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P5_1 "Everybody knows the Moon is made of cheese...": Return of the Cheddar

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Abstract

In this article we explore the repercussions of the Moon turning into cheddar, and find that when conserving angular momentum, with the same volume and lighter mass of $m = 2.49 \times 10^{22}$kg, it would escape the Earth's sphere of influence. We looked at two possible escape trajectories, prograde and retrograde, and found the new orbital distances to be between 0.73AU and 1.00AU, and between 1.00AU and 1.51AU, respectively. Thus potentially carrying the Moon very near to the orbits of Venus or Mars.

Introduction

This article focuses on the physical repercussions that would arise should the Moon suddenly be transformed into cheddar. We assumed that the volume and momentum of the Moon remained constant, with only the mass of the Moon changing, due to the shift in material density.

A similar approach was explored in a recent paper (*East et al*, 2013)[1] where they considered a Moon composed of varying cheeses, while keeping either the mass or volume constant. Their focus was primarily aesthetic, with some comments on the affect on the tidal forces under a fixed mass. We have undertaken a more mathematical approach to the orbital components of the Moon upon transformation to cheddar, and further explore the potential escape of the moon from Earth's sphere of influence.

Will The Moon Escape?

The Moon's current mass and orbital velocity are 7.35×10^{22}kg and $1,022$ms^{-1}, respectively. If the Moon transformed into cheddar, and maintained its volume, $V = 21.9 \times 10^{12}$m^3, its new mass would be $m = 2.49 \times 10^{22}$kg. This was found using a measured density of Waitrose Essentials Cheddar, 1134kgm^{-3} and the equation $\rho = m/V$. Using the conservation of momentum, $m_1 v_1 = m_2 v_2$, a new Lunar orbital velocity was calculated to be $v_{moon} = 3,018 ms^{-1}$.

By equating the centripetal force with the gravitational force, we have formed Equation (1). This gives the orbital distance, r, in terms of orbital velocity, v, and the mass of Earth, M.

$$\frac{GM}{v^2} = r \qquad (1)$$

Where G is the gravitational constant, 6.67×10^{-11}m^{3}kg^{-1}s^{-2}. We then calculated the escape velocity, v_{esc}, from the Moon's position in the Earth's gravitational field using Equation (2).

$$v_{esc} = \sqrt{\frac{2GM}{r}} \qquad (2)$$

As the escape velocity is $1,440$ms^{-1}, we can see that the new Moon's velocity far exceeds the escape velocity. Hence the Moon shall escape the Earth's sphere of influence.

This now introduces a variability in the orbit dependant on the point of escape. We shall now introduce two motion concepts: prograde, a manoeuvre along the positive velocity vector, causing an increase in velocity; and retrograde, a manoeuvre along the negative velocity vector, causing a decrease in the velocity.

Where Has The Moon Gone?

A prograde situation assumes the transformation occurs at a point in the orbit where the velocity vector at escape is in a prograde direction. The hyperbolic excess velocity, v_∞ - the velocity at the point at which a body escapes the gravitational pull of the Earth - was found using Equation (3)[2] to be $2,653$ms^{-1}.

$$\sqrt{v_{esc}^2 + v_\infty^2} = v_{moon} \tag{3}$$

As this scenario is prograde, we added the hyperbolic excess velocity to the orbital velocity of the Earth, $30,000$ms^{-1}, to get the velocity at periapsis, $v_p = 32,653$ms^{-1}.

$$\frac{v^2}{2} - \frac{GM}{r} = -\frac{GM}{2a} \tag{4}$$

We then used a rearranged form of Equation (4)[2], the 'vis-viva equation', to work out the semi-major axis, a in Equation (5), of the new Lunar orbit.

$$a = \left(\frac{2}{r} - \frac{v^2}{GM}\right)^{-1} \tag{5}$$

Using $a = 1.25$AU, and the periapsis, we calculated the apoapsis of the orbit.

$$a = \frac{1}{2}(r_p + r_a) \tag{6}$$

By rearranging Equation (6)[2] for r_a, the radius of the apoapsis, we calculated the value to be 1.51AU.

Here we investigate what happens if the Moon escapes on a retrograde trajectory. As worked out previously, the hyperbolic excess velocity is $2,653$ms^{-1}, which was subtracted from the Earth's orbital velocity to get a velocity of $27,347$ms^{-1}.

Using Equation (5) we calculated the semi-major axis to be 0.86AU. We again used Equation (6), but set the current Lunar location as the apoapsis of the new orbit under a retrograde burn, and found the new perihelion of the Moon to be 0.73AU.

Conclusion

We conclude that the cheddar Moon will escape the Earth's sphere of influence dramatically and begin an orbit around the Sun. In a prograde escape trajectory, the Moon will orbit with a perihelion and aphelion of 1.00AU and 1.51AU, respectively. For a retrograde escape trajectory the perihelion and aphelion would be at 0.73AU and 1AU, respectively. For note, Venus orbits at 0.72AU and Mars lies at 1.52AU, so it is possible that the Moon could come into very close proximity to either planet. As touched upon in *East et al*, 2013, the affects such a dramatic change on the moon would have on the Earth's tides would likely be catastrophic. As such, we firmly recommend the moon remains a rocky body for the time being.

References

[1] East, O., Longstaff, E., Li, C. and Fletcher, M. *P1_1 "Everybody knows the Moon is made of cheese..."*, PST 12, (2013).

[2] Vallado, D., (2007). *Fundamentals of Astrodynamics and Applications.* New York, New York: Springer