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A3_4 Jumping Into Orbit

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Abstract
The possibility of something (or someone) jumping from a moon or planet, directly into orbital motion, using only the initial thrust from the jump, is investigated. A physical model is created using IDL to plot paths of motion from a jump in order to do so. It was found that it is impossible to complete a full orbit of any body in space without landing or escaping gravitational pull.

Very little investigation has been done into the path of motion for an object with no additional thrust after takeoff (e.g. a jump). For possible future exploration of the universe it could be useful to explore the effects of a jump on a small moon, extraterrestrial or terrestrial body. In order to do so a computer program is written to create a model that will simulate this situation.

Basic equations of motion for orbit can be derived from Newton‘s law of gravitation [1],

$$F_g = \frac{GMm}{r^2}, \tag{1}$$

and centripetal force,

$$F_c = \frac{mv^2}{r}. \tag{2}$$

Combining Eq. 1 and Eq. 2 then rearranging gives,

$$v_o = \sqrt{\frac{GM}{r}}, \tag{3}$$

where v_o is orbit velocity and r is the radius of orbit.

This gives the velocity an object must have at a certain radius r, to orbit a second body of mass M. Additionally, it has long been known that if an object with an initial velocity from a large body exceeds a certain speed then it will escape the gravitational pull of said body [2].

$$v_{esc} = \sqrt{\frac{2GM}{r}} \tag{4}$$

Escape velocity is given as v_{esc}. This compares to Eq. 3 by a factor of the square root of two. It was decided as a first condition for the model that the initial velocity must be between Eq. 3 and Eq. 4. Any lower, orbit cannot happen and any larger, the gravitational pull will be escaped.

$$\sqrt{\frac{GM}{r_1}} < v_J < \sqrt{\frac{2GM}{r_1}} \tag{5}$$

Where v_J is the initial velocity for the jump and r_1 is the radius of the moon to be jumped from.

An additional, and fairly obvious, condition is that the radius of orbit cannot decrease to lower than the radius of the object being orbited (which is given as r_0).

$$r_1 < r_o \tag{6}$$

Some moons and planets are not uniform in shape and this condition can technically be violated for an elliptical orbit though an assumption made for the model used is that the moon being orbited is a perfect sphere.

The model creates plots (Fig.1 and Fig.2) of trajectories onto a two dimensional graph with a blue line representing the surface of Deimos and a black line plotting the path of motion. The axis of the plot shows an x-direction and a y-direction as though the planet is being viewed face down from a z axis (i.e. two dimensional, cartesian coordinates). No movement is plotted for this z-axis allowing the two-dimensional plot to function.

The 'jumper' starts at the top of the circle (y=radius of moon, x=0) and is given an initial thrust with whatever velocity wanted to be tested. With small increments of time acceleration due to gravity is calculated (Eq. 7) and split into x and y components (Eq. 8) using the cartesian coordinates of position to recalculate velocity (Eq. 9) and position in space (Eq. 10).

$$a_{g,t} = \frac{GM}{r_t^2} \tag{7}$$

$$a_{y,t} = a_{g,t}\cos\left(\arctan\left(\frac{x_t}{y_t}\right)\right) \tag{8}$$

$$v_{y,t+dt} = v_{y,t} + a_{y,t}dt \tag{9}$$

$$y_{t+dt} = y_t + v_{y,t}dt \tag{10}$$

The equations for calculating the y-axis components are shown though the equations for the x-axis are similar (with sine instead of cosine being used for the x component of acceleration in Eq. 8). Additionally for Eq. 8, for each quarter rotation of orbit of the jumper phasing terms had to be added though they are also not included.

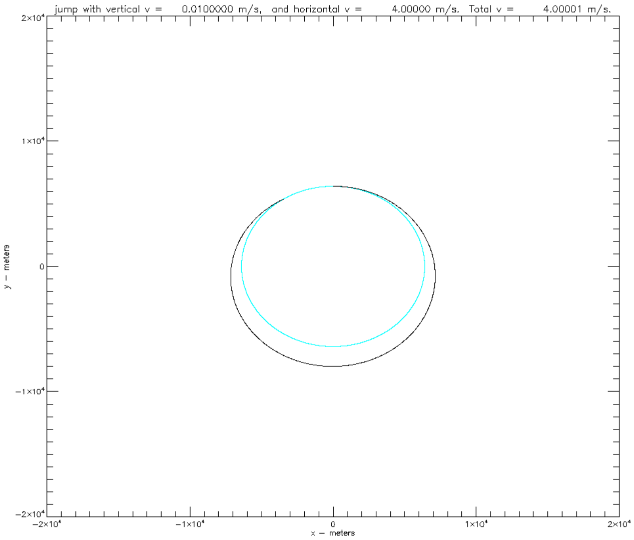

Fig. 1: *Plot showing a jump with very little thrust upwards though* $4ms^{-1}$ *in a horizontal direction.*

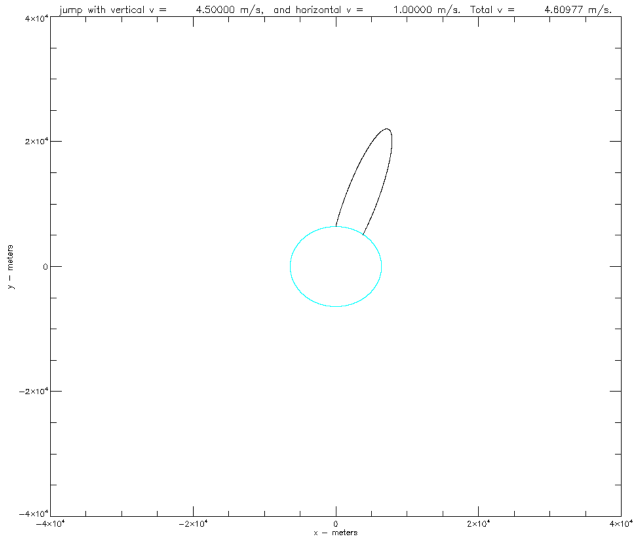

Fig. 2: *Plot showing a jump with larger upwards thrust and little horizontal thrust.*

For the purpose of this model the moon Deimos [3] was used due to it's mass (roughly 1.5×10^{15} kg) and radius (mean radius of 6.2 km), which give using eq.(5),

$$3.82ms^{-1} < v_J < 5.40ms^{-1}. \tag{11}$$

These velocity values are within reason for a human's jumping ability. Furthermore, Deimos does not have an atmosphere which allows the model to work without the requirement of drag forces to be considered. Additionally, Deimos is modelled as perfectly spherical which though inaccurate, also allows for a simplification.

Figure 1 shows a plot taken for an initial velocity of $0.01ms^{-1}$ upwards and $4ms^{-1}$ horizontally. These values were chosen as the horizontal velocity would be sufficient to satisfy Eq. 11, and the upwards velocity would be sufficient to give some lift. It was found that the trajectory which for an object above the surface (with the horizontal speed given) that should be able to achieve orbit does not for a jump from the surface. In this instance, the trajectory brings a landing having travelled around a majority of the moon.

Figure 2 explores a case where the upwards thrust is larger (in this case $4.5ms^{-1}$ with a horizontal velocity of $1ms^{-1}$). It was found that in this instance the trajectory of the jump causes a landing over less distance round the surface of Deimos in comparison to figure 1. It was found through the use of further plots that so long as the horizontal velocity was greater than the value given by Eq. 4 ($3.82ms^{-1}$ for Deimos), increases in the upwards thrust would lower the distance travelled round the surface before landing. This shows that adding height to a jump will decrease the chances of orbit and the results from figure 1 show that even with most of the velocity focused in a horizontal direction, orbit cannot be achieved. As of such it can be concluded that jumping into orbit is not possible.

Considering that a jump into orbit from the surface has been found impossible in this model, there could be ways to achieve orbit from a jump in reality. For example, Deimos is modelled as a perfect sphere in this model and any rotation it may have is neglected. Should someone be able to jump at a high altitude point on it's surface such that it's rotation will keep the person away from high points again throughout it's trajectory, orbit could be achieved. Additionally, if someone were able to build a platform on it's surface that could collapse after launch from it then again, orbit could possibly be achieved. A final thought to consider is if someone were to jump with a weighted object such that they could release it at a certain point in their trajectory in a certain direction to adjust velocity mid-flight and potentially maintain orbit.

Though seemingly useless, the information found in this article shows that should an astronaut jump on a perfectly spherical moon, as long as they do not reach escape velocity they will land back on the surface of said moon without being released into space.

References

[1] Tipler and Mosca, Physics for Scientists and Engineers 6th Edition, W.H Freeman, 2007, p367

[2] Tipler and Mosca, Physics for Scientists and Engineers 6th Edition, W.H Freeman, 2007, p375

[3] `http://www.space.com/20345-deimos-moon.html` viewed on 23.10.2014