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P6_5 The Flash and Quantum Tunnelling

A. Alam, J. Campbell, J. Phelan, B. Warne

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

The Flash, a popular DC Comics superhero, is regarded as the fastest human alive. He has the power to run faster than light and phase through solid objects while doing so. Since his power to run faster than light is disputed by Einstein's Theory of Special Relativity, for this article we will assume that he is able to achieve a maximum velocity of 0.99 c. Given the high kinetic energy, in this paper we look at the possibility of him quantum tunnelling through a solid object such as a wall.

Introduction

Comic books regularly feature characters with the ability to walk or phase through solid objects. This super-power is often explained using pseudoscience, for example the Flash is able to control the vibrational frequency of all the atoms in his body and match it that of other objects and pass through them. In reality two solid objects are prevented from passing through each other due to electrostatic repulsion, rather than a difference in atomic vibrational frequency. In this paper we look at whether or not we can attribute Flash's ability to phase through walls to the phenomenon of quantum tunnelling [1]. As in the comic books, we assume that the Flash's body is able to withstand the extreme forces that arises from moving at near-light speed [2].

Theory

Einstein's Theory of Special Relativity determines that the speed of light is the universal speed limit, and as result we will limit the Flash's maximum speed to 99% of the speed of light. In addition, the Flash is modelled as a spherical particle in order to simplify calculations. Therefore, his relativistic kinetic energy can be calculated [3]:

$$E = (\gamma - 1)mc^2 \tag{1}$$

where m is the rest mass, c is the speed of light in vacuum and γ is the Lorentz factor given by:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{2}$$

In this equation v is the velocity of the Flash, and using $v = 0.99\ c$, the Lorentz factor is 7.089.

In order to calculate the probability of quantum tunnelling, we have to consider the potential energy of the wall. The coulomb barrier or electrostatic repulsion prevents two nuclei from getting close enough such that they would undergo a nuclear reaction.

$$V = k\frac{Z_1 Z_2 e^2}{r} \tag{3}$$

where k is Coulomb's constant, Z_1 and Z_2 are the atomic numbers of the two nuclei and the r is the distance between the centers of the nuclei. Inside the nucleus the potential energy is given by the nuclear binding energy, however in order to simplify the model we only considered the electrostatic potential energy.

For a simple case of a constant rectangular potential barrier, the transmission probability is given by $T = e^{-2KL}$ where L is the width of the constant potential barrier, and $K = \sqrt{\frac{2m(V-E)}{\hbar^2}}$ [5]. However, Eq. (3) shows that the potential energy varies with r, therefore the exponent term was modified to:

$$\sqrt{\frac{2m}{\hbar^2}}\left(\frac{kZ_1Z_2e^2}{r} - E\right)^{1/2} \tag{4}$$

Eq. (4) was integrated over the width of the Coulomb barrier, given R_0 is the radius of the nucleus and R is the outer radius of the Coulomb barrier.

$$G = \int_R^{R_0} \sqrt{\frac{2m}{\hbar^2}}\left(\frac{kZ_1Z_2e^2}{r} - E\right)^{1/2} dr \tag{5}$$

$$= \sqrt{\frac{2m}{\hbar^2 E}} Z_1Z_2ke^2\left[cos^{-1}\sqrt{x} - \sqrt{x\Big(1-x\Big)}\right] \tag{6}$$

where G is the Gamow factor [4] and $x = \frac{R}{R_0}$. Thus, the transmission probability is:

$$P = e^{-2G} \tag{7}$$

Calculations

R_0 is calculated using the nuclear radius relation $R_0 = r_0A^{1/3} = 0.459 \times 10^{-14}$ m, where $r_0 = 1.2 \times 10^{-15}$ m and we assumed the wall is made up of only iron atoms (mass number $A = 55.845$ amu). R is the distance at which the Coulomb potential is equal to the energy of the incoming particle.

$$R = \frac{k^2Z_1Z_2e^2}{(\gamma-1)mc^2} \tag{8}$$

Around 99% of the human body is composed of the three elements hydrogen (65%), oxygen (24%), and carbon (10%) [6], and as a result we considered only these three for the calculations. Using the values of R, we calculated the Gamow factor and then the probability of tunnelling.

Element	Gamow Factor	Probability
Hydrogen	0.162538	0.722472
Oxygen	1.31949	0.0714341
Carbon	0.989660	0.138163

Discussion

The average human of 70 kg is composed of approximately 7×10^{27} atoms, which means that there are 4.55×10^{27} hydrogen atoms, 1.68×10^{27} oxygen atoms, and 7×10^{26} carbon atoms. If the quantum tunnelling of each atom is independent from the rest, and are not mutually exclusive, the total probability is $10^{-10^{28}}$.

Conclusion

The probability of 99% of the Flash's atoms quantum tunnelling through an iron nucleus is almost negligible, even though the model was simplified such that each atom tunnelling was considered an independent event. However, given that the human body does not consist of millions of atoms existing separately on their own, the probability will decrease even more if we take atomic bonds into account. This can be a topic of interest for the future, in addition to looking at how the tunnelling phenomena will occur inside the nucleus where the potential energy is given by the nuclear binding energy instead of the electrostatic potential energy.

References

[1] James Kakalios, *The Physics of Superheroes*, (Gotham Books)

[2] `http://bit.ly/2gK08BB` (17/11/2016)

[3] P. Tipler, G. Mosca, *Physics For Scientists and Engineers*, 6th ed., (W. H. Freeman)

[4] `http://bit.ly/2fVLrxj` (17/11/2016)

[5] David J. Griffiths, *Introduction to Quantum Mechanics*, 2nd ed., (Pearson Education)

[6] `http://bit.ly/2f2sSq2` (17/11/2016)