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P2_1 I Need a Day Between Saturday and Sunday

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Abstract

We investigate the changes that would need to be applied to the Earth's orbit to create a three-day weekend. We have approached this by extending the distance between the Earth and the Sun to $r = 1.63 \times 10^{11}$ m, creating a longer year with more days whilst keeping the length of days the same. We then calculated the orbital radius of the Moon at the time of an 8-day week, 767 million years ago where the days were around 21 hours long. This radius was 2.90×10^{7} m closer to the Earth than present day.

Introduction

Variations in the length of days happens frequently, although not enough to be noticeable - a recent change to a current in the southern ocean caused the orbit of the Earth around its axis to speed up due to the mass distribution of the ocean changing [1]. By examining the planetary motion, we can investigate how extreme alterations to the orbit of the Earth could add an extra day to our weekends. First we look at changing the radius of the Earth's orbit resulting in a year with extra days of the same length, before considering the radius of the Moon's orbit at a time when the Earth's days were shorter and a week contained 8 days.

Earth's Solar Orbit

We can describe the orbit of the Earth around the Sun, as seen in Figure (1), using Equation (1), Kepler's 3rd law:

$$T^2 = \frac{4\pi^2}{GM_s} r^3, \quad (1)$$

where T is the orbital period of Earth, r is the radius of that orbit, M_s is the mass of the Sun and G is the gravitational constant. By rearranging for r, as seen in Equation (2) we can find the new radius from the Sun at which the Earth would need to orbit to allow an extra 52 rotations about the axis of the Earth:

$$r = \left(T^2 \frac{GM_s}{4\pi^2} \right)^{1/3}. \quad (2)$$

Using T as the number of seconds in 417 days, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2} and $M_s = 1.99 \times 10^{30}$ kg, we find that $r = 1.63 \times 10^{11}$ m. This is an increase of 1.38×10^{10} m from the current orbital radius.

The Moon's Previous Orbit

In the past, the Moon had a smaller orbital radius around the Earth than currently observed today. This caused the Earth to rotate with a higher velocity around its axis following the principle of the conservation of angular momentum. This created shorter-houred days while maintaining the length of a year. We therefore decided to calculate this rotational velocity and hence find the Moon's previous orbital radius.

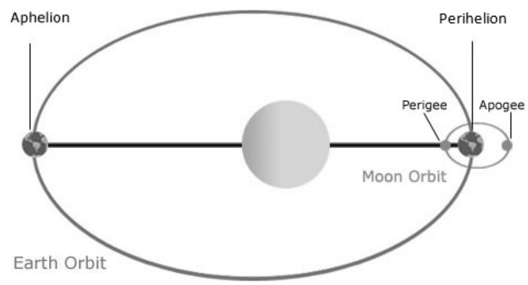

Figure 1: The Sun-Earth-Moon System [3].

As the Moon is currently retreating from the Earth at a constant rate, the number of years between the 8-day week and today's 7-day week could be calculated.

Firstly, using Equation (3), we could calculate the rotational velocity of the Earth in the past when the Moon was in a closer orbit:

$$\omega_{e,prev} = 2\pi/T, \quad (3)$$

where $T = 75400$ s the number of seconds in a day (roughly 21 hrs in a day in an 8-day week), therefore $\omega_{e,prev} = 8.33 \times 10^{-5}$ s^{-1}. From there, we could equate the angular momentum at present day to the angular momentum at the smaller Moon orbital radius, shown in Equation (4) and (5), in order to calculate the orbital radius of the Moon in the past:

$$L_{now} = \frac{2}{5} M_e R_e^2 \omega_{enow} + r_{mnow}^2 M_m \omega_{mnow}, \quad (4)$$

$$L_{prev} = \frac{2}{5} M_e R_e^2 \omega_{eprev} + r_{mprev}^2 M_m \omega_{mprev}, \quad (5)$$

where L_{now} and L_{prev} are the angular momentum values at present day and in the past respectively, M_e is the mass of the Earth, R_e is the radius of the Earth, ω_{enow} is the current rotational velocity of the Earth, r_{mnow} is the current radius of orbit of the Moon, M_m is the mass of the Moon, ω_{mnow} is the current orbital velocity of the Moon, ω_{eprev} is the past rotational velocity of the Earth, r_{mprev} is the past radius of orbit of the Moon and ω_{mprev} is the past orbital velocity of the Moon. These constants are given as: $M_e = 6.00 \times 10^{24}$ kg, $R_e = 6.38 \times 10^6$ m, $\omega_{enow} = 7.29 \times 10^{-5}$ s^{-1}, $r_{mnow} = 3.85 \times 10^8$ m, $M_m = 7.35 \times 10^{22}$ kg, and $\omega_{mnow} = 2.64 \times 10^{-6}$ s^{-1}. The values of ω_{mprev} and r_{mprev} were calculated using Equation (1), where T was substituted for $T = 2\pi/\omega$ where ω is the orbital velocity of the Moon in the past. Rearranging Equation (1) for r and using the required values gave $\omega_{mprev} = \sqrt{GM_e/r^3}$. This was then substituted into Equation (5) and all the values calculated to find $r_{mprev} = 3.56 \times 10^8$ m. This is 2.90×10^7 m closer to the Earth than present day.

The time taken for the Moon to regress from the past orbital radius to todays orbital radius was then calculated using a regression rate [3] of 3.78 cm yr^{-1}. We therefore found that the Moon had taken 767 million years to regress to its current position.

Assumptions

For this paper, we have assumed that the Earth possesses a spherical shape and have taken the true oblate spheroid shape as negligible. We have also assumed tidal forces to be negligible and have discounted all forces and effects felt outside of the Sun-Earth-Moon system.

Conclusion

In order to increase the year to an 8-day week in current times, you would need to increase the Earth's orbital radius around the Sun to 1.63×10^{11} m. However, 767 million years ago, the Earth's 8-day week was caused by a closer Moon orbital radius of 3.56×10^8 m.

References

[1] A. Wilkins, *Shifting Ocean Currents can (and do) actually speed up Earth's rotation*, 2012

[2] F. Wild, *What is an Orbit?*, NASA, 2017

[3] *Why the Moon is getting further away from Earth*, BBC, 2011