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A5_5 How Big are the Planets, Really?

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Abstract

In this paper, we investigate the discrepancies in circumference of the planets in the Solar System that arise due to relativistic length contraction caused by their angular velocities. We then compare these discrepancies with the aim of creating a new method of comparison for different celestial bodies.

Introduction

In special relativity, a phenomenon known as length contraction occurs, wherein a moving object is measured to be shorter (in its direction of travel) than in a rest frame (where its velocity is 0). Although this contraction only becomes noticeable at near light speeds, the effect occurs at all relative velocities. Length contraction, however, does not only apply to linear movements, but can be applied to any situation with a velocity differential between two points. One such situation is that of a rotating object, such as a planet. Length contraction would occur in the direction of the tangential velocity at any given point, and would cause the overall effect of reducing the circumference of the planet when compared to a reference frame in which the planet is not spinning. Length contraction would occur most noticeably at the planet's equator (the point on the surface with a radius normal to the axis of rotation).

Calculations

Length contraction, sometimes called Lorentz contraction, is calculated by dividing the proper length (length as measured in rest frame) by the Lorentz factor as shown below:

$$L = \frac{L_0}{\gamma} \tag{1}$$

The Lorentz factor γ is:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{2}$$

Where L & L_0 are the contracted and proper lengths respectively, v is the velocity of the object and c is the speed of light.

In this situation we are assuming that all planets are perfectly spherical, rigid bodies and, as such, have a definitive, uniform boundary. This assumption implies that each point on the equator of the planet is instantaneously travelling at the same speed and at the same angle to surface. This means we can treat the entire circumference as if it were a perfectly straight object of equal length travelling at the tangential velocity of the planet at the equator. We can then use Eq. (1) & (2), along with the calculated circumference of the planet's equator and the tangential velocity at the equator, to calculate the contracted circumference and the difference between that and the original circumference.

Results

Using data on the diameter and period (sidereal) of the planets [1], the difference in circumference (L-L_0) was calculated for each and the logarithm of the length was taken in order to allow us to create a graphic that could be used comparably. From the data it is clear that the significant differences occur in the gas giants predominantly with the innermost planets having almost no noticeable difference.

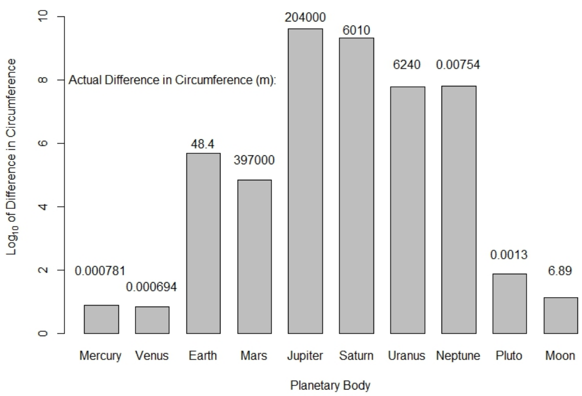

Figure 1: Bar chart of the logarithmic difference in circumference for each planet with the actual difference in metres above.

From the above graph, it can be seen that the difference in the circumference for Jupiter and Saturn is on the scale of 100 km in difference and is significantly greater than that of the other planets. It was found that a comparison between the logarithmic circumference difference and logarithmic mass difference produced a visible trend, as seen in Figure 2.

Although there are several planets that do not fall along the trend line, it is worth noting that all of them orbit within 0.85 AU of the Sun (the Moon orbits the Earth instead at 0.003 AU) and may indicate a minimum required distance for a noticeable difference to occur[1].

Discussion & Conclusion

In summary, Figure 2 shows a trend line and indicates the existence of a possible minimum re-

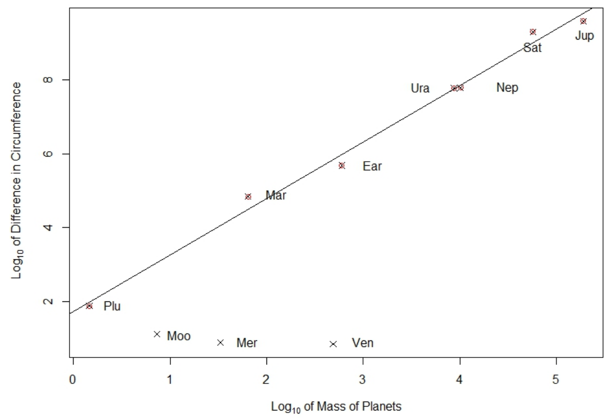

Figure 2: Scatter graph of the logarithmic difference in circumference against the logarithmic difference in mass. Red points indicate planets outside of 0.85 AU orbits.

quired value for the orbital radius before significant effects are seen. As such, this model could be the basis for a new method of comparison of celestial bodies. There are, however, several assumptions that may cause issues in the application of the model. The assumption of a uniform, rigid body is erroneous as, in actuality, planets tend to have uneven surfaces and are not rigid but in fact are subject to shrinking and expanding, particularly in the case of gas giants.

References

[1] `https://nssdc.gsfc.nasa.gov/planetary/factsheet/`[Accessed 10 October 2017]