Journal of Physics Special Topics

A3_3 Did the Solar System's Ice Giants drift outwards?

L.Ryan, D.Karazhov, T.Booth

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

February 23, 2011

Abstract

This article investigates whether it was possible for Uranus and Neptune to have migrated outwards from an orbit much closer to the Sun. There is a strong chance it could have occurred providing the surface density of planetesimals in the protoplanetary disk was continuous and sufficiently high.

Introduction

It is very likely that the giant planets in our solar system formed via the core accretion model. This model predicts that the timescale to form giant planets increases with increasing distance from the central star. We know from observations that the timescale for dust and gas in a protoplanetary disk which forms the planet, to dissipate is roughly about 10 Myr [1]. If giant planets form by the core accretion model, there is a critical radius beyond which bodies would not be able to form into fully grown giant planets. There is evidence for this in our solar system, from observations which show an absence of large bodies beyond Neptune, but a considerable number of planetesimals exist in this region.

The implication that this model has on Uranus and Neptune is that their current orbital distance from the sun is too large to have been able to grow to their current sizes.

Theory

The large orbits of Uranus and Neptune could be explained if we consider that they migrated outwards by a substantial amount whilst immersed in a planetesimal disk [1]. The idea is that a planet can gravitationally scatter planetesimals from the outer disk into lower angular momentum orbits. This loss in angular momentum is transferred to the planet which then moves outwards to a higher angular momentum orbit. In our solar system the giant planets could have formed within an exterior planetesimal disk [2]. Neptune would have then been large enough to have scattered bodies inwards to orbits between Uranus and Neptune. Uranus and Saturn could have continued to scatter these planetesimals inwards. Conservation of angular momentum then results in the orbits of Neptune and Uranus to increase. This means the ice giants could have formed into their current sizes, closer to the sun with no timescale problem and then consequently migrated outwards.

Investigation

Here we deduce the possibility of Uranus and Neptune drifting outwards from an initial smaller orbit closer to the sun and we examine to what extent they could have migrated outwards.

We first note that the current mass of the Kuiper belt is much smaller than during the early stage of the solar system. The current estimate is about 0.01-0.1 $M_\oplus$ [1]. Models have shown that for Kuiper belt objects to have grown within the lifetime of the protoplanetary disk, the required mass of the Kuiper belt must have been about 10 to 30 $M_\oplus$ [1]. This significant mass deficit shows that there must have been a mechanism in place at an earlier time that removed most of the mass from this region and this could be due to the scattering of planetesimals by the giant planets.

We can consider that a planet lies in an asymmetric disk of planetesimals, where these planetesimals are exterior to the planet's orbit. The planet will strongly attract these planetesimals within a distance equivalent to the radius of the planet's Hill sphere, Δr given by considering a circular sphere as [3]:

$$\Delta r = \left(\frac{M_p}{3M_*}\right)^{1/3} a \ , \qquad (1)$$

where M_* is the mass of the star, M_p the mass of the planet and a is the semi-major axis of the planet. We can calculate how much Uranus and Neptune migrate after interacting with planetesimals within one Hill radius. We first look at how much angular momentum, ΔJ is lost by the planetesimals within this radius due to scattering when they encounter the planet. This is [4]:

$$\Delta J = \Delta m\, \Delta r\, \Delta v\,, \tag{2}$$

where Δv is given by equating the centripetal force to the gravitational force of the planetesimals orbiting around the star of mass M_* [4]:

$$\frac{GM_*\Delta m}{\Delta r^2} = \frac{\Delta m \Delta v^2}{\Delta r}\,, \tag{3}$$

this can be rearranged to:

$$\Delta v = \sqrt{\frac{GM_*}{\Delta r}}\,, \tag{4}$$

and Δm is the total mass of planetesimals within the Hill sphere, given by multiplying the area of this zone of width Δr at an orbital distance of a by the surface density of planetesimals Σ [3]:

$$\Delta m = 2\pi a \Sigma \Delta r. \tag{5}$$

Where we have only considered the area of planetesimals exterior to the planet's orbit and a thickness of one layer for the planetesimal disk. We substitute (3) and (4) into (2) to get:

$$\Delta J = 2\pi a \Sigma \sqrt{GM_* \Delta r^3}\,. \tag{6}$$

The angular momentum lost by the planetesimals is gained by the planet of mass M_p and is given by (2) but Δr is now replaced by the distance the planet migrates outwards, Δa, Δv is now the change in angular velocity of the planet, and Δm is replaced with the mass of the planet M_p:

$$\Delta J = M_p\, \Delta a \sqrt{\frac{GM_*}{\Delta a}}, \tag{7}$$

rearranging gives:

$$\Delta J = M_p \sqrt{GM_* \Delta a}\,. \tag{8}$$

We can now equate (6) and (8) to give:

$$M_p \sqrt{GM_* \Delta a} = 2\pi a \Sigma \sqrt{GM_* \Delta r^3}. \tag{9}$$

Substituting (1) into (9) and rearranging to get an expression in terms of Δa gives:

$$\Delta a = \left(\frac{2\pi a \Sigma}{M_p}\right)^2 \Delta r^3\,. \tag{10}$$

We assume the initial orbit of Neptune was around 20 AU and of Uranus was 10 AU and Σ was 300 $\mathrm{kg/m^2}$ [1]. We also know Neptune's mass is 1.06×10^{26} kg[5] and that of Uranus is 8.61×10^{25} kg [6]. This gives a total distance moved by Neptune and Uranus within one Hill radius of 9.47 AU and 0.35 AU. These values show a significant drift outwards, especially for Neptune. For the planets to continue to migrate outwards the distance it moved Δa must be greater than Δr, so that it is able to scatter a new population of planetesimals. These values can be obtained if we assume that all the planetesimals within the Hill's radius interact strongly with the planet within the lifetime of the protoplanetary disk.

Discussion

Although we showed here that it is possible for Uranus and Neptune to have migrated outwards by interacting with planetesimals within a few Hill radii, we assumed the surface density of planetesimals was constant with radius in the outer regions where the Kuiper belt is. This isn't necessarily true and the density might fall with increasing radius, to a point where the surface density is too small and the planet might stall or slow down considerably. This is because there wouldn't be enough planetesimals for the planet to carry on interacting with. There could also be abrupt changes of surface densities locally, allowing a positive gradient to exist and increasing the migration rate. There's also the possibility of gaps in the planetary disk which would stop the planet from migrating outwards. We also didn't consider collisions between the planet and the planetesimals which could have affected the extent of the migration of the ice giants.

References

[1] L.F.Harold, et al, Icarus **196**, p. 258-273 **(2008).**

[2] J.A.Fernandez and W.H. Ip, Icarus **58**, p.109-120 **(1984)**.

[3] http://star-www.st-and.ac.uk/~iab1/teaching/lecnotes/exoplanets/exoplanf7.pdf (21/02/11).

[4] P. Tipler, G. Mosca, *Physics for Scientists and Engineers* (6th Ed.), (2007), p. 332-367.

[5]http://nssdc.gsfc.nasa.gov/planetary/factsheet/neptunefact.html (21/02/11).

[6]http://nssdc.gsfc.nasa.gov/planetary/factsheet/uranusfact.html (20/02/11).