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P1_1 The Fate of the Falling Feline

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Abstract

This article considers the potential energy of a falling cat and explores the maximum height from which the cat can fall without sustaining limb fractures, taking into account the drag due to air resistance. 2.3m was found to be the highest distance from which a cat can fall without fracturing all four of its legs. However, investigations into the typical number of limb fractures sustained indicated that cats are able to fall from greater heights. This is due to the assumptions made; cats have only one bone in their legs and their muscles do not contract upon landing.

Introduction

Arguably, the most curious aspect of a cat is its ability to not only land on its feet, but to survive when falling from great heights (e.g. 128m [1]). This article attempts to predict the maximum height a cat could fall from without breaking all four of its legs. Mehlhaff and Witney [1] completed a study of cats injured through high-rise syndrome (defined as a fall from a height of greater than 7-9m), and found that out of 132 incidents over a 5 month period, 39% resulted in limb fractures. Dvuk *et al* [2] found that out of 119 cats with high-rise syndrome, 46.2% had broken limbs from an average fall height of four storeys. Whilst other injuries were incurred in many of the incidents in both of these papers, the following investigation focuses on limb fracture sustained by the cat landing on all four of its legs.

Model

It is assumed a typical cat can be modelled as a cuboid of 45cm x 15cm x 35cm (*L* x *W* x *H*) and that the cat does not spread its legs throughout the fall (see *Figure 1*). This assumption neglects the cat's tail. The legs have been assumed to be 25cm in length and the average leg bone cross-section diameter is 2cm. The leg has been assumed to be without joints and the cat lands on all four legs.

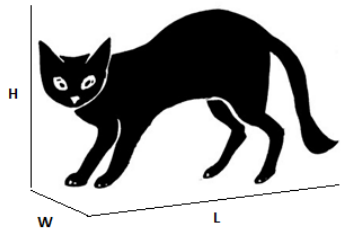

Figure 1: Diagram of a standing cat with dimensions *L*=45cm, *W*=15cm and *H*=35cm.

Equations

Considering the cat's potential energy, E_{PE}, the energy provided by the drag force over a distance *h* (due to air resistance), E_D, and the energy required to break the cat's legs, E_{BR} (the general equation taken from [3] is multiplied by 4 as the energy required to break all four of the cat's legs is desired) the potential energy minus the energy provided by the drag force and the breaking energy must be lower or equal to zero for the cat to not break all four legs, i.e.

$$E_{PE} - E_D - E_{BR} \leq 0 \qquad (1)$$

where,

$$E_{BR} = 4\,x\,\frac{UBS^2}{2\,Y}\,V, \qquad [3](2)$$

$$E_{PE} = mgh, \qquad (3)$$

$$E_D = \int_0^h \frac{C_D \rho A\, v^2}{2}\, dy. \qquad (4)$$

As the velocity, v, can be written as $v^2 = {v_0}^2 + 2gy$, where the initial velocity, v_0, is zero, *Equation (4)* becomes

$$E_D = \frac{C_D \rho \, A \, g \, h^2}{2}. \qquad (5)$$

In *Equations (1-5)*, UBS is the ultimate breaking stress of bone (1×10^8Pa [4]), Y is the Young's modulus of bone (1.4×10^{10}Pa [5]), V is the volume of bone in one leg (taken as 25cm in length and 1cm cross-sectional radius, giving a volume of 7.85×10^{-5}m^3 when modelled as a cylinder), m is the total mass of the cat (assumed 5kg), g is acceleration due to gravity (9.81ms^{-2}), C_D is the drag coefficient (0.75 as the cat is modelled as a cuboid [6]), ρ is the density of air (1.2kgm^{-3} [6]), A is the area of the plane perpendicular to the direction of motion ($W \times L$ = 0.068m^2) and h is the height of the fall. It has been assumed that the bone can be considered elastic up to this breaking point, so no damage is incurred until the breaking point is exceeded.

However, these assumptions will only hold for when the cat is not falling at its terminal velocity, v_t, as the potential energy and the energy provided by the drag force are equal in magnitude at this point. Equating the drag force to Newton's 2nd law of motion, the terminal velocity is given by

$$v_t = \sqrt{\frac{2mg}{\rho A C_D}}. \qquad (6)$$

Equation (6) is then substituted into the equations of motion to calculate the minimum height at which the cat reaches terminal velocity (81.7m or approximately 20 storeys when assuming 4m per storey). This means *Equation (1)* can then be calculated up to this point by setting the upper limit, h, to 81.7m.

Conclusion

This model predicts limb fracture occurs in all four legs of a cat when falling from heights above 2.3m (see *Figure 2)*; beyond 81.7m, the net energy has been calculated to remain constant. However, daily experience tells us cats can fall from a height of 2.3m (and indeed greater) without limb fracture. This mismatch is representative of the assumptions that the leg has only one bone, and that the muscle contraction can be neglected.

If these assumptions were not made, i.e. if muscle contraction and all three bones were included in the calculations, then the cat may survive from much greater heights than stated here. These considerations would cause the net energy to be reduced. If this is reduced by more than 1892J, the net energy would be negative. This would mean that a cat would not fracture all four limbs regardless of the height fallen. This would explain the fact that the number of fractures per cat from [1] and [2] does not equal 4 at any height (maximum height a cat fell was 128m).

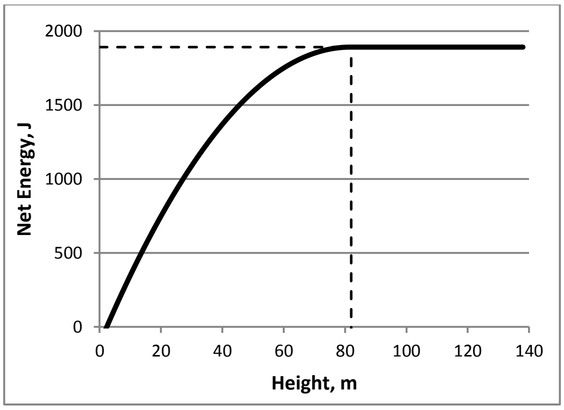

Figure 2: The Net Energy of the system plotted versus the height of the cat fallen to illustrate the asymptotic behaviour at a height of 81.7m.

References

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