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P3_13 Solar Sails

R. Pierce, A. Phong, M. McNally, T. Searle

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

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Abstract
The theory of a solar sail is introduced and a simple model of a 1000m^2 solar sail weighing 2000kg launched from near the Earth is considered. It is found that the solar sail can reach a speed of 26km s^{-1} in 6.33 years but beyond that acceleration is very small, making a solar sail viable for exploring the edge of the solar system.

Introduction

A 'solar sail' is a form of spacecraft propulsion that uses the radiation pressure from the sun to propel a spacecraft to high speeds. The spacecraft is built to have a large surface area, a 'sail' to catch the photons from the sun. Solar sails begin very slowly but they are accelerating for many years. This paper examines the use of a solar sail to propel a spacecraft to see what speeds could be obtained and in what time scale.

Theory

Photons, despite being massless do in fact have momentum, so a photon incident on a body will transfer a small amount of momentum to the body, causing the bodies velocity to increase. The momentum of a photon is given by [1]

$$p = \frac{E}{c}, \quad (1)$$

where E is the photon energy and c is the speed of light. Pressure is defined as [2]

$$P = \frac{F}{A} = \frac{dp/dt}{A}, \quad (2)$$

where F is the force and A is the area and force is equivalent to the rate of change of momentum. The radiation pressure for an incident photon that is completely reflected is then given by [1]

$$P = 2\frac{I}{c}, \quad (3)$$

where I is the intensity of the radiation in Wm^{-2}. The Sun has an average luminosity of 3.84×10^{26}W [3]. The intensity of the radiation pressure at a distance r from the sun is given by

$$I = \frac{3.84 \times 10^{26}}{4\pi r^2}, \quad (4)$$

Other than the radiation pressure the Sun outputs a stream of charge particles 'the solar wind', this wind also transfers momentum to the sail but this pressure is 3 orders of magnitude smaller than the radiation pressure [4] and so will be ignored in the calculation. There is also a drag force present on the sail from interstellar dust and gas particles and also from planetary atmospheres, which retards the sails velocity. The drag force is given by [5]

$$F_d = \frac{1}{2}\rho v^2 C_d A, \quad (5)$$

where ρ is the density of the medium, v is the velocity of the body, C_d is the drag coefficient and A is the cross-sectional area of the body.

Velocity Calculations

Consider a spacecraft with a solar sail attached, the total mass is 2000kg. The solar sail is made from aluminium and is assumed to have a reflectivity of 1 (in reality no material can be perfectly reflective). The solar sail has dimensions 1000m x 1000m with a thickness of 100nm. This design for a solar sail has been proposed before [6]. Aluminium has a density of 2700kg m^{-3} [1] and so the sail would weigh 270kg and would also require some sort of support structure. Still the payload could easily weigh around 1500kg [6].

The spacecraft is assumed to start from rest at a distance 1.501×10^{11}m away from the Sun, approximately 100 million metres from the Earth and effects from atmospheric drag gravitational effects are ignored. The drag force is assumed to come from interstellar hydrogen which makes up 70% of the local interstellar medium with a number density of 10^{-9} atoms m^{-3}[6] and so a mass density of $1.66 \times 10^{-27}\mathrm{kg\,m^{-3}}$. It should be noted that within the Solar System the spacecraft is not within the local interstellar medium but the heliosphere, but since the spacecraft is considered for an interstellar voyage it is reasonable to assume that the entire voyage can be considered as being in the interstellar medium.

At a distance of $1.501 \times 10^{11}\mathrm{m}$ the radiation pressure on the solar sail is equal to 9.04N which gives an change in velocity of $4.56 \times 10^{-3}\mathrm{ms^{-1}}$. The drag force is negligible at these low speeds but at higher velocities it becomes comparable to the radiation pressure. Since the drag force is a function of velocity and velocity a function of distance the value of the velocity at a given distant can be solved numerically. This is done by assuming that time can be discretised into individual steps at each point, and so constant acceleration can be assumed and the equation of motion for constant acceleration used to calculate velocity and distance at each time step. So from the acceleration applied by the radiation pressure the distance travelled in one step is calculated using [1]

$$s = ut + \frac{1}{2}at^2$$

where *u* is the initial velocity, *a* is the acceleration and *t* is the time step. After each time step a new distance and velocity is calculated. The updated distance and velocity is then used to re-calculate the radiation pressure and the updated velocity the drag force so the net force on the craft changes and hence the acceleration.

A computer program was used to iterate the discrete time calculation. The limit on the program was that only 200,000 iterations could be performed. So calculating velocities over large time scales requires a large time step for each iteration. With a time step of 10,000s, 6.33 years of travelling time can be calculated. After this time the spacecraft is travelling at 26km s^{-1}, faster than the voyager probe which is travelling at 17km s^{-1}[7]. The spacecraft has travelled approximately $6.9 \times 10^{11}\mathrm{m}$ or 4.8AU. Unfortunately for interstellar travel it is very much a case of diminishing returns and in 633 years the velocity has only increased to approximately 50km s^{-1}. The accuracy of the calculation decreases with longer time steps, but the terminal velocity, when factoring in mass changes due to special relativity appears to be around 0.015c, achieved around 10^5 – 10^6 years.

Conclusion

A simple solar sail could be used to reach the distant planets of the solar system with a reasonable payload. However interstellar travel is not particularly feasible. A possible remedy to this would be to have a solar sail that would fold itself up as it approached a nearby star and then redeployed to gain additional momentum from the star. Though this would require significantly more weight for the sail to survive the folding process intact and might be prone to failure.

References

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