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P3_1 Feeling Lucky?

A. Phong, M. McNally, R. Pierce, T. Searle

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

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Abstract

When playing the popular casino game Blackjack, the odds of getting a "*blackjack"* are quite low. The online gambling industry is growing rapidly, so when playing Blackjack online and the computer manages to win by getting multiple *blackjacks* in a row, was it due to "luck" or was the computer program cheating?

Introduction

Blackjack is a casino card game played with one or more standard decks of 52 playing cards. The basic outline of the game, without going into detail, is as follows. The player is dealt an initial two cards from the shuffled deck(s) and can then take additional cards, one at a time, to try and obtain a card total as close to 21 as possible without exceeding. Cards drawn are not replaced back into the deck, so after the first round, the game repeats with the remaining cards. The numbered cards have a value equal to its number, an ace has a value of 1 or 11 and a face card (Jack, Queen or King) has a value 10.

The quickest combination to achieve a total of 21 is if the initial two cards dealt to the player is an ace plus a ten or a face card, this is known as a *blackjack*. The player wins automatically when dealt a *blackjack* unless the dealer also has *blackjack.* Also winning with a *blackjack* pays the highest (1.5 times the bet) [1] therefore it is one of the most important aspects of the game. The probability of getting a *blackjack* can be calculated, but the number of decks used can vary from a single deck, up to a maximum of 8 decks shuffled together [2] and this can have an effect on the probability.

Probability

Obviously there is a greater chance of seeing *blackjacks* if half the cards have been drawn without any aces, 10s or face cards appearing. But we will consider the probability of obtaining multiple consecutive *blackjacks* at the start before any cards are drawn. Thus, the first two cards of the well shuffled deck(s) must form a *blackjack*. For the case in which only a single deck of cards is used, the probability of this is given by,

$$P_1(1) = 2 \times \left(\frac{4}{52} \times \frac{16}{52-1}\right). \quad (1)$$

This is simply the probability of the first card being an ace and the second card being a ten or a face card, plus the probability of the first being a ten or face card then followed by an ace. Hence, when n number of decks is used, the probability is given by,

$$P_1(n) = 2 \times \left(\frac{4n}{52n} \times \frac{16n}{52n-1}\right). \quad (2)$$

The probability that, on the m^{th} round of cards drawn from a set of cards containing n decks, is the m^{th} consecutive *blackjack* in a row is given by,

$$P_m(n) = 2 \times \left(\frac{4n-(m-1)}{52n-(2m-2)} \times \frac{16n-(m-1)}{52n-(2m-1)}\right). \quad (3)$$

Discussion

As shown in Table 1, the probability of a *blackjack* in the first two of cards decreases with more decks, but the subsequent probabilities of a *blackjack* increases with more decks. The total probability of getting consecutive *blackjacks* is calculated by multiplying the individual probabilities of the number of consecutive *blackjacks* given that the previous draw of cards resulted in *blackjacks.* For example, the actual chance of getting two *blackjacks* at the start of playing is equal to the probability of a *blackjack* from the first draw, multiplied by the probability of a *blackjack* from the second draw given that the first draw was a *blackjack*. These are listed on the right hand side of Table 1 and shows that the probability increases with the number of decks used. It is very rare to see consecutive *blackjacks* no matter how many decks are used. You can expect to see two *blackjacks* in a row about 0.2% of the time. That is once in every 500 sets of two draws.

Fig. 1 shows that if a *blackjack* was drawn, then the chance was getting the next *blackjack* drops significantly the fewer number of decks you use. This is because, for example after the first *blackjack* is seen with a single deck there will only be 75% of aces left, but for 5 decks there will be 95% of aces left, so the loss of the cards that make up a *blackjack* becomes less important. The graph also flattens out as you increase the number of decks so its effect becomes insignificant in high numbers.

Conclusion

Clearly the likelihood of encountering consecutive *blackjacks* is dependent on the number of games you play. As with all games of chance, it is hard to differentiate between cheating and "luck" if the level of cheating employed is very subtle. A large sample of games will need to be played against the computer program that is under scrutiny and the results can then be compared to those in Table 1 to give an insight into the accuracy of the program. Then the standard deviation of the probability distribution can be analysed and a chosen confidence interval can be used to determine whether the program is likely to be rigged. The results of this paper should give a good base for further study on different Blackjack programs.

References

[1]http://www.pagat.com/banking/blackjack.html#betting

[2]http://www.casinoguide.com/blackjack-rules.html

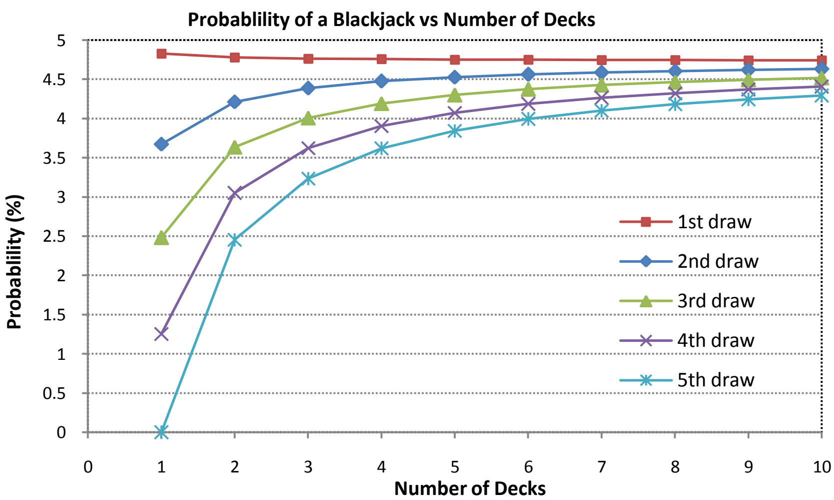

Figure 1. A graph showing the probability of a next consecutive *blackjack* given previous *blackjack* vs. the number of decks.

Appendix

Number of Decks	Chance of *Blackjack* From First Two Cards (%)	Chance of 2nd Consecutive *Blackjack* Given First (%)	Chance of 3rd Consecutive *Blackjack* Given Second (%)	Chance of 4th Consecutive *Blackjack* Given Third (%)	Chance of 5th Consecutive *Blackjack* Given Fourth (%)	Total Cumulative Probability (%)			
						2 Black jacks	3 Black jacks	4 Black jacks	5 Black jacks
1	4.83	3.67	2.48	1.26	0	0.177	0.0044	0.00006	0
2	4.78	4.21	3.64	3.05	2.46	0.201	0.0073	0.00022	5.49E-06
3	4.76	4.39	4.01	3.62	3.24	0.209	0.0084	0.00030	9.83E-06
4	4.76	4.48	4.19	3.91	3.62	0.213	0.0089	0.00035	1.26E-05
5	4.75	4.53	4.30	4.07	3.84	0.215	0.0093	0.00038	1.45E-05
6	4.75	4.56	4.37	4.19	4.00	0.217	0.0095	0.00040	1.58E-05
7	4.75	4.59	4.43	4.26	4.10	0.218	0.0096	0.00041	1.69E-05
8	4.75	4.61	4.46	4.32	4.18	0.219	0.0098	0.00042	1.76E-05

Table 1. Calculated probabilities for values of n from 1 to 8.