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P4_9 Tether propulsion to Geosynchronous Transfer Orbit

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Abstract
A tether system is proposed as a means of lifting payloads from low Earth orbit (LEO) to geosynchronous transfer orbit (GTO). The electromagnetic interaction of such tethers with the Earth's magnetic field and plasma has been tested on orbit and is not discussed. The stresses on such a tether are found to be manageable and the mass less than that of the ISS. Although possibly challenging, this technique would produce significant savings on the mass needed to be placed on orbit.

Background

Tethers propulsion experiments have been conducted in space for decades. Tethers exhibit two properties in low-Earth orbit that makes them useful: Firstly, tidal forces can be used to cause the tether to align itself perpendicular to the direction of orbit and become taut. Secondly, their movement through the Earth's magnetic field causes them to generate current which can be utilised by the spacecraft, or alternatively the tether can be used as a motor to change the orbit of a spacecraft without expending propellant.

The first deployment of a space tether was on the Gemini 11 mission [1]. This non-conducting tether was used for gravity-gradient stabilisation and for artificial gravity experiments. A 20km long electrodynamic tether system (TSS) was used with varying success on two space shuttle missions, STS-46 and STS-75 [2]. This was used as a generator rather than a motor, but the symmetry of Maxwell's equations means that this demonstrates both uses.

Geosynchronous transfer orbit (GTO)

To achieve geosynchronous orbit, a satellite must first be transferred from a low-Earth orbit (LEO) using a geosynchronous transfer orbit (GTO). This is a highly elliptical orbit whose perigee intersects LEO and whose apogee intersects geosynchronous orbit. A satellite is inserted into this orbit with a rocket burn, and then when it reaches apogee the orbit is circularised into a geosynchronous orbit with a smaller rocket burn.

The orbit considered for this application is not strictly speaking a traditional GTO: because its perigee is at a substantially higher altitude, the orbit reaches apogee faster and is less eccentric, so requires less circularisation.

A tether can be used to access GTO by having the payload climb the tether mechanically. The engineering challenges of this are beyond the scope of this paper.

Tether size

In order for a gravity gradient stabilised tether to release a satellite into a GTO (altitude approximately 36000km), the tangential velocity of the tip of the tether must be (using the vis-viva equation[3])

$$v = \left(GM\left[\frac{2}{r} - \frac{1}{a}\right]\right)^{1/2} \qquad \textbf{(1)}$$

If the centre of mass of the tether is at an altitude of 500km (easily accessible to launch systems) and in a circular orbit then the angular velocity of the whole system is given by

$$\omega = \frac{v_0}{r_0} = \frac{1}{r_0}\sqrt{\frac{GM}{r_0}} = 0.001107 rads^{-1} \textbf{(2)}$$

So the required altitude of the top of the tether can be found by combining the two equations

$$\frac{r}{r_0}\sqrt{\frac{GM}{r_0}} = \left(GM\left[\frac{2}{r} - \frac{1}{a}\right]\right)^{1/2} \qquad \textbf{(3)}$$

which has a single real solution at *r*=8371km, or an altitude of approximately 2000km.

There must be an equal amount of force acting on the centre of mass from below, but clearly there is not room for an equal length tether so a counterweight must be added. In order to minimise drag issues, this counterweight is assumed to be placed at a minimum altitude of 400km

Forces acting on the tether

An object in orbit normally experiences a centripetal force equal to the gravitational force. For any mass in the tether system, the deviation from this equilibrium is given by

$$\Delta F = mr\omega^2 - \frac{GMm}{r^2} \quad \textbf{(4)}$$

Integrating this over the portion of the tether above the centre of mass gives

$$F_{upper} = \left[\frac{1}{2}\sigma\omega^2 r^2 + \frac{GM\sigma}{r}\right]_{6871km}^{8371km} \quad \textbf{(5)}$$

$$F_{upper} = \sigma \times 5.31 \times 10^6 \; m^2 s^{-2} \quad \textbf{(6)}$$

where σ is the mass per unit length of the tether. Multiplied by the mass per unit length, this gives the highest tension generated by the upper part of the tether, at the point where it meets the centre of mass. This force must be balanced, thus the mass of the counterweight can then be calculated.

$$F_{upper} = F_{cw} + \left[\frac{1}{2}\sigma\omega^2 r^2 + \frac{GM\sigma}{r}\right]_{6771km}^{6871km} \quad \textbf{(7)}$$

$$F_{cw} = \sigma \times (5.31 \times 10^6 - 1.189 \times 10^5) \textbf{(8)}$$

$$m_{cw} = F_{cw}\left(r\omega^2 - \frac{GM}{r^2}\right)^{-1} \quad \textbf{(9)}$$

which gives a value for $m_{cw} = \sigma \times 5.078 \times 10^6$ – as with the values for the total force.

The mass density of an electrodynamic reboost tether designed for the ISS is 0.02kg/m and it has a cross-sectional area of $1.25\text{x}10^{-5}$ square metres[4]. Substituting these values gives

$$m_{cw} = 101560kg \textbf{(10)}$$

$$F_{total} = 2F_{upper} = 1.062 \times 10^7 N \; \textbf{(11)}$$

$$F_{total}/A = 8.496 \times 10^{11} Pa \quad \textbf{(12)}$$

which would pose serious obstacles to implementation of this system. A tether geometry must be considered with lower mass per unit length, and higher area at the point of greatest tension. Assuming tether is tapered to have an area of 1m^2 at the point it meets the centre of mass, then

$$F_{total}/A = \frac{2\times5.31\times10^6 m^2 s^{-2}}{1m^2} = 10.62MPa \textbf{(13)}$$

Assuming that the tether is made of pure aluminium (tensile strength 90MPa, density 2.7gcm^{-3}[5]), this would require the mean area of

$$\bar{A} = \frac{0.2gcm^{-1}}{2.7gcm^{-3}} = 0.074cm^2 \quad \textbf{(14)}$$

corresponding to a radius of just under 1mm. Finally, the total tether mass is given by

$$m = \sigma l = 40000kg \quad \textbf{(15)}$$

Conclusion

The total mass required on orbit is 142 tonnes, less than the current mass of the ISS[6] and so clearly within the current launch capabilities of humanity, and the tether requires no exotic materials for its construction. This of course does not address any practical implementation issues.

If such a system can translate LEO capacity of launchers into GTO capacity it would save 10000kg payload mass for a typical heavy lift launch[7] thus recovering its own mass cost in less than 15 launches.

References

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