Journal of Physics Special Topics

P1_2 The Temperature of Jupiter

D. Treacher, C. Michelbach, L. Holmes

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

October 10, 2014

Abstract

It is well known that there is a discrepancy between Jupiter's observed surface temperature, quantified through infrared emissions, and the theorised temperature based on the approximation that Jupiter acts as a black body. This paper will attempt to assess the additional contribution of two popular factors to the excess heat output of Jupiter; gravitational collapse and the differentiation of He and H [1].

Gravitational collapse

In all celestial bodies, one of the many forces that are at play is the 'Kelvin-Helmholtz mechanism' [2]. This details how the gravitational collapse of a massive body compresses the inner structure, resulting in an increase in internal energy which in turn translates to thermal energy. For a planet like Jupiter, which is often approximated as being in hydrostatic equilibrium, the contraction in the radial direction due to gravity is of the order of millimetres a year [3]. The calculation requires the total gravitational potential energy of the shell that has undergone contraction. Taking the outer and inner limit of the contracting shell as R_1 and R_2 respectively, in spherical polar co-ordinates, the standard total gravitational potential energy, U, is [4]:

$$\mathrm{U} = -\int_{\mathrm{R2}}^{\mathrm{R1}} \frac{G(M(r))(4\pi r^2\rho)}{r}\,dr\,, \quad (1)$$

where G, r, M(r), $(4\pi r^2\rho)$dr and ρ are Newton's gravitational constant, radial distance from the centre of Jupiter, mass held within radius 'r', the mass element of the shell and the density (Taken to be $1.33\text{x}10^3$ kgm^{-3} [5]) respectively. Here we can assume that the total mass of Jupiter remains roughly constant and replace M(r) by volume (of radius 'r') multiplied by density in order to condense and solve equation (1) into equation (2):

$$\mathrm{U} = \frac{-16G\pi^2\rho^2}{15}\left({R_1}^5 - {R_2}^5\right). \quad (2)$$

Here, we take R_1, the initial radius of Jupiter to be $6.99\text{x}10^7$ m [8], and R_2, the final radius after a one thousand year period of contraction to be R_1-($1\text{x}10^{-3}$m)(1000yr), or R_2 = 69899999m, giving a value of $1.4165\text{x}10^{29}$J of gravitational potential energy in the shell. Next, considering the virial theory [6], only half of this energy is radiated out into space (making it available for remote sensing), so over an arbitrary 1000 year period, Jupiter has radiated $7.08\text{x}10^{28}$J of energy as a consequence of gravitational contraction, which equates to $2.29\text{x}10^{18}$ W of power. Through the use of the Stefan-Boltzman law [7],

$$\mathrm{P} = \mathrm{A}\varepsilon\sigma T^4, \quad (3)$$

which relates the power radiated by a body to the fourth power of its temperature (equation (3), where P, A, ε, σ and T denote power emitted, area of Jupiter ($6.14\text{x}10^{16}$ m^2 [8]), emissivity, the Stefan-Boltzman constant and temperature), and the assumption that Jupiter's emissivity is equal to unity, we calculate that as a result of this radiated gravitational power, an additional temperature in the region of 160 K should be seen.

This is clearly an overestimate on the accepted values of 50K – 60K, which is probably a result of approximations made during the calculations shown above. Primarily, the authors have assumed that the shell that has been pulled toward the core is totally uniform. As Jupiter is an oblate spheroid as oppose to a perfect sphere, it is more likely that it is the equatorial radius would contract by 1m over this period, whilst the polar caps would contract by a lesser amount in order to keep Jupiter's eccentricity at its constant value, resulting in an over estimate of the volume of the shell being considered translating to more internal energy present than is true.

Differentiation of H and He

Another possible cause for this temperature difference is the frictional heat generated by the differentiation of hydrogen and helium: as liquid helium rains in the form of precipitation radially inward toward the core, the friction generated between this and the metallic hydrogen composing Jupiter's more solid centre produces heat energy. In order to calculate this, we first need to be aware that, by mass, helium makes up 24% of the composition of Jupiter [9]. Now, assuming that this is mixed evenly through the entire planet, we can see through trivial calculation that Jupiter contains $4.56\text{x}10^{26}$kg of helium. If it is assumed that a quarter of this is moved from a distance r_0 (at the planet's surface) to a distance r_1 (at the region which hydrogen begins to act as metallic hydrogen, which will be taken to be 1,000km below the surface [10]), we can calculate the energy change as follows:

$$\Delta E = G M_J M_{He} \left(\frac{1}{r_1} - \frac{1}{r_0}\right), \qquad (4)$$

Where G is the gravitational constant, M is the mass of Jupiter and helium respectively, r_0 is the radius of Jupiter and $r_1 = r_0 - 1\text{x}10^6$m. From this it can be seen that the energy change is equal to $2.99\text{x}10^{33}$J. When divided by the approximate age of Jupiter, this gives a power output, $P = 2.08\text{x}10^{16}$W. Again, equation (3) can be used to obtain an approximate temperature change due to these processes. This method provides a temperature change of 49.4K.

Conclusion

The calculation for the differentiation of H and He provides a temperature value much closer to that which is seen naturally in Jupiter and may be a possible cause for the temperature difference. However, it should be noted that many assumptions have been made due to the scope of this paper and therefore, the values calculated have relatively wide margins of error, possibly necessitating some combination of both mechanisms to fully explain the discrepancy. It is worth noting that the contribution from radio-isotope heating in Jupiter's core was also calculated. However, it requires accurate knowledge of the core's composition and structure. As this information is still a matter of speculation, many sweeping assumptions, considered too extensive to include in this paper were made, resulting in a rough contribution of 1.3K to the surface temperature. Additionally, residual primordial heat from Jupiter's formation is also considered to be a candidate for the temperature difference. This third mechanism would make a valid route for continued study.

References

[1] http://physics.le.ac.uk/journals/index.php/pst/article/view/19/19 - 10/10/14.

[2] http://nineplanets.org/jupiter.html - 10/10/14.

[3] http://burro.astr.cwru.edu/stu/advanced/jupiter.html - 10/10/14.

[4] An Introduction to Modern Astrophysics (2nd Ed.). BW Carroll & DA Ostlie (2007) pp. 296 –298 – 10/10/14.

[5] http://nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.html - 10/10/14.

[6] http://burro.astr.cwru.edu/Academics/Astr221/StarPhys/gravity.html - 10/10/14.

[7] http://en.wikipedia.org/wiki/Stefan%E2%80%93Boltzmann_law – 10/10/14.

[8] "Solar System Exploration: Jupiter: Facts & Figures". NASA. May 7, 2008 - 15/10/14.

[9] "The helium abundance of Jupiter from Voyager", Gautier, D et al, Journal of Geophysical Research 86, 1981.

[10] "Jupiter and Saturn". Elkins-Tanton, Linda T. (2006).