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A1_2 Single Photon Vision

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Abstract
This paper considers a hypothetical human eye capable of registering a single photon as a conscious image in the brain. To such an eye, a receding light source would not grow faint to the point of vanishing, but instead would eventually register in the brain as flickering of individual photons. The psychophysical concept of flicker fusion frequency is used to determine the threshold distance at which the receding light source ceases to appear continuous. Simple lasers with varying angular divergence are used to model the scenario.

Introduction

The human eye collects light through the aperture of the pupil which is then received by photoreceptor cells in the retina [1]. Although it is possible for the retina to respond to a single photon, at least five to nine photons within 100 ms must be received before neural filters will allow the signal to pass to the brain triggering a conscious response. The filter is a necessary adaptation to exclude the high unwanted visual noise that would otherwise be present in low light [2]. Hence, as a constant light source recedes from the eye, the image becomes gradually fainter with the decreasing intensity, until it no longer produces a conscious response in the brain.

This paper examines the hypothetical scenario in which neural filters are removed, and a conscious response to a single photon is possible. A receding light source would still grow fainter with distance, but eventually the brain would register the light simply flickering on and off as single photons are incident on the retina. The rate of flickering will decrease with distance, as the intensity of light at the point of the pupil continues to fall.

Flicker fusion threshold

The rate at which an intermittent light stimulus ceases to appear continuous to the average human is known as *flicker fusion threshold* [3]. This depends on a number of factors both physiological and environmental, but for the purposes of this paper a reasonable value of the flicker fusion threshold is 60 Hz [4]. Therefore, by considering a receding light source, it is possible to calculate the theoretical distance at which flicker fusion threshold occurs, and the human observer no longer registers a steady image.

To simplify the problem, monochromatic laser light sources with varying angular divergences will be considered. The geometry of which is shown in Fig. 1.

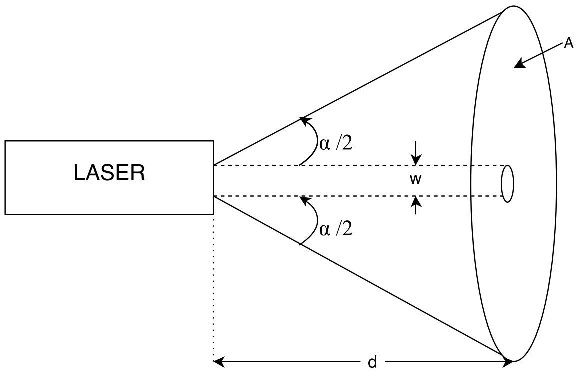

Fig. 1: Geometry of a laser beam of width w, diverging out by angle α, over a distance d, at which point the cross-sectional area of the beam is, A.

For a laser of a given power, P, producing light of wavelength, λ, the number of photons, N, emitted per unit time is

$$N = \frac{P\lambda}{hc}, \quad (1)$$

where h is Planck's constant and c is the speed of light in a vacuum. Fig. 1 reveals that the area of the beam, A, due to divergence at distance d is

$$A = \frac{\pi}{4}\left(2d\tan\left(\frac{\alpha}{2}\right) + w\right)^2, \quad (2)$$

where the terms d and α relate to Fig. 1, and w is the diameter of the non-diverged laser beam. The photon flux at distance d is given by $F = \frac{N}{A}$. Combining Eq. (1) and Eq. (2) this is

$$F = \frac{4P\lambda}{\pi hc\left(2d\tan\left(\frac{\alpha}{2}\right) + w\right)^2}. \quad (3)$$

The required flux, F_t is equal to the flicker fusion threshold through the area of a pupil:

$$F_t = \frac{60\text{ Hz}}{A_p}, \quad (4)$$

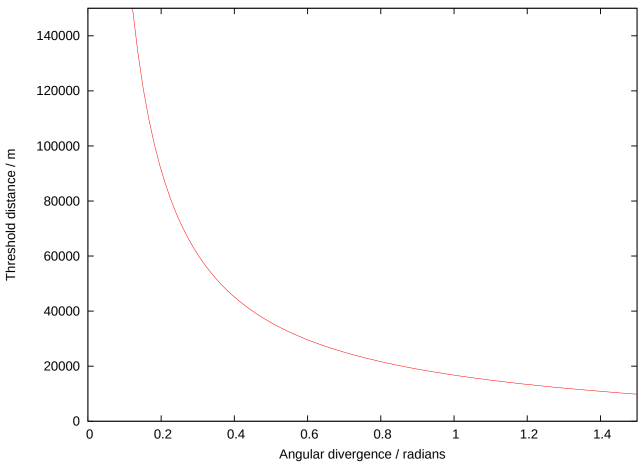

Fig. 2: A plot of Eq. (6), with angular divergence, α, as the independent variable. Approximate values for a typical green laser pointer were used: $P = 2$ mW; $\lambda = 500$ nm; $R_p = 1$ mm. [5]

where A_p is the area of the pupil. Equating Eq. (3) and Eq. (4) and writing $A_p = \pi R_p^2$, where R_p is the radius of the pupil gives:

$$\frac{60}{\pi R_p^2} = \frac{4P\lambda}{\pi hc\left(2d_t \tan\left(\frac{\alpha}{2}\right) + w\right)^2}, \tag{5}$$

where d_t is now the distance at which flicker fusion threshold occurs. Clearly, for any significant distance, w is negligibly smalll in comparison with the diverged beam area. Therefore, discarding w and rearranging Eq. (5) leads to an expression for the threshold distance:

$$d_t = \frac{1}{2}\cot\left(\frac{\alpha}{2}\right)\left[\frac{P\lambda R_p^2}{15hc}\right]^{\frac{1}{2}}. \tag{6}$$

Eq. (6) indicates that for a given laser with angular divergence, α, the distance from the eye at which flicker fusion threshold occurs is proportional to $\cot(\frac{\alpha}{2})$. To demonstrate the magnitude of the distances, d_t, values for a typical laser and pupil radius, R_p, can be substituted into Eq. (6), while allowing α to vary. The results are plotted in Fig. 2.

Realistic values of α are very small for most lasers (often of the order milliradians). For example, a laser with the same specifications as used in Fig. 2, but with a fixed angular divergence of $\alpha = 1$ mrad would not begin to appear to flicker until it was 18308 km away from the eye.

Conclusion

An eye capable of registering single incident photons would never experience a divergent light source becoming invisibly faint. Instead, there would come a point - the flicker fusion threshold - where the image is no longer continuous, but rather flickers on and off due to low photon flux through the pupil.

By considering a laser light source, an expression was obtained to calculate the distance at which flicker fusion threshold would occur in terms of the specifications of a laser and the radius of the pupil. The threshold distances for realistic angular divergence of the laser beam appear to be very high. However, this is a reasonable finding since the required photon flux through the pupil is very small.

References

[1] http://www.ic.ucsc.edu/~bruceb/psyc123/Vision123.html.pdf, accessed on 06/10/2014

[2] http://math.ucr.edu/home/baez/physics/Quantum/see_a_photon.html, accessed 06/10/2014

[3] A. R. Seitz et al, *Visual experience can substantially alter critical flicker fusion thresholds*, Hum Psychopharmacol Clin Exp **20**, 55-60 (2005)

[4] R. Remington et al, *Introduction to Humans in Engineered Systems*, 2012, pp.141

[5] http://www.roithner-laser.com/laser_pointers_532.html, accessed on 06/10/2014