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A4_6 Honey I Shrunk the Blood Vessels

David Winkworth, Vlad Tudor, Elliott Shaw

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

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Abstract
The 1989 family film ‘Honey I Shrunk the Kids’ describes a number of children being reduced to ¼ inch high after an accident involving an ‘electromagnetic shrink ray’. This paper considers the effect this phenomenon would have on the circulatory system. By considering the blood vessels it is found that, in order to maintain adequate flow rate, the blood pressure drop throughout the body would need to increase by a factor of 200. The heart rate would also be required to increase by a factor of 200. It is not considered plausible that the body would survive this.

Introduction

In the 1989 hit film ‘Honey I Shrunk the Kids’, struggling inventor Professor Wayne Szalinski creates an ‘electromagnetic shrink ray’ which is accidentally used to reduce a group of children to ¼ of an inch high [1]. It is explained that this technique is possible based on the principle of reducing the ‘empty volume’ within atoms. This is ridiculous however, so it is hypothesised that the scientist is incompetent and instead the process simply reduces the number of atoms in the body, linearly. This may be further confirmed by the fact that their children’s mass does not remain the same (as displayed by their ability to climb plants etc. without them collapsing).

This paper does not examine the validity of this theory, but instead considers the effect this process might have on the human heart and circulatory system.

Discussion

By shrinking the children to ¼ inch (0.7cm), Szalinski’s shrink ray has reduced them by a factor of approximately 200. This means that their blood vessels will also be reduced by a factor of 200. It is known via the Hagen-Poiseuille equation [2], however, that flow rate is not linearly proportional to a vessel’s radius. For a viscous and incompressible fluid with dynamic viscosity η flowing through a pipe of radius R, the flow rate Q is given by

$$Q = \frac{\pi R^4}{8\eta}\frac{|\Delta P|}{L} \qquad (1)$$

where ΔP is the pressure drop along the length of pipe L. This means that a reduction of the vessels radius by a factor of x, along with a reduction in the length by factor x, will result in a reduction in flow rate (for constant pressure and viscosity) of

$$Q \propto \left(\frac{R}{x}\right)^4 \left(\frac{x}{L}\right) = \frac{R^4}{Lx^3}.$$

When x is taken to be 200, the flow rate is reduced by a staggering factor of $8\text{x}10^6$!

Assuming that the reduced body will require $\frac{1}{x}th$ of the original oxygen, it can also then be deduced that the change in blood pressure will be required to significantly increase. This increase can be found by considering

$$\frac{Q}{x} \propto \frac{R^4|\Delta P|}{Lx^3}$$

before rearranging for pressure drop and only considering the variables

$$\mapsto \qquad |\Delta P| \propto x^2. \qquad (3)$$

In order to ensure that the necessary flow rate is maintained following the shrinking, the pressure drop must increase by $4x10^4$.

This approximation can, however, be improved upon by considering the variation in mass and how this relates to oxygen requirement. As mass is a function of the cube of length, reducing the children's height by a factor of x will reduce both their volume and mass by a factor x^3. The basal metabolic rate (BMR) in Watts of a mammal of mass *M* is given by [3]

$$BMR = AM^{2/3}\ , \qquad (4)$$

where *A* is a constant of proportionality. A reduction in the bodies mass by a factor x^3 will then result in a reduction in BMR of x^2. The BMR is directly proportional to the amount of oxygen required.

Inserting this more accurate variation in oxygen into equation 3, as shown in

$$\frac{Q}{x^2} \propto \frac{R^4|\Delta P|}{Lx^3},$$

displays the necessary pressure drop to then be

$$|\Delta P| \propto x. \qquad (5)$$

This may appear to be a surprising result, considering the initial relationship between flow volume and radius shown by equation 1. Equation 5 indicates that the pressure drop throughout the body would need to increase by a factor of 200.

To achieve the correct flow rate whilst the heart's volume *V* is reduced by a factor of x^3, the heart rate *N* must increase to *N'*. The relationship between these is found by considering the initial heart rate

$$N = \frac{Q}{V}\ ,$$

before modifying to include the reduced size

$$N' = \frac{Q^2}{x^2}\frac{x^3}{V} = Nx. \qquad (6)$$

For an estimated initial heart rate of 60bpm, equation 6 indicates that the heart rate would then increase by a factor of 200 and resulting in a rate of approximately 1200bpm.

The validity of this result can be tested by comparing a smaller mammal to a human. A rat can be considered approximately 10 times smaller than a person, which indicates a heart rate which is 10 times greater. Considering a human's heart rate of 60bpm, we expect that a rat might have a heart rate of 600bpm. This is certainly of the same order as accepted values which are in the region of 250-500bpm [4].

Conclusion

In conclusion, it has been determined that upon reducing the size of the body by such an extreme factor as Szalinski's shrink ray, a number of unexpected consequences are experienced by the cardiovascular system. Both the blood pressure drop and the heart rate would be required to increase by a factor of 200. It is considered unlikely that the body could survive such an extreme phenomena.

References

[1] *http://www.imdb.com/title/tt0097523/*, accessed 5/12/2012

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[3] C. R. White, "*Mammalian basal metabolic rate is proportional to body mass $^{2/3}$*", Proceedings of the National Academy of Sciences, 2003.

[4] *http://ratguide.com/health/basics/advanced_health_check.php* accessed on 20/11/2012