introduced to an extended problem which they have ~4 weeks to study, without tutor support.

Table 1: Problem scenarios (referred to as "Missions") which make up the assessed components of the current workshop. Missions used as analysis case studies in the current work are shown in italics. The number of marks available in formative assessment is noted in the final column.

Mission	Description	Marks
Polar Orbit	Simple closed orbit & exploration of basic keplerian laws, R2BP.	2
Sun Synchronous Orbit	Introduction of perturbations, J2 effect.	2
Geosynchronous Orbit	Exploration of east-west drift, stable/unstable points & drag paradox; reference frame choices.	3
Critically Inclined Orbit	Precession of the argument of periapsis, generation of Molniya orbits	3
Hohmann Transfer	Introduction to propulsive manoeuvres, orbit transfers and goal seeking.	2
Bielliptic Transfer	Demonstration of Oberth Effect; deep space transfers which exceed Hohmann transfer efficiency.	3
One Tangent Transfer	Introduction to flight path angle; fast transfers; ΔV versus transfer time.	5
Inclination Change	Introduction to out-of-plane manoeuvres. Multiple goal seeking.	4
Combined Inclination & RAAN Change	Identification of common points in orbits; efficiency of sequences vs single manoeuvres.	5
Aerobraking	Planetary atmosphere models; atmospheric drag; apoapsis lowering manoeuvres; planetary capture; entry corridors.	5
Gravity Assist	Hyperbolic orbits; introduction to gravitational assist manoeuvres; relationship between eccentricity, periapsis and turning angle.	6

The topic is sufficiently challenging that the groups tend to self-organise, allocating different phases or techniques to specific individuals, then working to fit the elements into a single coherent mission. The topics to date have been:

- A Phobos sample-return mission, calculating trajectories to/from Mars, and a series of rendezvous operations in Mars space allowing spacecraft to land on the moon Phobos then return to Earth.
- Studying and reproducing specific Apollo missions to explore the difference between the various trajectories used in the moon programme, understanding the need for trajectory correction manoeuvres and comparing actual flight data (for e.g. burn times, directions, re-entry locations) with the predictions of the student's GMAT scenario. This challenge ended with a final stretch goal to design an Apollo-like transfer for the present day, testing

student's ability to apply techniques to find new solutions, rather than relying on historical parameters known to give the required behaviour.

The Final Challenge is assessed during a Mission Presentation morning, in which each group presents their solution to the rest of the class and a small panel of 2 - 3 academic staff. A key feature of this event is the prohibition of Powerpoint presentations; students instead present their work directly in GMAT, showing supporting analysis in e.g. MATLAB or Excel as required. This is an important approach which the author was introduced to during a visit to the European Space Agency's Concurrent Design Facility (see e.g. Bandecchi, Melton, Gardini, & Ongaro, 2000); practicing scientists and engineers who use this facility to design real missions, present solutions in this way because it is more efficient than copying work into a presentation slide, which is by its nature non-interactive; it facilitates a higher level of discussion by enabling interactive "what if" questions to be explored in the session, and enables subject experts to probe aspects of the models which the presenter may have overlooked. Effective use of this presentation style is a transferrable skill and a specific learning outcome of the workshop.

Student Feedback and Future Development

Student feedback is solicited at the end of each year's workshop, and the results for the first two years of operation are summarised in figure 10. Significant changes were made after the 2015

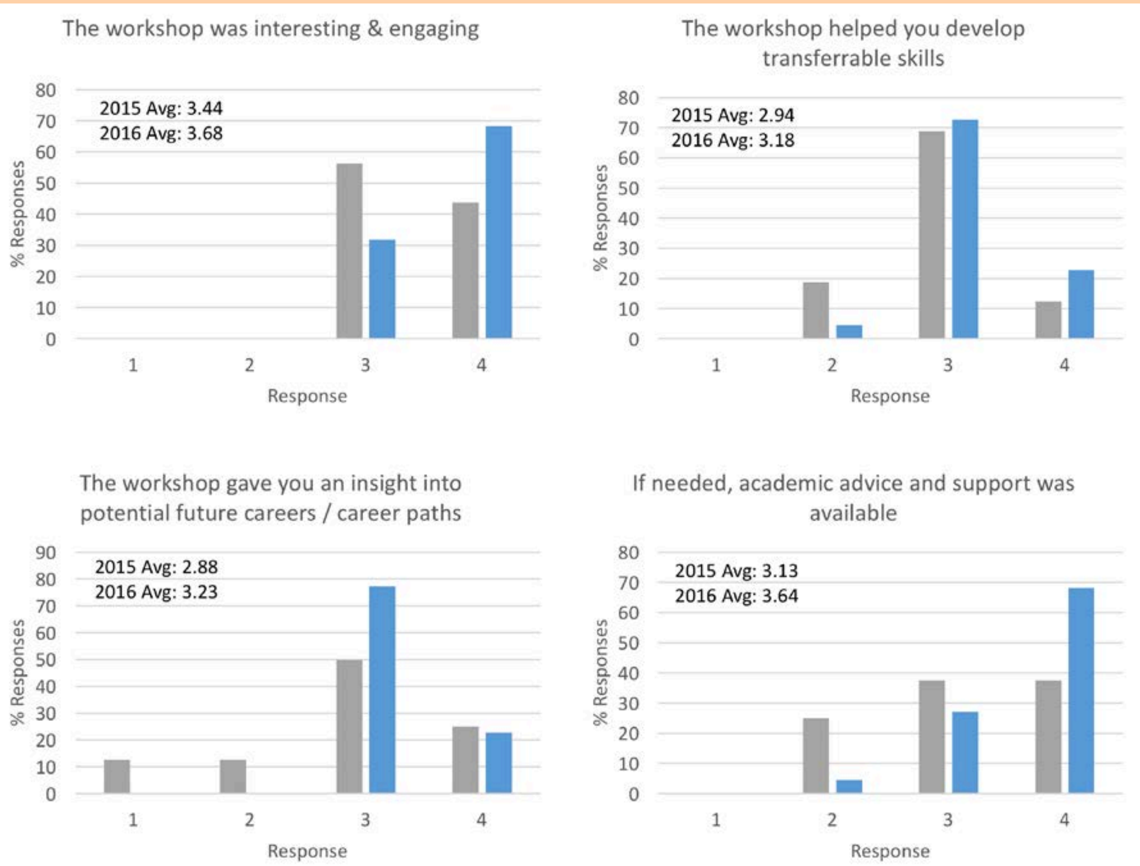

Figure 10: Feedback from students at the end of the 2015-16 (grey) and 2016-17 (blue) sessions, normalised to the total number of respondents in each year (16 in the 2015-16 session, 22 in 2016-17). Responses correspond to "Definitely Disagree" (1); "Mostly Disagree" (2); "Mostly Agree" (3) and "Definitely Agree" (4).

workshop in response to feedback, including the introduction of "walk-through" sessions to help students gain familiarity with the basic software architecture; short breakout sessions to cover specific astrodynamics concepts in a more conventional lecture-like form while remaining in the workshop room, and a final year PhD student facilitator to increase the level of support available. While the statistics are based on a small number of respondents, the results suggest that these changes have led to an improvement in satisfaction in each of the four areas questioned.

As reflected in these results, a majority of respondents viewed the workshop very positively. The most common requests in response to the question "How would you suggest the workshop could be improved?" were to increase the number of walkthroughs provided, and increase the number of contact sessions.

It is not viable to increase the number of contact sessions to a level which keeps students satisfied. But as a means of developing insight into the concepts which make up the undergraduate astrodynamics ILOs, and helping students develop a more intuitive understanding of the subject, the computational workshop appears to be more engaging and effective than the traditional lecture delivery, with the added advantage that it provides students with experience in the use of an analytical tool that is being adopted in the professional community. Consequently, it is planned to address the student's desire for additional contact sessions by adopting a flipped approach to the overall astrodynamics module in future, increasing the use of screencasts to cover some of the "bookwork" currently delivered in conventional lectures, and freeing up lecture contact time for the workshop and other interactive GMAT-enabled sessions.

Summary and lessons learned

NASA's General Mission Analysis Tool is enabling students to engage in active learning in astrodynamics, celestial mechanics, and elements of spacecraft systems engineering and mission design. Based on feedback from survey responses, conversations with students, and the level of discourse in the workshop, it is evident that they gain much from the ability to visualise and test fundamental concepts of astrodynamics in the GMAT environment; further, they have gone on to apply the skills developed in the workshop to other areas of their studies, particularly in project work.

Using software simulation in teaching requires more than identification of a package and timetable slots in which to use it. A number of lessons have been learned during preparation and implementation of the GMAT astrodynamics workshop, which may benefit those seeking to incorporate simulation-based workshops in their own teaching.

Synchronisation with taught modules

When using simulations to support a conventionally delivered taught module, the timing of the activities must be carefully planned so that prerequisite knowledge can be accessed by the student before using simulation to support their learning. This not only affects the starting dates of the two kinds of activity, but can also drive the pace of the taught module (item 2, below). An alternative approach, planned for adoption in the University of Leicester Astrodynamics course, is to use a flipped structure in which the simulation sessions represent the contact time, and lectures are replaced by screencasts and directed reading in a schedule that is published in advance. The ability to track a student's access of online resources such as screencasts within a VLE can be useful for monitoring private study behaviour, and can be discussed in class if necessary.

Do not over-estimate the pace of progress through the workshop

The use of simulation requires the student to gain familiarity with the software platform as well as the fundamental concepts to be explored. This overhead tends to be greatest at the beginning of the workshop, but is present throughout the duration of the work as the student is required to use the system in increasingly sophisticated ways. This overhead must be allowed for in the workshop schedule. Specific training sessions based around techniques such as "walk-through" tutorials, are likely to be required at various points in the workshops to introduce new layers of simulator capability (student feedback suggests that reliance on self-guided tutorials is less effective). Nevertheless, in the author's experience these overheads are a price worth paying for the improved clarity of understanding that can come from the use of simulations, and, designed correctly, the walkthroughs can still be used to provide insight into fundamental concepts rather than being simple "driving lessons".

Test thoroughly before deployment

Course leaders will have excellent familiarity with the subject, and using time to test simple problems in software may appear needless. But simulation platforms can introduce unexpected complications. For example, the targeting algorithms used in GMAT can occasionally lead to a model failing to converge on a solution which could easily be derived from first principles on the blackboard. Here, the tutor may wish to spend time explaining the principles and potential problems of numerical methods such as Newton-Raphson iteration, where oscillations around local minima and maxima can prevent a solution being reached, or adjusting problem initial parameters to avoid regions of such behaviour. Similarly, minor software bugs and "features" can lead to surprising results. In GMAT, two apparently equivalent methods of setting up a multi-spacecraft model can lead to different final outcomes, one of which is obviously wrong. Adequate preparation and bug-testing of the workshop (preferably with the use of a volunteer who may make the same mistakes as a student) can identify these issues, and workshop instructions can be updated to avoid the problem disrupting the taught sessions. Note however, that allowing students to experience these behaviours can lead to interesting discussions about the construction and limitations of numerical simulations. Although extensive testing was carried out before deployment of the GMAT workshop, new problems are occasionally encountered, and the workbook produced to accompany the activity is regarded by both facilitators and students as a "living document" updated regularly to reflect these issues.

Simulation is not a replacement for more conventional learning methods

A simulation environment such as GMAT is a powerful system for visualising and exploring the consequences of concepts. But it cannot replace the effort needed to understand the scientific principles underpinning the system being modelled. Even the most simple tutorial or walk-through is little more than an exercise in entering numbers into a black box, without first of all providing the instructional scaffolding that gives meaning to those numbers and their origin. Referring again to lesson (1), the scaffolding may be provided in a variety of formats including screencasts or conventional lectures, but should be provided separately from the simulation, and should start before the first simulation session. Only in this way can simulations fulfil their potential as environments which support learners to make meaning, rather than becoming black boxes that achieve little other than filling contact time.

Insufficient data are currently available to determine whether use of GMAT is leading to a significant long-term improvement in examination results for the conventional astrodynamics module, but adoption of this white / grey box tool within the course has enabled students to explore and test the relationships developed in texts and lectures in a way that cannot be matched in more conventional teaching sessions, helping students to make meaning in their studies. From this perspective alone, introduction of GMAT as a core learning tool has been of substantial benefit. In addition, GMAT is not simply an educational platform available only within the Higher Education ecosystem: it is a professional tool, produced by NASA and used by the agency for mission planning and development, and is being increasingly adopted in the academic research community. Gaining a working knowledge of GMAT therefore has the additional benefit of enabling students to develop skills which are directly transferable to employment in the academic and industrial space sector.

Finally, while this paper has focused on the use of GMAT in a Higher Education environment, instructors in upper secondary education (ISCED 3), should find it an asset in science classes, whether to demonstrate fundamental principles by showing the outputs of simulations to students, or by leading groups of students through the construction and basic analysis of e.g. simple closed orbits for spacecraft, or exploiting the ability of GMAT to model the Earth's orbit, helping students to understand concepts such as seasons and moon phases.

References

Alfieri, L., Brooks, P.J., Aldrich, N.J., & Tenenbaum, H.R. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology*, *103*(1), 1-18. https://doi.org/10.1037/a0021017

Acton, C.H. (1996). Ancillary data services of NASA's navigation and ancillary information facility. *Planetary and Space Science*, *44*(1), 65-70. https://doi.org/10.1016/0032-0633(95)00107-7

Bandecchi, M., Melton, B., Gardini, B., & Ongaro, F. (2000). The ESA/ESTEC concurrent design facility. In *Proceedings of the 2nd European Systems Engineering Conference*, *EuSec*, 1, 329-336. Retrieved from: https://www.researchgate.net/profile/Massimo_Bandecchi/publication/268424314_The_ESAESTEC_concurrent_design_facility/links/55a6125208ae81aec91389dc.pdf

Beshore, E., Sutter, B., Mink, R., Lauretta, D., Moreau, M., Boynton, W., Dworkin, J., Everett, D., Shinohara, C., & Gal-Edd, J. (2015). The OSIRIS-REx asteroid sample return mission. *Aerospace Conference, 2015 IEEE*, 1-14. https://dx.doi.org/10.1109/AERO.2015.7118989

Biggs, J.B., & Tang, C. (2011). *Teaching for quality learning at university: What the student does*. Maidenhead, England: McGraw-Hill Education (UK). GS SEARCH

Bishop, J.L., and Verleger, M.A. (2013). The flipped classroom: A survey of the research. In *ASEE National Conference Proceedings*, Atlanta, GA (Vol. 30, No. 9). Retrieved from: https://www.asee.org/public/conferences/20/papers/6219/view

Bruer, J. T. (1993). *Schools for thought*. Cambridge, MA: The MIT Press. GS SEARCH

Bruner, J.S. (1966). *Toward a theory of instruction* (Vol. 59). Cambridge, MA: Harvard University Press. GS SEARCH

Buchberger, B. (1990). Should students learn integration rules*? ACM SIGSAM Bulletin*, *24*(1), 10-17. https://doi.org/10.1145/382276.1095228

Dori, Y.J., and Belcher, J. (2005). How does technology-enabled active learning affect undergraduate students' understanding of electromagnetism concepts? *The Journal of the Learning Sciences*, *14*(2), 243-279. http://dx.doi.org/10.1207/s15327809jls1402_3

Falconer, J.L., DeGrazia, J.A.N.E.T., Medlin, J.W., and Holmberg, M. (2009). Using screencasts in ChE courses. *Chemical engineering education*, *43*(4), 302-305. GS SEARCH

Foster, C. (2013). Trajectory Browser: An online tool for interplanetary trajectory analysis and visualization. *Aerospace Conference, 2013 IEEE*, 1-6. https://doi.org/10.1109/AERO.2013.6497313

Furinghetti, F., & Paola, D. (2003). To Produce Conjectures and to Prove Them within a Dynamic Geometry Environment: A Case Study. *International Group for the Psychology of Mathematics Education*, *2*, 397-404. GS SEARCH

George, L.E., & Kos, L.D. (1998). *Interplanetary mission design handbook: Earth-to-Mars mission opportunities and Mars-to-Earth return opportunities 2009-2024*. NASA/TM—1998–208533. Retrieved from: https://ntrs.nasa.gov/search.jsp?R=19980210557

Hoyles, C., & Lagrange, J.B. (eds.). (2009). *Mathematics Education and Technology-Rethinking the Terrain: The 17th ICMI Study (Vol. 13)*. London, England: Springer Science & Business Media. GS SEARCH

Hughes, S.P. (2016). *General Mission Analysis Tool (GMAT)* [software tutorial and presentation]. NASA Goddard Space Flight Center. Retrieved from: https://ntrs.nasa.gov/search.jsp?R=20160003520

Izzo, D. (2010). Global optimization and space pruning for spacecraft trajectory design. *Spacecraft Trajectory Optimization*, *1*, 178-200. https://doi.org/10.1017/CBO9780511778025.008

Jakosky, B.M., Lin, R.P., Grebowsky, J.M., Luhmann, J.G., Mitchell, D.F., Beutelschies, G., Priser, T., Acuna, M., Andersson, L., Baird, D., & Baker, D. (2015). The Mars atmosphere and volatile evolution (MAVEN) mission. *Space Science Reviews*, *195*(1-4), 3-48. https://doi.org/10.1007/s11214-015-0139-x

Jonassen, D.H. (1997). Instructional design models for well-structured and III-structured problem-solving learning outcomes. *Educational Technology Research and Development*, *45*(1), 65-94. https://doi.org/10.1007/BF02299613

Kirschner, P.A., Sweller, J., & Clark, R.E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational psychologist*, *41*(2), 75-86. http://dx.doi.org/10.1207/s15326985ep4102_1

McGlynn, E. (2007). Introducing gyroscopes quantitatively without putting students into a spin. *European Journal of Physics*, *28*(3), 479. https://doi.org/10.1088/0143-0807/28/3/008

Neill, A. and Maguire, T. (2006). An Evaluation of the CAS Pilot Project. *Evaluations of the 2005 Secondary Numeracy Pilot Project and the CAS Pilot Project*, 69. Retrieved from: https://nzmaths.co.nz/sites/default/files/Numeracy/References/Comp05/SNPReport05_neill_maguire.pdf

Perkins, D.N. (1992). *Smart schools—From training memories to training minds*. New York, NY: FreePress. GS SEARCH

Rutten, N., Van Joolingen, W.R., & Van der Veen, J.T. (2012). The learning effects of computer simulations in science education. *Computers & Education*, *58*(1), 136-153. https://doi.org/10.1016/j.compedu.2011.07.017

Sappington, J., Kinsey, K. & Munsayac, K. (2002). Two studies of reading compliance among college students. *Teaching of Psychology*, *29*(4), 272-274. https://doi.org/10.1207/S15328023TOP2904_02

Simon, H.A. (1973). The structure of ill structured problems. *Artificial intelligence*, *4*(3-4), 181-201. https://doi.org/10.1007/978-94-010-9521-1_17

Stelzer, T., Gladding, G., Mestre, J.P., & Brookes, D.T. (2009). Comparing the efficacy of multimedia modules with traditional textbooks for learning introductory physics content. *American Journal of Physics*, *77*(2), 184-190. https://doi.org/10.1119/1.3028204

Tooley, C.R., Houghton, M.B., Saylor, R.S., Peddie, C., Everett, D.F., Baker, C.L., & Safdie, K.N. (2010). Lunar Reconnaissance Orbiter mission and spacecraft design. *Space Science Reviews, 150*(1-4), 23-62. https://doi.org/10.1007/s11214-009-9624-4

Vallado, D.A. (2013*). Fundamentals of astrodynamics and applications* (4th ed.). Hawthorne, CA: Microcosm Press. GS SEARCH

Windschitl, M. and Andre, T. (1998). Using computer simulations to enhance conceptual change: The roles of constructivist instruction and student epistemological beliefs. *Journal of Research in Science Teaching*, *35*(2), 145-160. https://doi.org/10.1002/(SICI)1098-2736(199802)35:2<145::AID-TEA5>3.0.CO;2-S

Wood, D., Bruner, J.S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of child psychology and psychiatry*, *17*(2), 89-100. https://doi.org/10.1111/j.1469-7610.1976.tb00381.x

Zhang, Z.H., & Linn, M.C. (2011). Can generating representations enhance learning with dynamic visualizations? *Journal of Research in Science Teaching*, *48*(10), 1177-1198. https://doi.org/10.1002/tea.20443