A3_6 Pandora's Floating Mountains: Rotational Momentum

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Abstract
This report follows a previous article [1], which looked into the physics of how to cause a mountain to float as depicted in the James Cameron movie "Avatar". This article seeks to prove this possible through the fictitious centrifugal force, as experienced in the inertial frame of the planet's surface generated by the rotation of the planet. It was discovered that the necessary period of rotation, if that planet were Earth-like, was 13.4 minutes.

Introduction

All planets' gravity is dependent on latitude. Specifically, the rotation of a planet can cancel out a planet's gravitational pull. This article's aim was to find out to what extent this phenomenon could be responsible for the floating mountains depicted in the film "Avatar" [2]. These mountains are shown as free-floating, apparently unaided by any conventional flight systems. The gravity field of the planet will then be balanced against the centrifugal force, experienced by objects on the surface in a rotating reference frame, caused by the planet's rotation, and therefore, the gravity gradient ranging from the equator to the poles.

Theory

The centrifugal force is a fictitious force used in the description of rotating reference frames. It is dependent on the rotational velocity of the planet in question [3]:

$$F = m\frac{d^2r}{dt^2}, (1)$$

$$F = \frac{mv^2}{R_p}, (2)$$

where R_p is the radius of the planet, and all the other values have their usual meaning.

This force is balanced against that of the centripetal force from the planet's gravitational field. The force due to gravity is given by the Newtonian gravity equation:

$$F = \frac{GM_pm}{{R_p}^2}, (3)$$

where M_p is the mass of the planet in question.

The combination of these equations gives the relation between the planet's properties and the required conditions to counteract it:

$$v = \sqrt{\frac{GM_p}{R_p}}. (4)$$

The velocity of a point on a spherical object depends on the rotational velocity and the latitude of the point, assuming a perfectly spherical Earth:

$$v = r\omega = R_p\omega\cos\theta, (5)$$

where ω is the rotational speed, in cycles per second, and R_p is the radius of the earth at the equator, and θ is the latitude. The resultant force in the radial direction is therefore given by taking the force from rotation from the force from the planet's gravity. This cancels down from the sum of total forces to the general form for Δg, where Δg is the difference between earth gravity with no rotation, and the experienced gravity: $\Delta g = g_0 - a_{rotation}$:

$$F = \frac{GM_pm}{{R_p}^2} - \frac{m(R_p\omega\cos\theta)^2}{R_p}, (6)$$

$$a = g_0 - \frac{\left(R_p \omega \cos\theta\right)^2}{R_p}, (7)$$

$$R_p(\omega\cos\theta)^2 = \Delta g, (8)$$

$$\text{where } g_0 = \frac{GM_p}{{R_p}^2}. (9)$$

Therefore, when $\Delta g = g$, the gravity of the centrifugal force balances the gravity, giving no net forces. Under these conditions, mountains could theoretically float. Ideally, this should occur around the equator to minimise the required rotational velocity; additionally, this would avoid the scenario where mountains are floating at latitude, causing objects at lower latitudes to escape into higher altitudes.

For an Earth-like planet $\left(g{\sim}9.81\text{ms}^{-1},\ R_p{\sim}6371\text{km}\right)$, the required rotational velocity would be

$$\omega = 1.24 \times 10^{-3}\text{s}^{-1},$$
$$t = 13.4\text{mins.}$$

This translates to a significant distribution of the planet's gravity, depending on latitude (Figure 1).

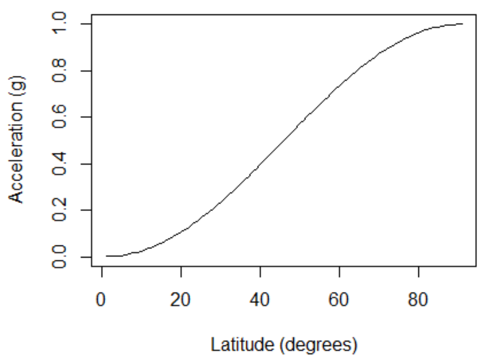

Figure 1: Strength of gravity on the surface of the planet, as a fraction of the non-rotational gravity field strength, as a function of latitude.

Conclusion

The speed calculated for an earth-like planet seems unlikely, as no known cases of such rapid rotation. The density of the planet is the main factor for deciding the required rotational speed, as the gravitational strength is a function of mass, and both forces are a function of planetary radius. Therefore, a large, low-density planet would be required to lower the necessary angular rotation to more sensible values.

References

[1] http://james-camerons-avatar.wikia.com/wiki/Pandora Accessed on 15/10/2014
[2] "Pandora's Floating mountains"; McQuade G, Walker M, Garland L, Bradley T; Journal of Physics Special Topics, Vol. 13 2014
[3]http://en.wikipedia.org/wiki/Centrifugal_force_%28rotating_reference_frame%29 Accessed on 27/11/2014