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P1_5 Martian Muon decay

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Abstract

One of the first pieces of evidence for Einstein's theory of special relativity was the apparent detection of muons at the surface of earth. We consider the same effect but using a Martian atmosphere. We calculate that the time taken to traverse the Troposphere is $t_{actual} = 1.3 \times 10^{-5}$s after allowing for relativity. Lastly, we find that the number of muon's detectable is $\sim 0.26\%$.

Introduction

One physical phenomena that stems from Einstein's theory of special relativity [1] is the detection of muons at the surface of the earth. Einstein released his theory in 1905 that completely changed our views of what happens to objects at high speeds. His theorem was built on two postulates: the speed of light is the same for all observers and the laws of physics are invariant for any inertial reference frame. From this, two powerful modifications to classical dynamics arose: length contraction and time dilation. In this paper we show the Einsteinian effects of muon decay in the Martian atmosphere.

Special Relativity and cosmic rays

In relativity we are told that clocks moving at a high speed with respect to an inertial reference frame will appear to tick slower. Although this effect is insignificant at sub relativistic speeds, when an object travels at a significant fraction of the speed of light, we observe large time differentials. The time dilation equations is [1]:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{1}$$

where t' is the proper time of the clock and t is the reference frame of the muon. Note that $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is defined by the relativistic γ factor. Length contraction occurs under the same conditions for (1): the length of an object is reduced in its direction of motion when travelling at relativistic speeds:

$$L' = L\sqrt{1 - \frac{v^2}{c^2}} \tag{2}$$

where L is the frame of the moving object and L' is the inertial reference frame. In the muon's reference frame, the distance it traverses is shorter.

When cosmic rays enter the atmosphere they collide with the nuclei of an atom. This scatters to produce secondary muons (and pi mesons) which in turn, decay. Upon entrance, the muons emit Cherenkov radiation due to their initial velocity in the new medium being faster than the local speed of light.

Our assumptions are that the muons are extremely energetic (1 - 10GeV) and that 1 million are released instantaneously. The velocity of these muons is accepted to be $0.995c$ [1]. These short living particles have a mean decay time of

$\sim 2.2\mu$s [2]. We investigate the possibility of observing muons on Mars.

Calculations and discussion

The Martian atmosphere is very thin compared to the Earth, meaning that the likelihood that cosmic rays collide with atoms at high altitude is unlikely.

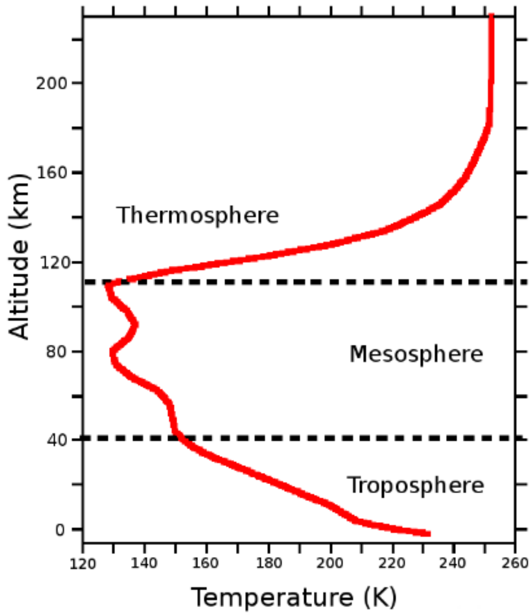

Figure 1: Martian Atmosphere as a function of temperature against height [3]

Figure 1 displays different sections of the atmosphere and demonstrates that in the Mesosphere, the temperature is on average the lowest. In the Troposphere the temperature increases with decreasing altitude which resembles the earth closely. Therefore, we take the height from the top of the Troposphere $L' = 40$km.

Now, we calculate the relativistic factor γ by using formula stated earlier: $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, using $v = 0.995c$. This yields a results of $\gamma \sim 10$. By subbing the values for γ and L' into equation (2) we find that the length travelled in the muon's reference frame is $L = 4$ km. Thus, the faster the muon is travelling, the higher the length contraction.

However, we must also account for time dilation. This is done by subbing the mean decay time into equation (1), resulting in $t = 22\mu$s. Therefore, as long as the muon decays after 22μs on its 4km path, it will be detectable.

By using the speed time relation, the time it takes a muon to traverse the 4km distance is $t_{actual} = 1.3 \times 10^{-5}$s. Therefore, since $t_{actual} < t$ the muon is detectable on the surface of Mars.

Lastly, we calculate the approximate number of muons that have reached the surface. By arbitrarily assuming that there are 1 million muons incident, we use the equation [4]:

$$N(t) = N_0(\frac{1}{2})^{\frac{t}{t_{1/2}}}, \tag{3}$$

where $N(t)$ is the number of muons left at time t ($t = 22\mu$s) N_0 is the initial number of muons and $t_{1/2}$ is the half life of a muon. Using $t_{1/2} = 1.56\mu$s [4], the recorded number of muons would be $N(t) \sim 57$.

Conclusion

We have shown that muons on Mars can be detected when one accounts for special relativity. If Newtonian mechanics had only been considered, we add that muon detection from the surface would be essentially impossible. Furthermore, we deduced that roughly 57 of the original 10^6 muons would be detected, providing the instrumentation is sensitive enough. We note that the sensitivity of the device and the inability to release 10^6 muons at once are our limitations, (assuming one could travel to Mars).

References

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