Journal of Special Topics

A2_9 None Like It Hot

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Mar 08, 2011

Abstract

This paper looks into the use of a Hohmann transfer to help combat global warming by moving the Earth further away from the Sun to reduce its surface temperature. It was found that the total delta-v required to move the Earth from its current orbit to one which would lower the temperature by 1.1 ^{o}C, would be 118.7 ms^{-1}.

Introduction

Global warming is the increase in the Earth's average surface temperature. This temperature increase is at least partly down to the increasing concentrations of greenhouse gases (e.g. water vapour, carbon dioxide) resulting from burning fossil fuels and deforestation [1]. One way of reducing the 'carbon footprint' (greenhouse gas emissions) is by means of alternate energy sources such as solar power or wind turbines, which are currently seeing use already [2] [3].

Outlined in this paper is an alternate method to help combat global warming, through the use of a Hohmann transfer to move the Earth further away from the Sun in an attempt to reduce its surface temperature.

Investigation

A Hohmann transfer is an orbital manoeuvre using two timed engine burns to move an object (usually a spacecraft) between two circular orbits (see Fig. 1) [4]. Ideally the engine burns would be impulsive (i.e. instantaneous) so extra fuel is required in practice to perform such a manoeuvre. For the purposes of this investigation, it is assumed that the burns are instantaneous.

To find the delta-v required to move the Earth into a wider orbit, the new temperature must first be decided. To work out the temperature of the Earth, the Stefan-Boltzmann Law (see Eq. 1) can be used.

$$L = 4\pi R^2 \sigma T^4. \qquad (1)$$

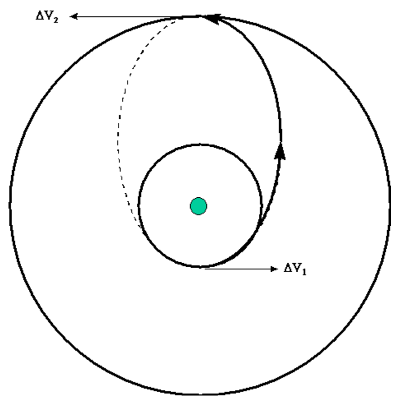

Fig. 1. Diagram outlining a Hohmann transfer orbit [5]

Here, L is the luminosity (power) of the object, R is the radius of the object, σ is the Stefan-Boltzmann constant (5.67 × 10^{-8} Wm^{-2}K^{-4}) and T is the effective temperature of the object. The power intercepted by the Earth, P_E from the Sun is given by [6]

$$P_E = \frac{L_S \pi R_E^2}{4\pi a^2}, \qquad (2)$$

where L_S is the luminosity of the Sun, R_E is the radius of the Earth and a is the semi-major axis of the Earths orbit (1.496 × 10^{11} m [7]).

Now it is possible to form an equation relating the temperature of the Sun to the temperature of the Earth. Using the luminosity, radius and effective temperature of the Earth in Eq. 1, it is possible to equate this with Eq. 2 and rearrange to find T_E, the effective temperature of the Earth (to remove L_S from the equation simply use Eq. 1 with the Sun's values instead):

$$T_E = T_S \sqrt{\frac{R_S}{2a}}, \qquad (3)$$

where T_S is the effective temperature of the Sun (5778 K)[8] and R_S is the radius of the Sun (6.96×10^8 m) [8]. This gives the temperature of the Earth to be ~278 K. The latest report by the Intergovernmental Panel on Climate Change (IPCC) projects that the temperature of the Earth will rise by 1.1 ^{o}C during the 21st Century [9].Therefore it is assumed that the temperature of the earth must drop by 1.1 ^{o}C when it reaches its new orbit. This puts the new semi-major axis of the orbit at a distance of 1.508×10^{11} m.

With the new distance found, it is now possible to calculate the delta-v required to move the Earth to a wider orbit. As the Hohmann Transfer uses two impulses, there will be two delta-v's, Δv_1 and Δv_2 given by Eq. 4 and Eq. 5 [10]:

$$\Delta v_1 = |v_P - v_E|, \qquad (4)$$
$$\Delta v_2 = |v_A - v_{NE}|, \qquad (5)$$

where v_P is the velocity at perihelion of the transfer orbit, v_A is the velocity at aphelion of the transfer orbit, v_E is the orbital velocity of the Earth (assumed to be circular) and v_{NE} is the orbital velocity of Earth in its new orbit (also assumed to be circular). Finding v_P and v_A is simply a matter of solving the equation for the total energy of the system E:

$$E = \frac{1}{2} m v_E^2 - \frac{GMm}{r} = -\frac{GMm}{2a}, \qquad (6)$$

where m is the mass of the Earth, M is the mass of the Sun [8] and r is the radius of orbit. Solving for the velocity gives:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right), \qquad (7)$$

where a is the semi-major axis of the transfer orbit (the average of the two circular orbits semi-major axes).

Using Eq. 7 yields a velocity at perihelion of 29838.7 ms^{-1} and a velocity at aphelion of 29601.3 ms^{-1}. The orbital velocity of the Earth in both orbits can be found by equating the centripetal force and the gravitational force to give

$$v^2 = \frac{GM}{r}, \qquad (8)$$

where the terms are defined as before. This gives the velocity of the Earth as 29779.3 ms^{-1} in its current orbit and 29660.6 ms^{-1} in its new orbit. The delta-v's therefore are 59.4 ms^{-1} and 59.3 ms^{-1} respectively, bringing the total delta-v to 118.7 ms^{-1}.

Conclusion

It is found that the total delta-v required to move the Earth to an orbit that would lower its surface temperature by 1.1 ^{o}C, is 118.7 ms^{-1}. This is a around an order of magnitude smaller than a Hohmann transfer from low-earth orbit to geosynchronous orbit, which requires a delta-v of 3950 ms^{-1} [11]. This however does not make this manoeuvre easy to perform, as there are other factors to consider, mainly the mass of the Earth, and the fact that the Earth is rotating.

References

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