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P5_8 Listening to Metal

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Abstract

We considered whether constructing an unknown metal into a tuning fork of known dimensions and identifying its fundamental frequency is accurate enough to find what the metal is. Metals have specific modulus with differences of$\pm$ 10^5 m^2 s^{-2}, and the tuning for gives the specific modulus to an accuracy of $\pm$ 8X10^4 m^2 s^{-2} so metals can be told apart.

Introduction

The frequency of a tuning fork depends on its dimensions and the material the tuning fork is made from. Each type of metal would therefore produce its own unique sound which could be used to identify it, which is the reason why different materials are used to make guitar strings [4]. We consider how accurately we can determine the frequency of a tuning fork made of an unknown metal with known dimensions and how this can be used to determine the material.

Theory and Results

A tuning fork is an acoustic resonator used to produce a fixed tone. Unlike other types of resonators, such as a plucked wire, the tuning fork has most of its vibrational energy at the fundamental frequency, with little energy going to the overtone frequencies. The tuning fork's overtones die out fast, leaving a pure sine wave at the fundamental frequency. By comparison, the first overtone of a vibrating wire is twice the fundamental, so when the wire is plucked its vibrations tend to mix the fundamental and overtone frequencies.

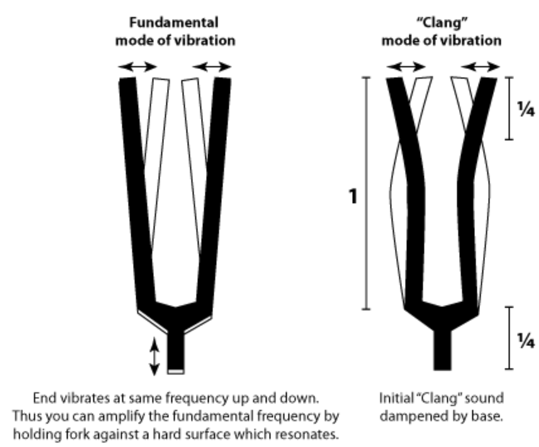

Figure 1: A tuning fork's vibrational modes [2].

the frequency produced by a tuning fork with cylindrical prongs of known material and dimensions can be found from the following:

$$f = \frac{1.875^2 r}{4\pi\ell^2}\sqrt{\frac{E}{\rho}} \tag{1}$$

Where f is the frequency the fork vibrates, 1.875 is the smallest positive solution of $\cos(\mathrm{x})\cosh(\mathrm{x}) = 1$ [3], ℓ is the length of the prongs in m, E is the Young's modulus of for material, ρ is the density of the fork material, and r is the radius of the circular cross-section of the cylindrical prongs. Re-arranging equation 1 for the ratio $\frac{E}{\rho}$, we find:

$$\frac{E}{\rho} = \frac{16\pi^2\ell^4}{1.875^4 r^2} f^2 \tag{2}$$

This ratio is known as the specific modulus, and can be seen in the graph below:

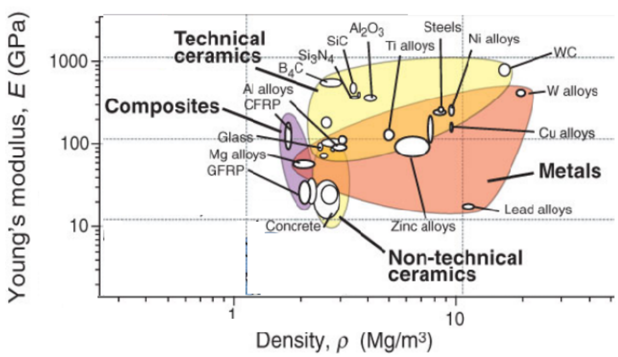

Figure 2: The density of different materials against its Young's modulus [1].

We will take our known dimensions arbitrarily: $\ell = 0.15 \pm 5\text{x}10^{-5}$ m, and $r = 0.005 \pm 5\text{x}10^{-5}$ m, where the error is that from a standard caliper in the construction of the fork [7]. Assuming the frequency measurements are taken at 20 °C temperature, it can be found that the error in the frequency will be about 1/10000 th of the measurement for an average frequency counter [6]. We can Propagate these errors using the following error propagation equations:

$$\text{If: } Z = X^n \text{ then: } \frac{\Delta Z}{Z} = n\frac{\Delta X}{X} \tag{3}$$

$$\text{If: } Z = X/Y \text{ or } Z = XY$$
$$\text{then: } \frac{\Delta Z}{Z} = \sqrt{(\frac{\Delta X}{X})^2 + (\frac{\Delta Y}{Y})^2} \tag{4}$$

Equation 3 can be used to propagate the known errors to the powers that are on the variables in the equation. Equation 4 is then used to combine these errors propagated to the powers to find the error of the specific modulus over a given specific modulus to be $8.0069\text{x}10^{-3}$. the specific modulus for a given metal will be of the magnitude 10^7 m^2 s^{-2} [5], giving the error to the order of $\pm$ 8X10^4 m^2 s^{-2}. This is small enough that metals can be easily differentiated, as they are roughly $\pm$ 10^5 m^2 s^{-2} difference between each metal [5].

Discussion

At different temperatures the tuning forks will have slightly different values of frequency, as the material will have a different specific modulus for different temperatures, so there will be a deviation of 86 ppm per °C from the standard temperature of 20°C.

Conclusion

We found by making an unknown metal into a tuning fork of known dimensions we could find what type of metal it was from the frequency of the tuning fork.

References

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