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A5_5 Don't go to the Dark Side!

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Abstract

This paper investigates the assumptions made in "A5_2 Living in the Dark" as to whether a satellite at the second Lagrangian point will be completely in the shadow of its host planet. The paper finds that while this assumption holds to some degree for satellites in orbit about Mercury and a hot Jupiter at Mercury's orbital distance, a satellite at Earth's L2 point would still receive a portion of its flux directly from the Sun.

Introduction

The paper "Living in the Dark" [1] found that a satellite at the second Lagrangian point (henceforth denoted by "L2 point") could reach a temperature necessary for liquid water (273 K) from the re-emitted flux of the host planet. The paper contained a graph that used a range of values for the radius of the satellite, plotted against the length of time that it would take to reach such temperatures. Another figure displayed the time that such a satellite would be in a temperature range that would be deemed habitable. The maximum satellite radius for both graphs was taken to be the radius of Mercury, whereby it was assumed that the planet would block all of the radiation from the Sun and re-emit it towards the satellite. This paper investigates how much of the radiation would be blocked by the planet, and whether a smaller radius for the satellite would be needed.

Theory

Simple trigonometric relations can be used to determine the radius necessary for the planet to completely obscure the light from the Sun, shown in Figure 1. The radius of the Sun, host planet and satellite are denoted by R_{Sun}, R_p, and R_s respectively, while the distance from the Sun to the planet, or the radius of orbit, is given by R_o. The angle between the top of the body to the one adjacent, α, is the same for both triangles.

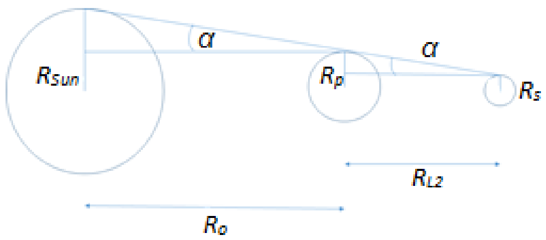

Figure 1. Describes the system in terms of the radius of each object and its position in relation to other objects.

First, the separation between the Sun and the satellite, r, is given by Equation (1):

$$r = R_o + R_{L2} \ , \qquad (1)$$

where R_{L2} is the distance of the L2 point. This can be calculated in the same way as seen in the previous paper, shown in Equation (2):

$$R_{L2} = R_o \sqrt[3]{\frac{M_2}{3M_1}} \ , \qquad (2)$$

where M_2 is the mass of the planet and M_1 is the mass of the Sun. The angle α for both triangles can be calculated through Equation (3):

$$\tan\alpha = \frac{R_{Sun} - R_S}{r} \ , \qquad (3)$$

which gives the triangle between the sun and the satellite. This can be equated to Equation (4) for the triangle between the Sun and the planet:

$$\tan\alpha = \frac{R_{Sun} - R_P}{R_O} . \qquad (4)$$

Equalising and rearranging the equations for R_S, an expression for the maximum radius of the satellite can be obtained, shown below in Equation (5):

$$R_S = R_{Sun} - \frac{r(R_{Sun} - R_P)}{R_o}. \qquad (5)$$

Values for the mass, planetary radius and orbital radius [2] [3] [4] are used to compute the maximum permitted size of the satellite for each of the three systems noted in the paper.

Results and Conclusion

Planet	Orbital Radius ($x10^9$m)	Distance to L2 point ($x10^9$m)	Maximum radius of satellite (km)
Mercury	57.91	0.22	153.71
Hot Jupiter	57.91	4.00	26800
Earth	150	1.5	-523

Table 1. Calculations of the maximum radius of the satellite for the assumption that no radiation from the sun is directly incident upon its surface.

The minimum radius for the satellite is far smaller than that assumed in the previous paper for the satellite situated at the L2 point for Mercury. Previously, the radius of the satellite had been cut off at the radius of Mercury (2440km), but this new calculation shows that the satellite must be much smaller to remain completely in the shadow of the planet.

The radius for the satellite orbiting the hot Jupiter, which in the previous paper was assumed to be at the same orbital radius as Mercury, is much larger than the assumed maximum radius. Additionally, the maximum radius is beyond the radius of Earth, which produces the possibility of an Earth-like satellite being situated at the L2 point. This result suggests that in other solar systems, it would be possible to place a large satellite in orbit about a hot Jupiter and support life for lengthy periods.

Finally, Table 1 shows that there is no possibility of a satellite existing at the L2 point of Earth without the Sun’s radiation being directly incident upon it, shown by the negative sign for the radius of the satellite. This result implies that the convergence point of the Sun’s radiation is much closer to the Earth than the L2 point. This shows that it is physically impossible to place a satellite at the L2 point at the Earth’s current orbit such that there is no direct radiation from the Sun.

Thus, the assumption made in the previous paper has some shortcomings, but to some degree it is still possible to place a satellite at the L2 point of a planet in close proximity to the Sun such that liquid water can exist on the surface.

References

[1] Penney J. I, et al, “Living in the Dark”, 09/10/2014

[2] http://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html, Ed Grayzeck, last updated 01/07/2013

[3] http://nssdc.gsfc.nasa.gov/planetary/factsheet/mercuryfact.html, Ed Grayzeck, last updated 01/07/2013

[4] http://nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.html, Ed Grayzeck, last updated 01/07/2013

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