# Journal of Special Topics

## P4_1 Brownian Motors

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**Abstract**
The idea of using a Brownian motor mechanism to propel oneself in fluids at very small scales is explored. It is discussed whether the random motion of particles could produce enough useful work to be a possible mode of transport.

### The Microscopic World

The microscopic world has many physical properties that are strange and different from a macroscopic perspective. For example, swimming is a just a regular activity when considering it on a macroscopic scale, but if you were to be shrunk to a size of the order of microns then it would become non-trivial; the most notable issues would be the apparent viscosity of the fluid and the random thermal motions of the particles. Fluids appear to be much more viscous at small scales [1] and this coupled with the random movements of surrounding particles (that are approximately the size of you) would make it very difficult to move in any direction. It is the purpose of this paper to investigate whether the Brownian motion of the particles could somehow be harnessed to form a motor that could in theory propel you through the fluid at small scales. This is a significant question since it is thought that biological motors in cells could work in a similar way [2], and it has a clear relevance to the field of nanotechnology.

### Brownian Ratchet and Pawl

Let us introduce a thought experiment in which we have a paddle that is connected to a ratchet and pawl (where the pawl restricts the ratchet so it can only rotate in one direction). Both the paddle and ratchet are isolated in separate boxes of fluid (see Fig. 1). It can be shown that if the device is small enough to let Brownian motion influence its movement and if there was also a temperature gradient it allows useful work to be done by acting as a heat engine [3][4].

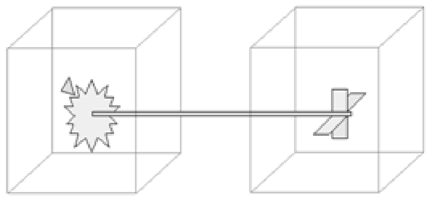

**Figure 1 – Brownian ratchet and pawl:** The random Brownian motion of the particles imparts momentum on the paddle. The ratchet experiences motion in both directions but the pawl allows motion in only one direction, thus creating a motor.

The energy of the Brownian motor is analogous to switching on and off a saw-tooth potential as in Fig. 2 [5]. This shows how the random motion of the particles can allow the motor to have unidirectional motion; in this case, it is moving in the positive x-direction.

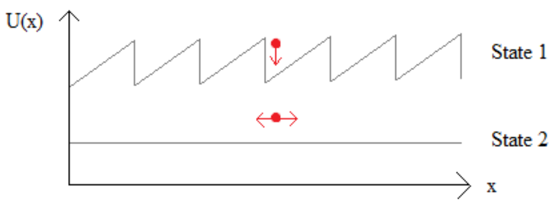


**Figure 2:** The energy of the system can be described with a saw-tooth potential. When the potential is oscillated between ‘on’ and ‘off’ (states 1 and 2 respectively) the particle experiences different forces. In state 1 it tends to the bottom of the potential well, in state 2 it diffuses due to its random motion. In this diagram it will drift in the x-direction.

Unfortunately, naturally found temperature gradients at small scales would not be large enough to have a significant effect, but could be created from non-equilibrium effects such as chemical reactions.

**Using the Motor**

A possible real-life Brownian motor could be the protein myosin. The interactions of myosin are powered by ATP molecules, which may introduce a thermal gradient that allows the Brownian motor motion [6].

The drag force of the particles can be described by the Stokes equation [7] (used for low Reynolds numbers instead of the Navier-Stokes equation)

$$F_{drag} = -6\pi r\eta v, \qquad (1)$$

where $v$ is the velocity of the particles, $r$ is the radius of the (spherical) particles and $\eta$ is the viscosity of the fluid. Using this we can calculate the work that needs to be done to overcome the drag force experienced in the fluid:

$$W = \int F_{drag} \cdot ds = 6\pi r\eta vs, \qquad (2)$$

where $s$ is the length of each potential step (around the order of µm). Let us model the motion of the myosin molecule. An estimated amount of energy one ATP molecule could produce through chemical reactions is $10^{-19}$J [8]. Using this value, an estimated 10nm radius, and the viscosity of blood (~$10^{-3}$ Pas) the maximum speed of the motor through the fluid can be calculated to be around 5mms$^{-1}$.

However, this maximum velocity is not the limit to the motor – its velocity is limited by its diffusion when it is in state 2. Using the Einstein-Stokes relation [7]

$$D = \frac{kT}{6\pi r\eta}, \qquad (3)$$

and using the temperature of the human body (37$^{o}$C) the diffusion rate can be found to be estimated at around $10^{-2}$ nm$^{2}$s$^{-1}$. This makes the motion of the motor very slow, but possible. To make the motor faster, the temperature would have to be increased – possibly by some other chemical reaction.

**Experimental Support**

The theory behind the Brownian motor has been shown to work at a macroscopic level using granular gas to simulate Brownian motion [9]. Instead of the temperature gradient or chemical reaction providing the energy to do the work, the energy comes from the constant vibration of the particles in order for them to behave like an ideal gas.

**Conclusion**

The concept of using Brownian motion to power a motor has been shown to be theoretically correct as long as it is a non-equilibrium system. The most likely source of energy for the system is a chemical reaction that causes a suitably large temperature gradient. It has been shown how it would work in theory using myosin proteins as a motor, with ATP molecules providing the necessary temperature gradient. The concept of a Brownian motor has been shown to work experimentally. Understanding this effect further could improve knowledge of biological motors, and present many interesting ideas for future nanobot designs.

**References**

[1] G. I. Barenblat , A. J. Chorin, *Small viscosity asymptotics for the inertial range of local structure and for the wall region of wall-bounded turbulent shear flow*, Proc. Natl. Acad. Sci. USA, **93** pp. 6749-6752, **(1996)**

[2] G. Oster, *Brownian ratchets: Darwin's motors,* Nature **417**, 25 **(2002)**

[3] P. Reimann, P. Hänggi, *Introduction to the physics of Brownian motors* , App. Phys. A **75**, 2, 169-178 **(2002)**

[4] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics (*Addison Wesley, Reading, MA, 1963), Vol.1, p. 46-1–46-2

[5] R. Astumian, *Thermodynamics and Kinetics of a Brownian Motor,* SCIENCE, **276** pp. 917-918, **(1997)**

[6]http://www.phy.uab.edu/Research/reu/Projects/Darnell.pdf (viewed 02.11)

[7] G. Woan, *The Cambridge Handbook of Physics Formulas,* Cambridge University Press (2003 Edition)

[8]http://hypertextbook.com/facts/2000/AmberIqbal.shtml (viewed 02.11)

[9] P. Eshuis, K. van der Weele, D. Lohse, and D. van der Meer, *Experimental Realization of a Rotational Ratchet in a Granular Gas,* Phys. Rev. Lett. **104**, 248001 **(2010)**