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P2_6 A Solar Diet Plan

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Abstract

The solar mass loss per year from fusion and solar wind outflow was estimated, and found to be 1.836x10^{17} kg. Calculations showed that the contribution from fusion amount to approximately three times that of the solar wind. The effect on Earth was determined to be an increase in orbital radius of 1.38 cm per year.

Introduction

The Sun is a dynamic body exhibiting short term changes such as flares that can have a direct effect on Earth's magnetic field and atmosphere. This study quantifies a more subtle change over time, the inexorable solar mass loss as nuclear fusion proceeds in its core, and parts of the solar corona are lost to space via the solar wind. Earth's orbit must change due to conservation of angular momentum and in the following the magnitude of this effect is determined.

Mass loss via fusion

The entire solar luminosity, L, is generated by fusion processes taking place in the core. For each reaction, a small amount of mass is converted directly to energy and there are no other sources of L [1]. Details of the various reaction pathways do not have to be considered since the total associated mass loss, ΔM_f, over time t, can be directly related to L by

$$\Delta M_f = \frac{L\,t}{c^2}, \qquad (1)$$

where c is the speed of light. Note that this assumes the present day luminosity of 3.9x10^{26} Js^{-1} [2] remains constant over t. In reality L increases with time due to stellar evolution on the main sequence [1] but over short timescales the effect is negligible. Equation (1) gives a mass loss of 1.368x10^{17} kg yr^{-1}.

Mass loss via the solar wind

Collisions in the extremely high temperature (≈10^{6} K) plasma [1] that constitutes the solar corona enable particles to reach escape velocity. The resulting continuous outflow into the solar system is the solar wind, the second significant mass loss mechanism for the Sun. A estimation of the mass flow rate from first principles is beyond the scope of this study, hence empirical values for the average solar wind velocity, v_s, of 350 kms^{-1} and proton number density, n_p ,of 9 cm^{-3} (as measured at 1 AU in the ecliptic plane) are used [3]. Due to the large proton to electron mass ratio and the low number density of ions heavier than H^{+} in the solar wind, most of its mass is in the form of protons and thus the total mass flux per unit area, F, can be approximated as

$$F = v_s n_p m_p, \qquad (2)$$

where m_p is the mass of a proton. Assuming v_s and n_p are constant, the total mass loss over time t due to the solar wind, ΔM_{sw} , is therefore

$$\Delta M_{sw} = 4\pi\, d^2 v_s n_p m_p\, t, \qquad (3)$$

where d is 1 AU, i.e. 1.496x10^{11} m, giving 4.674x10^{16} kg yr^{-1} for ΔM_{sw}. In reality, the quoted values are overestimates for the solar wind above and below the plane of ecliptic [4].

Neglecting their latitudinal variation thus results in an overestimated mass loss rate. Note however that the contribution from coronal mass ejection events is also neglected, reducing the effect of the overestimate of solar wind mass loss.

Adding the result of (1) and (3) gives a total mass loss, ΔM, of 1.836x10^{17} kg yr^{-1}. While this is over ten times the mass of the moon Phobos [5], it only amounts to 9.23x10^{-14} M_0, a vanishingly small fraction of the current mass of the sun M_0. Note that the contribution of fusion is larger than that of solar wind by a factor of 3.

Effect on Earth's orbit

The relationship between Earth's orbital radius, r, and the mass of the Sun at a given time, M, can be found by considering S, the magnitude of the angular momentum of Earth's orbital motion at velocity v.

$$S = m\,r\,v = m\,r\left(\frac{G\,M}{r}\right)^{1/2}, \qquad (4)$$

where m is the mass of the Earth and G is the gravitational constant. Rearranging gives

$$r = \frac{S^2}{G\,m^2}\,M^{-1}. \qquad (5)$$

Since angular momentum is conserved and m is constant, r is inversely proportional to the mass of the Sun. Hence Earth's orbital radius at time t can be expressed as

$$r(t) = r_0\frac{M_0}{M(t)} = r_0\frac{M_0}{M_0 - \Delta M}, \qquad (6)$$

and the increase in orbital radius as

$$\Delta r = r(t) - r_0 = \frac{r_0}{1-\frac{\Delta M}{M_0}} - r_0 \approx r_0\frac{\Delta M}{M_0}, \qquad (7)$$

where the current value of Earth's orbital distance, r_0, is 1.496x10^{11} m and the binomial expansion was used to simplify the expression, valid for small $\Delta M/M_0$. Substituting the value of ΔM per year found above (in units of M_o) gives a Δr of 1.38 cm per year.

Conclusion

Although the Sun loses a large amount of mass per year in absolute terms, relative to its total mass the loss is insignificant. Hence the associated change in Earth's orbit is small on timescales appreciable by humans. On geological timescales, the effect becomes more significant: per Gyr the total change in r would amount to 1.38x10^{7} m, or 0.0001 AU. Over such periods of time the change in solar luminosity may become significant and further work is required to include this in the calculation of ΔM_f. In addition, the assumption of a constant Earth mass should be revisited by quantifying losses from atmospheric escape and increases from the continuous flux of extraterrestrial material onto Earth's surface over long timescales.

References

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