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P2_11 Fuselage Holes

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Abstract

This article looks at how long it would take the air to evacuate an aeroplane if the fuselage developed a hole, imposed under incompressible constraints. We find that the time for depressurisation, as a function of hole size, follows a power law.

P2_3 General Physics

Introduction

This article looks at how long it would take the air to evacuate an aeroplane (for the inside pressure to equal the outside pressure) if the fuselage developed a hole and how this varies as a function of hole size. We formulate this problem by approximating air to be an incompressible fluid, the validity of which is debatable. We assume that it is suitable for this simple model.

Deriving an expression for the flow rate, v

The Bernoulli equation for an Incompressible fluid is given as

$$B = \frac{P}{\rho} + \frac{v^2}{2} + gz \qquad (1.0)$$

where *B* is a constant, *P* is the pressure, v is the velocity of the air, ρ is the density of air and *z* is the displacement in the z direction (which will be zero assuming horizontal flow).

The problem is set up as so:

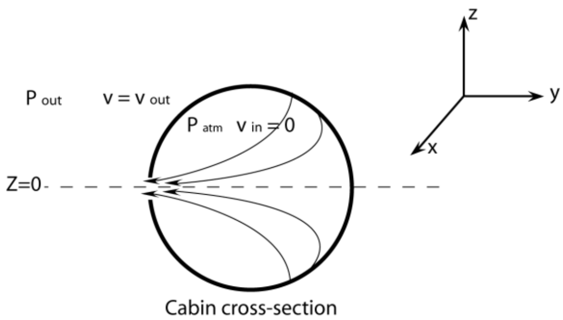

Figure 1.

Outside the plane

$$B = \frac{P_{out}}{\rho_{air}} + \frac{v^2}{2} \qquad (1.1)$$

Inside the plane

$$B = \frac{P_{atm}}{\rho_{air}} + \frac{v_0^2}{2} \qquad (1.2)$$

where v_0 =0

Equating the flows and rearranging w.r.t. *v* gives us:

$$v = \left[2\frac{(P_{atm} - P_{out})}{\rho_{air}} \right]^{1/2} \qquad (2.0)$$

Equating the velocity to the mass flow rate

The Mass flow rate is $\frac{dm}{dt} = \rho a v$ (3.0)

where *a* is the cross-sectional area of the hole.

In our simple approximation of an incompressible fluid, $dm/dt = constant$. For a constant mass flow rate $\Rightarrow$

$$\frac{m}{\rho a v} = \Delta t \qquad (3.1)$$

Treating the left side of equation (3.1)

$$\frac{m}{a\rho} = \frac{V}{a} = \frac{AL}{a} \qquad (3.2)$$

$$\Rightarrow \Delta t = \frac{V}{a} v^{-1} \qquad (3.3)$$

where *V* is the volume of the plane.

Substituting for the velocity in equation (3.3) using equation (2.0):

$$\therefore \Delta t = \frac{V}{a}\left[2\frac{(P_{atm} - P_{out})}{\rho_{air}}\right]^{-1/2} \quad (4.0)$$

Determination of Constant Values

Pressure

Using the ideal gas law it can be shown that,

$$P_{out} = P_{atm}e^{-\frac{Z}{H}} \quad (5.0)$$

where the scale height is

$$H = \frac{KT}{m_{air}g} \quad (5.1)$$

T is the temperature at 35,000ft and K is Boltzmann's constant.

Z=10,668m (35,000ft) [1]

$T_{10,668} \approx -50°\text{C} = 223K$ [2]

A Good approximation for the mass of air

$0.99m_{air} = 0.78m_{N2} + 0.21m_{02}$ [3] (5.2)

$$m_{air} \approx (0.78m_{N2} + 0.21m_{02})(100/99)$$

$$m_{air} \approx 2.396 \times 10^{-26}kg \text{ (4 s.f.)} \quad (5.3)$$

Inserting these values into equation (5.0) we find the pressure outside of the aeroplane to be $\therefore P_{out} = 44.85\ kPa$ (4 s.f.)

Density of the air inside the plane

$\rho_{air} = 1.205\ kg \cdot m^{-3}$[4] (At 20 C, assumed to be a realistic cabin temperature).

Volume of the Plane

Assuming the fuselage of a plane to be approximately cylindrical,

$$V_{plane} = (\pi r^2)L \quad (5.4)$$

For a *Boeing 747-8* $V_{plane} = 2230\ m^3$ [1]

∴ constant, *c* = 7.282 (4 s.f.)

Inserting these values into equation (4.0) we find that

$$t = \frac{c}{a} \quad (5.5) \quad \text{where } c = V\left(2\left(\frac{P_{atm}-P_{out}}{\rho_{air}}\right)\right)^{1/2} \quad (5.6)$$

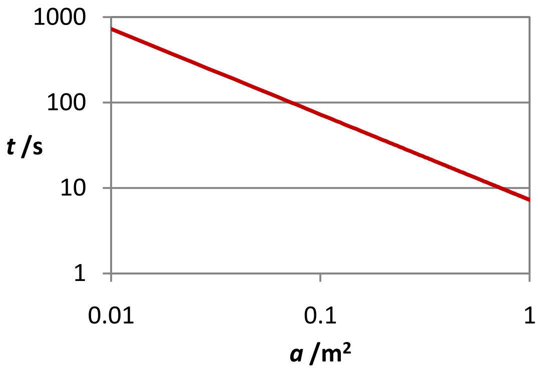

Figure 2. t vs. hole size.

Discussion

In reality the flow rate won't be constant. The velocity is a function of pressure, which is a function of the density within the fuselage. However, these functions will still be linear so we assume that the depressurisation time will still be of this form. The model produces plausible answers, therefore implying that the model is feasible. For a standard magnum .28 calibre round [5] with radius, *r* 3.5mm, assuming it leaves a hole of the order 10 larger, corresponds to a depressurisation time, *t* of 32 mins.

Conclusion

Within this simple model we have shown that the time for depressurisation, as a function of hole size, follows a linear, inverse power law. Depressurisation times for specific hole sizes can subsequently be read off figure 2.

References

[1] 747-8 Technical Characteristics, *http://www.boeing.com/commercial/747family/pf/pf_400_prod.html*

[2]Temperature in the Stratosphere, *http://www.windows.ucar.edu/tour/link=/earth/Atmosphere/stratosphere_temperature.html&fr=t%20t*

[3] Terrestrial Atmosphere, *http://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html*

[4]Density of air, *http://www.engineeringtoolbox.com/air-properties-d_156.html*

[5]7 MM Rem.Mag *www.blue-star-inc.com/catalog/ammo*