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A4_1 Space Travel Using Relativity

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Abstract

This paper compares two different ships that use time dilation phenomenon to travel across space. One ship travels close to the speed of light at constant velocity while the other travels under constant acceleration of 1g. It was found that the constant acceleration ship can travel 2,000,000 ly in half the ship time it takes the other to travel 200 ly. However only the constant velocity ship is feasible in its application due to fuel requirements.

Introduction

As space travel evolves so does the distance in which humanity will want to explore. One of the current theories of how inter-stellar travel could be achieved is with a ship travelling close to the speed of light. As an object travels close to the speed of light, time within its reference frame slows down compared to a stationary external observer. There are two different ways in which to exploit this phenomena, first is a ship at constant velocity close to light speed the other is a ship that is experiencing a constant acceleration of 1g (9.81 m/s^2). The question therefore is how far could each ship travel within one lifetime of the crew onboard?

Constant Velocity

For constant velocity the equations that will be needed come straight from the theory of special relativity and the Lorentz transformation [1]. The time difference is given by equation 1, where T_0 is the time in the ship and T is the time for the stationary frame, the time for an observer on Earth.

$$T = \gamma T_0 \quad . \qquad (1)$$

Gamma is the Lorentz factor given by equation 2, where v is the velocity of the object and c is the speed of light.

$$\gamma = (1-(v/c)^2)^{-(1/2)} \quad . \qquad (2)$$

Constant Acceleration

For a ship that is travelling under a constant acceleration the equations of motion come from an object following a line of hyperbolic motion. By starting at the Lorentz formulas it is possible to find an equation for the acceleration, a, of an object [2]. This can then be rearranged for the velocity and put back into equation 1, resulting in equation 3.

$$T_0 = (c/a)\sinh^{-1}(aT/c) \quad . \qquad (3)$$

For a ship travelling under constant acceleration, its velocity will always be approaching an event horizon or travelling at the speed of light, which it will never quite reach.

Comparison

To compare the two different types of travel the constant velocity ship is set to move at 0.99c, while the constant acceleration ship is accelerating at 1g after starting at rest. All calculations are done assuming no external force on the ships, e.g. gravity. In this case the constant velocity ship will initially travel a larger distance, however rate of time progression within the constant acceleration ship is constantly slowing down while its speed increases so it will reach a point at which it overtakes the constant velocity ship. This point was calculated to be at an Earth time of 30 years, as the mission time is approaching 4 years.

To calculate this and to make a comprehensive comparison then the distance travelled, d, is needed. For a constant velocity ship this is just the velocity multiplied by the Earth time that has passed. While for a constant acceleration ship distance is given by equation 4,

$$d = (c^2/a)[\cosh^{-1}(aT/c) - 1] \quad . \qquad (4)$$

As well as observing the differences in distance that can be travelled in relative time, it can also be calculated how much fuel they use. This is done by using conservation of energy and momentum from relativistic energy and momentum equations, equations 5 and 6 [3].

$$E_i = (M+m)c^2 \quad , E_f = \gamma mc^2 + E_l \quad . \quad (5)$$

$$P_i = 0 \quad , \quad P_f = \gamma mv + E_l / c \quad . \quad (6)$$

Where E is the energy and P is the momentum, the subscripts i and f denote initial and final respectively. M is the mass of fuel and m is the mass of the payload. E_l is the energy of light where it is assumed all the mass of the fuel is converted into light. By setting them equal and substituting into each other, equation 7 is obtained as the mass of fuel for a given velocity. Using the parametric equations to replace velocity with acceleration, equation 8 is derived for a constant acceleration ship. This is assuming a 100% efficient fuel.

$$M/m = \gamma(1 + v/c) - 1 \quad . \quad (7)$$

$$M/m = \exp(aT_0/c) - 1 \quad . \quad (8)$$

By using all the equations stated, the mass of fuel used in the journeys of the ships can be compared with time on the ship and distance travelled. In 28 years the constant velocity ship will have travelled 197 ly in 199 years Earth time and to do so would have required 13 kg of fuel per 1 kg payload to get up to the required velocity of 0.99*c*. While the constant acceleration ship will have travelled 2,000,000 ly (most of the way to the Andromeda galaxy [4]) in only 15 years ship time or just over 2,000,000 years Earth time. To do this would require 4,100 tonnes of fuel per 1 kg of payload. As the distances travelled increase past this point then the equations used are not reliable, due to the expanding nature of the universe. All these values are for travelling without stopping. For stopping at a point, these values would be larger as one has to reverse the engines halfway through the journey.

Conclusion

As can be seen a ship travelling at constant acceleration would be better for inter-stellar travel in terms of distance it could travel in one lifetime of a crew (60 years). However propelling the ship does start to become an issue at greater distances. Under constant velocity fuel is only expended in reaching the maximum speed while for constant acceleration an increasing amount of fuel is needed to accelerate the ship. This is because as the speed nears the speed of light the mass of the ship also increases, which results in an exponential growth in the need of fuel.

The constant acceleration ship is perfect in the sense that it can travel to almost any point in the observable universe within a crew's single lifetime but it would be almost impossible to fuel with current physical knowledge. Within a crew's lifetime the constant velocity ship would make it outside the local area of the Milky Way (421 ly), it may be slower but it is more feasible as it is possible to fuel it. In conclusion the constant velocity ship would be the most suitable for inter-stellar travel.

References

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[3] http://johanw.home.xs4all.nl/PhysFAQ/Relativity/SR/rocket.html – 19/10/11
[4] http://www.krysstal.com/scale.html - 19/10/11