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A4_14 How to score a goal

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Abstract

This paper examines the force causing a football to curve when an angular velocity, or spin, is imparted. A relationship between the distance travelled by the football, in the direction it is kicked, and the amount that the path bends is found.

Introduction

For an object with angular velocity moving in a fluid, there exists a force perpendicular to the object, called the Magnus force [1]. Footballers commonly take advantage of this force, by imparting angular velocity to a football, so that it will bend around a wall of players. This paper aims to examine the relationship between the angular velocity and the perpendicular force acting on the football.

The Magnus Force

When an object is rotating in a fluid, there exists a thin layer of fluid which is moving with the surface. By considering the air the football is moving in as a fluid, the situation can be illustrated as shown in figure 1. This shows that there is a difference in the velocity of air at the surface of the football between the upper and lower sides, due to the combination of the angular velocity and the air flow velocity. The difference in velocities causes a pressure difference, thus resulting in a net force. This is the Magnus force.

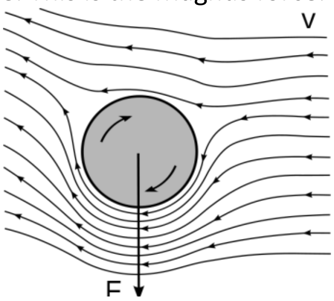

Figure 1. A diagram showing a rotating football moving at velocity, v, through air. There is a net force, F, acting on the football, which is the Magnus force [2].

By considering the air flow on each side of the football, the magnitude of the Magnus force can be determined. Only the directions perpendicular to the fluid flow are considered, because for any force, acting in a direction other than perpendicular to the flow, there is another equal and opposite force. Bernoulli's equation is used to equate the pressure, flow velocity and height from the ground, P_1, v_1 and z_1 respectively for the lower side of the ball, and P_2, v_2 and z_2 for the upper side. It is noted that the flow velocity is equal to the velocity of the ball through the air. By using these values, Bernoulli's equation is [1]:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho v_2^2 + +\rho g z_2, \quad (1)$$

where ρ is the air density and g is the gravitational acceleration. By assuming that the air has a constant density, and the height difference is negligible, equation (1) becomes:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2. \quad (2)$$

Figure 1 shows that the velocity on the lower side of the football, v_1, is equal to $v + \omega R$, and the speed on the upper side, v_2, is equal to $v - \omega R$, where v is the fluid flow, ω is the angular velocity of the ball and R is the radius of the football. By substituting for v_1 and v_2 into equation (2) and rearranging, it can be found that:

$$P_2 - P_1 = 2\rho\omega R v.$$

The resulting force on the football is equal to the product of the pressure difference, $P_2 - P_1$, and the effective area of the football, πR^2 [2]. Therefore, the Magnus force, F, acting on the football is:

$$F = 2\pi R^3 \rho \omega v. \qquad (3)$$

This shows that the Magnus force is linearly dependent on the velocity of the football through the air and its angular velocity.

Amount of bending

For a football moving with constant velocity and spin, a relationship for how much the ball bends with distance can be determined. It is assumed that the ball is kicked in the *x*-direction and the distance that the ball travels in the direction of the Magnus force, *D*, will be considered as the *y*-direction. Therefore, the Magnus force, *F*, and acceleration in the *y*-direction, $a_y = F/m$, where m is the football mass, is constant. By considering constant acceleration with an initial zero velocity in the *y*-direction, *D* is found to be [3]:

$$D = \frac{1}{2}\frac{F}{m}t^2, \qquad (4)$$

where *t* is the time from kicking (i.e. time spin was imparted to football). By considering the motion in the *x*-direction, the time can be expressed as [3]:

$$t = \frac{x}{v}, \qquad (5)$$

where *x* is the distance travelled in the *x*-direction. By substituting for *t*, using equation (5), and *F*, using equation (3), into equation (4), the distance is:

$$D = \frac{\pi R^3 \rho \omega}{vm} x^2. \qquad (6)$$

Therefore, a relationship between the distance travelled in the direction the ball was kicked, and the amount of deviation from this path is found.

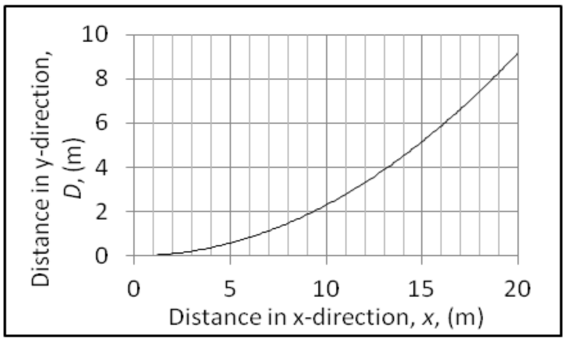

Figure 2. A plot of the distance travelled in the y direction, D, as a function of the distance travelled in the x direction, x, for a typical football.

By using equation (6), *D*, which represents the distance deviated, is plotted as a function of x. For an average football, the parameters can be approximated as 35ms^{-1} for the ball velocity [4], 0.11m ball radius [5], 0.42kg mass [5], and 10 rev/s angular velocity [6]. The density of air, ρ, is taken to be 1.3 kgm^{-3}. The resulting plot is shown in figure 2.

Discussion

It has been found that the amount a football bends depends linearly on the speed of the ball and the amount of spin. Therefore, the football player would need to kick the football with a certain speed and spin depending on the distance to the goal.

By reference to equation (6), the amount of bending also depends on the characteristics of the football. This analysis has assumed that the ball is completely spherical. However, footballs that are less spherical have turbulence effects, which reduce the bending of the ball.

Conclusion

This paper has determined the relationship between the angular velocity of a football, and how this affects its path. This analysis could be improved by accounting for drag forces on the football as it moves through the air, which would alter the trajectory.

Further investigations on this topic could include analysing the Magnus force in relation to other rotating objects, such as in ballistics and other ball sports.

References

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[6] http://www.wired.com/wiredscience/tag/air-resistance/page/2 (Accessed 14/11/2011).