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S2_1 Getting comfortable on Neptune

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Abstract

The effects of long term exposure to high gravity on human health are not well understood but will likely prove to be problematic. Therefore, if humans wish to colonise planets with significantly higher gravity than Earth then it may be necessary to engineer solutions to artificially reduce the effects of gravity. This paper assesses the admittedly elaborate idea to use the centrifugal and Coriolis effects by increasing a planet's rotation to artificially reduce the effects of gravity. We decided to use Neptune as an example but the concepts and maths used in this paper should be applicable to any planet. We worked out that we would need to speed up Neptune's rotation by 6.12×10^{-5} rad s^{-1} to simulate a downwards acceleration of 9ms^{-2} at the equator and that this would require an impulse of 6.2×10^{28} Ns.

Introduction

The net force that an object experiences on the surface of a planet is not only due to gravity. Other sources such as the centrifugal and Coriolis effects arise from the planet's rotation and act away from, and perpendicular to, the axis of rotation. This paper aims to calculate how fast a planet with higher gravity than Earth would have to rotate for an object at the equator to experience a net downwards force similar to the gravitational force on Earth. We then proceeded to calculate the impulse that would be required to be applied at the planet's equator to speed up the planet's rotation to achieve this goal.

Initial approximations

As there are no terrestrial planets in our solar system larger than Earth, we chose Neptune as an example planet but the concept of this paper could be applied to any planet. We also chose 9 ms^{-2} as our desired value for the effective downwards acceleration at the equator. This is because the centrifugal and Coriolis effects decrease with distance from the equator so it would be practical to aim for a value slightly lower than the gravity on Earth. We also used the equatorial radius of Neptune as this would be the most appropriate.[1]

Method

The equation for the net effective force ($\boldsymbol{F_{eff}}$) experienced by an object of mass m on the surface of a planet rotating with angular frequency $\boldsymbol{\omega}$ is [2];

$$\boldsymbol{F_{eff}} = \boldsymbol{F_g} - m\,\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) - 2m\boldsymbol{\omega}\times\boldsymbol{v}, \quad (1)$$

where $\boldsymbol{r}$ is the distance from the axis of rotation to the surface, v is the velocity of a point on the surface and $\boldsymbol{F_g}$ is the force of due to gravity. There are two possible extra terms on the right hand side but these depend on angular acceleration and an accelerating radius, both of which are not applicable in this scenario and are therefore ignored. To generalise the equation for any object equation (1) was divided by m to get the effective acceleration ($\boldsymbol{a_{eff}}$);

$$\boldsymbol{a_{eff}} = \boldsymbol{g} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) - 2\boldsymbol{\omega} \times \boldsymbol{v}, \qquad (2)$$

where g is the acceleration due to gravity and approximately 11.14ms^{-2}. The second term on the right hand side of equation (2) can be simplified using the vector identity;

$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) = (\boldsymbol{\omega} \cdot \boldsymbol{r})\boldsymbol{\omega} - (\boldsymbol{\omega} \cdot \boldsymbol{\omega})\boldsymbol{r} = \boldsymbol{\omega}^2\boldsymbol{r}, \quad (3)$$

where $(\boldsymbol{\omega} \cdot \boldsymbol{r})\boldsymbol{\omega} = 0$ as $\boldsymbol{\omega}$ and $\boldsymbol{r}$ are always perpendicular and $(\boldsymbol{\omega} \cdot \boldsymbol{\omega})\boldsymbol{r} = \boldsymbol{\omega}^2\boldsymbol{r}$. The third term on the right-hand side of equation (2) can be simplified to $2\boldsymbol{\omega}^2\boldsymbol{r}$ as $\boldsymbol{v}$ and $\boldsymbol{\omega}$ are perpendicular and $\boldsymbol{v} = \boldsymbol{\omega}\boldsymbol{r}$. This allows equation (2) to be rearranged and simplified to:

$$\boldsymbol{g} - \boldsymbol{a_{eff}} = 3\boldsymbol{\omega}^2\boldsymbol{r}, \qquad (4)$$

which can be rearranged to make ω the subject;

$$\boldsymbol{\omega} = \sqrt{\frac{\boldsymbol{g} - \boldsymbol{a_{eff}}}{3\boldsymbol{r}}}. \qquad (5)$$

As the planet is already rotating, the change in $\boldsymbol{\omega}$ was calculated;

$$\Delta\boldsymbol{\omega} = \boldsymbol{\omega} - \boldsymbol{\omega}' = 6.12 \times 10^{-5}\text{rad s}^{-1}, \qquad (6)$$

where ($\boldsymbol{\omega}'$) is the initial angular velocity of Neptune at the equator calculated from the period of rotation (T) $\boldsymbol{\omega}' = 2\pi\frac{1}{T}$ [1]. Finally, the impulse required to achieve this change in angular velocity was calculated;

$$\boldsymbol{F} = \frac{\boldsymbol{\tau}}{\boldsymbol{r}} = \frac{1}{\boldsymbol{r}} I \frac{d\boldsymbol{\omega}}{dt} = \frac{1}{\boldsymbol{r}} \frac{d\boldsymbol{L}}{dt} = \frac{1}{\boldsymbol{r}} \frac{2M\boldsymbol{r}^2}{5} \frac{d\boldsymbol{\omega}}{dt}, \qquad (7)$$

where I is the moment of inertia and equal to $\frac{2M\boldsymbol{r}^2}{5}$ for a sphere. $\boldsymbol{L}$ is the angular momentum and $\boldsymbol{\tau}$ is torque. Therefore, the impulse required is:

$$Impulse = \boldsymbol{F}\Delta t = \frac{2M\boldsymbol{r}}{5}\Delta\boldsymbol{\omega} = 6.2 \times 10^{28} \text{ Ns}. \qquad (8)$$

Conclusion

It was calculated that an impulse of 6.2×10^{28} Ns would need to be applied at the equator of Neptune to speed up the planets rotation enough to simulate the effects of Earth's gravity at the equator. While this idea is clearly eccentric and would have enormously hazardous effects on the atmosphere and climate of the planet in question, the fact remains that there are few if any viable solutions for reducing the effects of gravity. Therefore, if humans in the future wish to colonise planets with a higher gravitational force than the Earth, it is not completely unthinkable that the ideas presented in this article may be implemented. Further possible research could be to calculate the portion of a planets surface within a range of 'desirable' effective downwards forces as the centrifugal and Coriolis effects will decrease with distance from the equator.

References

[1] Hamilton, Calvin J. (2001). "Neptune". Views of the Solar System. Available at http://solarviews.com/eng/neptune.htm

[2] Thornton, Stephen T. Marion, Jerry B. (2003), "Classical Dynamics of particles and systems" 5th Edition, Pg-392. (Accessed: 27 September 2017).