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A1_1 A Leak in Space

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Abstract

In the interest of risk management, it is important that we consider all possible eventualities that could endanger the crew of future manned spaceflights. In this paper, we derive the rate of loss of oxygen when a hole compromises a ship and determine how long a crew have to act before oxygen reaches critical levels. We find the relation, $t = 2.8 \times 10^{-3} \frac{V}{A}$ s, where V is the volume of the cabin, t is the time till critical levels are reached and A is the area of the exposed hole. We conclude that, for a reasonable V, the crew have a very limited amount of time to act. This makes it clear that the best solution is to compartmentalise a ship, as there is enough time vacate the area and seal off the leak.

Introduction

Space is where the future of mankind lies. We are getting more ambitious with regards to the boundaries of civilisation, particularly with colonising missions to Mars. It is, therefore, even more important to look at the dangers of space travel and the risk management involved. One possible eventuality is a hole in a spacecraft. This could be due to a mechanical fault, accident or asteroid impact, among other reasons. How much time does the crew have once this happens?

Theory

We derive a equation for the time it takes for air to leak out of the cabin. We assume that:

- The volume of the cabin is constant.
- The air in the cabin behaves as an ideal gas, and thus kinetic energy is only attributed to the translational motion of the molecules.
- The temperature in the cabin is constant; all particles of all energies escape at rates proportional to their speeds. This acts to maintain a constant average kinetic energy.

Oxygen levels are important for human survival; and thus, we want an equation only in terms of it. This can be done because the rate of loss of oxygen is only dependent on its concentration.

Considering the oxygen in the cabin, 1/6 of the velocity distribution will be traveling towards the hole (one of six directions). $\rho_o(t)Av_o$ is the mass flow rate, where v_o is the mean speed of the oxygen; A is the area of the hole; and $\rho_o(t)$ is the density of oxygen at time, t. Dividing by the molecular mass of oxygen, m_o, we find:

$$\frac{dN_o(t)}{dt} = -\frac{1}{6}\frac{\rho_o(t)}{m_o}v_o A, \tag{1}$$

Now, using the ideal gas law, we get:

$$\frac{dP}{dt} = \frac{d}{dt}\left(\frac{NkT}{V}\right) = \frac{kT}{V}\frac{dN(t)}{dt} \tag{2}$$

$$\implies \frac{dP_o}{dt} = -\frac{1}{6}\frac{kTv_o}{m_o}\frac{A}{V}\rho_o(t) \tag{3}$$

To solve this for time, we need to find an equation for $\rho(t)$, which can be done using the following definition:

$$\frac{mN(t)}{V} = \rho(t) \tag{4}$$

Therefore:

$$\frac{d\rho_o(t)}{dt} = \frac{m_o}{V}\frac{dN_o(t)}{dt} = -\frac{v_o A}{6V}\rho_o(t) \tag{5}$$

Solving this differential equation and substituting in for the constant, gives us:

$$\rho_o(t) = \rho_o(0)\exp\left(-\frac{v_o A}{6V}t\right) \tag{6}$$

The Maxwell speed distribution states that the velocity of an ideal gas is given by:

$$v = \sqrt{\frac{8kT}{\pi m}}. \tag{7}$$

Now, using equations (3), (6) and (7), we can now write:

$$\frac{dP_o}{dt} = -P_o(0)\frac{A}{V}\alpha\exp\left(-\frac{A}{V}\alpha t\right) \tag{8}$$

Where,

$$\alpha = \frac{1}{6}\sqrt{\frac{8kT}{\pi m_o}} \tag{9}$$

Solving Eq. (8):

$$P_o(t) = P_o(0)exp\left(-\frac{A}{V}\alpha t\right) + \kappa \tag{10}$$

Where κ is a constant of integration, found to be 0 from substituting $t = 0$. Rearranging for t:

$$t = -\frac{V}{A\alpha}\ln\left|\frac{P_o(t)}{P_o(0)}\right| \tag{11}$$

Results

In order to see how long the crew have to react, we take $P_o(t)$ to be the pressure at which humans cannot survive. This is taken to be 6% of atmospheric pressure [1]. This gives us, $P_o(t) = 0.06 \times P(0)$. The initial Oxygen pressure will be taken to be $P_o(0) = 0.21 \times P(0)$ according to the oxygen makeup by concentration [2]. Now, substituting in RTP and the other known values into Eq.(11), we can get a final relation in terms of V and A:

$$t = const\frac{V}{A} = 2.8 \times 10^{-3}\frac{V}{A}s \tag{12}$$

So, if we assume a hole of 5 cm^2 in size, and a cabin volume of 10 m^3 , the crew present will have 57 seconds to plug the hole before they reach lethal conditions. With a hole 4 times the size they will have a quarter of that(inverse proportionality).

Conclusion

In summary, they do not have too long to act, but, in the case of small holes, they will still have enough time to take some form of action. With any hole in the order of a decimetre squared or larger, the time would not be significant at all and the crew would have insufficient time to stop the leakage; they would only have the air in their lungs, which shouldn't be held onto when the pressure gets comparable to vacuum levels (the pressure difference could cause fatal damage to your lungs as the air inside pushes out). The best way to make sure the entire ship isn't endangered is to keep backup oxygen and compartmentalise the ship, so you can isolate the leaking area and the hole can be repaired later in more controlled circumstances.

References

[1] `https://sciencing.com/minimum-oxygen-concentration-human-breathing-15546.html` [Accessed 5 Oct. 2017]

[2] `http://eesc.columbia.edu/courses/ees/slides/climate/table_1.html` [Accessed 26 Oct. 2017]