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P4_3 Shrinking the Earth

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Abstract
This article investigates the energy requirements to compress the Earth to within 1 km. This was done using a model assuming the compression to be elastic, and another model estimating the energy to overcome the Coulomb repulsion between the constituent protons. The energies found were 3.1×10^{32} J and 8.3×10^{38} J for each model respectively. It was also shown that to compress the Earth to within 1125 km you would have to overcome electron and neutron degeneracy pressure, and if compressed to a radius of 9 mm, the Earth would be a black hole.

Introduction

This paper will consider a thought experiment of the energy requirements of shrinking the Earth to within 1 km. We will consider two models: one where the compression of the Earth is assumed to be elastic, and the second considering the energy requirements to overcome Coulomb repulsion.

Elastic model

Let us assume the Earth to be a perfect solid sphere and to be made entirely out of iron; the crust, mantle and core are composed mostly of iron [1]. Iron has a Young's modulus of about 190 GN/m^2 [2]. We shall assume this to be the pressure to compress the Earth and that it is applied equally over the whole surface. Hence the force, F, is given by

$$F = PA = 4\pi P R_E^2, \tag{1}$$

where P is the pressure, and A is the surface area, which in this case is for a sphere, giving $A = 4\pi R_E^2$ with R_E the radius of the Earth.

This force is applied equally over the whole surface area of the Earth over a distance R_E. The Earth is being modelled as elastic with the elastic potential energy, U, given by

$$U = \frac{1}{2} F x, \tag{2}$$

where x represents the distance over which the object is being stretched or compressed. In this case x is equivalent to R_E. Therefore, substituting for F from (1) and $x = R_E$ into (2) gives

$$U = 2\pi P R_E^3. \tag{3}$$

Taking $P = 190$ GN/m^2 and R_E as 6371 km, using equation (3) we find an estimate of the energy for compressing the Earth to be 3.1×10^{32} J.

As one would expect, the energy requirements are very large. It should be noted that R_E has been given to within an accuracy of 1 km, hence the energy required to shrink the Earth any less than this would be higher. In this calculation it was assumed that the collapse of the Earth would be symmetrical.

Coulomb repulsion model

The electrostatic potential energy, U, between two point charges, q_1 and q_2, with separation, r, is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}, \tag{4}$$

where ϵ_0 is the permittivity of free space. Consider two protons of charge e. If we wish to shrink the Earth then some of the constituent protons in the atoms will be brought closer together. We estimate this distance of the order of 1 fm or 10^{-15} m since the strong force only operates at this distance or smaller. Hence using the usual values for the constants e and ϵ_0, and taking $r = 10^{-15}$ m gives the energy, U, as 2.3×10^{-13} J. The number of constituent protons and neutrons in the Earth, N, can be estimated by

$$N = \frac{M_E}{m_p}, \tag{5}$$

where M_E is the mass of the Earth and m_p is the mass of a proton which is approximately the same as that of a neutron. Taking M_E as 5.97×10^{24} kg and m_p as 1.67×10^{-27} kg, N is 3.6×10^{51}. This calculation assumes an approximately equal number of protons and neutrons in the Earth. Hence,

$$U = 3.6 \times 10^{51} \times 2.3 \times 10^{-13} = 8.3 \times 10^{38} \text{ J}.$$

That is, the energy requirements to overcome the Coulomb repulsion is about 8.3×10^{38} J. This about six orders of magnitude larger than the previous estimate.
Of course, each proton will have an associated electron for the Earth to be charge neutral, but this extra mass has been ignored since the mass of the electron is 1/2000 that of the proton. For a more reliable estimate you would need to know what fraction of the Earth's mass is in each constituent material.

Earth as a star
The energy requirements would probably have to be larger due to the electron degeneracy pressure that would act to prevent the Earth from collapsing due to gravity. When trying to collapse a massive object, gravity will be one of the dominant forces. In the case of the Earth, this would mean electrons would be forced to occupy higher energy states and attain faster velocities. Electron degeneracy occurs at densities of about 10^6 kg/m^3 [3]. Therefore, for the Earth, electron degeneracy pressure would start at a volume of 5.97×10^{18} m^3, which means a radius of about 1125 km. If the massive body has a mass less than 1.44 solar masses then electron degeneracy pressure would be enough to support the gravitationsl collapse [4]. The mass of the Earth is much less than a solar mass so it would take a very large amount of energy to overcome electron degeneracy pressure.
In fact if it were possible to crush the Earth to such a significantly small size, by also overcoming neutron degeneracy, it would have sufficient density to form the astronomical object known as a black hole. The equation for the Schwarzchild radius, R, of a black hole is

$$R = \frac{2GM}{c^2}, \tag{6}$$

where G is the gravitational constant, M is the mass of the black hole, and c is the speed of light. We can use this equation to find the size of a black hole if it were to have the mass of the Earth. Using $M = 5.97 \times 10^{24}$ kg, and the other constants having their usual values, R is then about 9 mm. This is about the size of a marble, which demonstrates just how significantly dense black holes are. Therefore, returning to our investigation, if we were to shrink the Earth to about 9 mm, it would take at least between 10^{32} J to 10^{38} J of energy to do this based on our models.

Conclusion
In summary, two different energy requirements to shrink the Earth have been found, based on separate assumptions. One model treated the compression of the Earth as elastic, and the other dealt with overcoming the Coulomb repulsion between the protons when they are compressed. It was found the elastic model gave an energy of 3.1×10^{32} J, and the Coulomb repulsion model about 8.3×10^{38} J. It was then deduced that if the Earth were to be compressed to a radius of 9 mm it would form a black hole. However, to get the Earth to this size would require an enormous amount of energy to overcome electron and neutron degeneracy pressures. Our calculations neglected the rotation of the Earth and therefore the centripetal force.

References

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