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A1_6 Do you want to hang out?

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Abstract

This paper addresses the simple question: how much time will it take for wet clothes to dry, if they are hung up on a washing line, based on the current weather conditions? The metric form of the Penman equation is used to calculate the evaporation rate of water from a plane surface, which is dependent on the meteorological conditions of ambient temperature, wind speed, relative humidity, and the various properties of air and water at said ambient temperature. Empirically measuring the surface area of, and the mass of water contained in, a wet piece of clothing enables the time taken t for total evaporation to be determined. This paper concludes with an equation for t, provided the above factors are known, and gives example results for different items of clothing at a certain date and location.

Introduction

During the winter months, when we are not blessed with a sunny climate, drying clothes naturally becomes a chore. With inclement weather, cold temperatures, and fewer daylight hours, it is difficult to work out whether hanging clothes outside in the morning will dry before the evening. This paper addresses that question, defining an expression for the time taken for a simple piece of clothing to dry as a function of weather conditions (easily available from the internet), location, and of certain measurable properties of the piece of clothing.

Discussion

Dalton's law gives the evaporation rate in unit time E as

$$E = (e_a - e_d)f(u), \qquad (1)$$

where e_a is the saturated vapour pressure at the evaporation surface (this is known for water if the surface temperature is known), e_d is the vapour pressure in the atmosphere above, and $f(u)$ is a function of the horizontal wind velocity [1]. Penman expands on this with an equation for the evaporation rate as a function of the air conditions only; and Shuttleworth re-writes this equation in metric units as

$$E = \frac{\Delta A'\lambda + \gamma[6.43(1 + 0.536\,u)D]}{\lambda(\Delta + \gamma)} \qquad (2)$$

(in $\mathrm{mm\ day^{-1}}$), where $\Delta = \mathrm{d}e_a/\mathrm{d}T_a$ is the rate of change of saturated vapour pressure (in $\mathrm{kPa\ K^{-1}}$) at air temperature; $\lambda = [2.501 - 0.002361(T_s - 273)]\ \mathrm{MJ\ kg^{-1}}$ is the latent heat of vaporization in water (T_s is the surface temperature in Kelvin); A' is the estimated energy available for evaporation from the free water surface (in $\mathrm{mm\ day^{-1}}$); u is the wind speed (in $\mathrm{m\ s^{-1}}$); D is the vapour pressure deficit (in kPa) measured at 2 m; and $\gamma = 0.0016286\,P/\lambda\ \mathrm{kPa\ K^{-1}}$, where P is the atmospheric pressure (in kPa) [2].

The product $R_n = A'\lambda$ is the *net irradiance* of sunlight incident on the surface (in $\mathrm{MJ\ m^{-2}\ day^{-1}}$) [3]. The vapour pressure deficit D is given by [1, 2]

$$D = e_a - e_d = (1 - h)e_a, \qquad (3)$$

where $h = e_d/e_a$ is the *relative humidity* (a measurable quantity). We can make an approximation for e_a, the saturated vapour pressure, as a function of air temperature T_a [4]:

$$e_a = \exp\left(21.07 - \frac{5336}{T_a}\right), \tag{4}$$

and therefore the rate of change of saturated vapour pressure $(\mathrm{mmHg\,K^{-1}})$ at air temperature is [4]

$$\begin{aligned}\Delta &= \frac{\mathrm{d}e_a}{\mathrm{d}T_a} \\ &= \frac{5336}{T_a^2}\exp\left(21.07 - \frac{5336}{T_a}\right).\end{aligned} \tag{5}$$

To get Δ in the required units, we use the fact that $1\ \mathrm{mmHg} = 0.1333\ \mathrm{kPa}$.

If we make the assumptions that the water in a piece of clothing quickly reaches thermal equilibrium with the air, then $T_s = T_a = T$; the net irradiance R_n is a function of latitude and time of year [5]; and that the atmospheric pressure $P = 101.3\ \mathrm{kPa}$; then Eq. (2) shows that the evaporation rate E is clearly a function of T, h, u, latitude, and time of year only.

An item of clothing of total surface area A gains an extra mass m of water when washed, and so the amount of time t taken for the item to dry (for all of the water to evaporate) is given by

$$t = \frac{m}{\rho E A}, \tag{6}$$

where $\rho = 1{,}000\ \mathrm{kg\,m^{-3}}$ is the density of water, needed to get E in terms of the mass of water per unit area per unit time. Eq. (6) uses SI units of $\mathrm{m\,s^{-1}}$ for E.

Application

As an example, if we take measurements of $T = 286\ \mathrm{K}$, $h = 0.87$, $u = 0\ \mathrm{m\,s^{-1}}$, and $R_n \cong 117\ \mathrm{W\,m^{-2}} = 10.08\ \mathrm{MJ\,m^{-2}\,day^{-1}}$ (at 13:00 10/11/2011 in Leicester, UK)[1], we find that $E = 3.957\ \mathrm{mm\,day^{-1}}$ and $\rho E = 4.58 \times 10^{-5}\ \mathrm{kg\,m^{-2}\,s^{-1}}$. A t-shirt of surface area $A = 0.5\ \mathrm{m^2}$ gains an extra mass $m = 0.075\ \mathrm{kg}$ of water when washed (measured by the author, 09/11/2011), and so using Eq. (6) $t = 3{,}280\ \mathrm{s} \cong 55\ \mathrm{min}$. Under the same weather conditions, a towel of surface area $A = 1.44\ \mathrm{m^2}$ gains an extra mass $m = 0.25\ \mathrm{kg}$ of water when washed (again measured by the author, 10/11/2011), and so $t = 3{,}790\ \mathrm{s} = 1.05\ \mathrm{hours}$.

Conclusions

An equation for the estimated time taken for an item of clothing to evaporate has been found. However, it is clear from the two results discussed, for a t-shirt and a towel, that there are missing factors from this equation, as you would expect the evaporation rate to depend on the thickness of the fabric – it is quite commonly known that bath towels take significantly longer to dry than t-shirts. This paper has modelled evaporation from a plane surface (in fact Penman's equation is for evaporation from an open water source) and so does not take into account the availability of water molecules on the surface of the clothing. Also, the wind speed has a very large effect on E, and as it may vary substantially within the timescales for drying discussed, errors may creep in. If clothes are being dried in a room or an enclosed courtyard or garden, then it may be valid to assume the localized wind speed is close to zero.

References

[1] *Natural evaporation from open water, bare soil and grass.* Penman, H. L. 1948. Proc. Roy. Soc. London. pp. 120-145.
[2] Shuttleworth, W. J. *Putting the 'vap' into evaporation.* Dept. of Hydrology and Water Resources, University of Arizona. Tucson : SAHRA, 2007. p. 211.
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[4] Merva, G. E. *Physioengineering principles.* California : Avi Pub. Co., 1975. ISBN 0870551817.
[5] ITACA. [Online] 18 November 2005. [Cited: 13 November 2011.] http://itacanet.org/eng/elec/solar/sun2.pdf.

[1] Calculation in [5] shows that the solar energy available at the Earth's surface is dependent on latitude and the month of the year. The value of the mean solar radiation for November, $0.7\ \mathrm{kWh\,m^{-2}}$, is divided by the number of sunlight hours in the more northerly parts of the northern hemisphere (6) to give R_n in $\mathrm{W\,m^{-2}}$.